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ROBOTIC NAVIGATION USING HARMONIC FUNCTION-BASED PROBABILISTIC ROADMAPS

A THESIS

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IN PARTIAL FULFILLMENT OF THE REQUIREMENTS

FOR THE DEGREE OF

MASTER OF APPLIED SCIENCE

IN

INDUSTRIAL SYSTEMS ENGINEERING

UNIVERSITY OF REGINA

By

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FACULTY OF GRADUATE STUDIES AND RESEARCH

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Moslem Kazemi, candidate for the degree of Master of Applied Science, has presented a thesis titled, *Robotic Navigation Using Harmonic Function-Based Probabilistic Roadmaps*, in an oral examination held on December 15, 2004. The following committee members have found the thesis acceptable in form and content, and that the candidate demonstrated satisfactory knowledge of the subject material.

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Abstract

A new hybrid motion planning approach based on Harmonic Functions (HF) and Probabilistic Roadmaps (PRM) is presented. The proposed approach consists of building a Probabilistic Roadmap using information obtained about the workspace topology through a Fluid Dynamic (FD) paradigm based on HFs. The crux of our approach is to identify narrow passages using a FD paradigm and pass them over to a PRM method to build a roadmap that efficiently captures the connectivity of the free configuration space (C-space) especially in narrow regions. We present two techniques to implement the proposed approach: (1) the original Harmonic Function-based Probabilistic Roadmap (HFPRM) technique where HFs are computed over the explicit representation of the robot's C-space, and (2) the Incremental HFPRM (IHFPRM) technique which is an extension of the original HFPRM, and can be applied to virtually any type of robot. Simulation results obtained using these techniques show that the combination of the HF and the PRM works better than each individual in terms of finding a collision free path in environments where narrow passages

exist. Inspired by the promising results obtained through the original HFPRM and incremental update of it, we also present a sensor-based motion planning technique for robotic navigation in environments not known a priori. Experimental results obtained using this technique for a mobile robot equipped with ultrasonic range finders moving in an unknown environment are also presented.

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Nomenclature

$\mathcal{A}$:	the robot.
$\mathcal{A}(q)$:	the physical space occupied by the robot at configuration q .
$\mathcal{V}_s$:	the field of view of the robot's sensor.
$\mathcal{W}$:	the physical space.
$\Delta\mathcal{W}$:	the additional information obtained about the physical space due to a scan.
$\mathcal{C}$:	the robot's C-space.
$\mathcal{C}_{free}$:	the entire free C-space.
$\mathcal{C}_{unkbnd}$:	the boundary of the unknown C-space.
$\Delta\mathcal{C}_{free}$:	the additional free C-space obtained due to a scan.
$\mathcal{R}$:	the roadmap.
$\mathcal{V}$:	the velocity profile of the potential flow.
q_s :	the robot's start configuration.
q_g :	the robot's goal configuration.
q_{cur} :	the robot's current configuration.

q_v : the view configuration.

Acronyms

API: Application Program Interface

C-space: Configuration Space

DOF: Degree Of Freedom

FD: Fluid Dynamic

FDM: Finite Difference Method

FEA: Finite Element Analyzer

FEM: Finite Element Method

GUI: Graphical User Interface

HF: Harmonic Function

HFPRM: Harmonic Function-based Probabilistic Roadmap

IHFPRM: Incremental Harmonic Function-based Probabilistic Roadmap

MP: Motion Planning

MBMP: Model Based Motion Planning

OHFPRM: Original Harmonic Function-based Probabilistic Roadmap

PF: Potential Field

PID: Proportional Integral Derivative

PRM: Probabilistic Roadmap

SBMP: Sensor Based Motion Planning

Chapter 1

Introduction and Literature Survey

Given an environment cluttered with obstacles the basic robot Motion Planning (MP) problem is established as: moving a robot from a start configuration to a goal configuration without colliding with obstacles. The robot motion planning can be divided into two main categories: (1) Model-Based MP (MBMP) where the environment is completely known and the robot's task is to find a collision free path from the start configuration to the goal configuration, and (2) Sensor-Based MP (SBMP) where the robot, equipped with on-board sensors, is to explore the environment not known a priori.

We present a hybrid motion planning technique based on Harmonic Functions (HF), which are the solutions to the Laplace's equation, and Probabilistic Roadmaps (PRM). It will be shown through computer simulations and experimental results that this technique can solve MBMP and SBMP problems efficiently. The main idea of the

proposed approach is to identify critical regions, i.e. narrow passages and/or hard-to-navigate regions, using information obtained about the environment topology through a Fluid Dynamic (FD) paradigm. Furthermore, these information will be passed to a Probabilistic Roadmap (PRM)-based method to build a roadmap¹ which efficiently captures the connectivity of the free C-space². In the following section we give an overview on our proposed approach.

1.1 Overview

A new hybrid robot motion planning technique, combining the potential field approach based on Harmonic Functions (known as harmonic potential fields) and the Probabilistic Roadmaps (PRM), which computes collision-free paths for robots moving in static workspaces, is presented. Harmonic potential field is a sub-class of the Potential Field (PF) approach, where the potential satisfies Laplace’s equation [1]. The PRM is based on a random sampling scheme to construct an implicit representation of the robot’s C-space as a roadmap [2]. The efficiency of the PRM in solving the classical robot motion planning problem is reduced in environments where narrow passages exist. Simulation results show that the combination of the potential field approach based on the HF and the PRM can overcome each individual technique’s

¹Roadmap is usually referred to as one connected component but in our studies it can be any type of graph.

²Here C-space refers to the configuration space of the robot where the robot can be represented by a point (or automaton).

shortcoming in finding a collision-free path between start and goal configurations in environments where narrow passages exist. This proposed MP approach can be utilized in autonomous robot MP with applications in robotic search-and-rescue, and unmanned space exploration and construction. Two techniques are proposed: (1) the Harmonic Function-based Probabilistic Roadmap (HFPRM) that is applicable to MP of robots with low Degrees Of Freedom (DOF's), and (2) the Incremental HFPRM (IHfPRM) technique which is applicable to MP of robots with high DOF's.

The HFPRM technique comprises three phases: in phase one, the Laplace's equation, pertinent to potential flow, in an environment cluttered with obstacles is solved. In phase two, a PRM-based method with a novel sampling scheme is constructed based on the information obtained about the environment topology through the HF technique developed in phase one. The roadmap is then searched for the shortest path connecting the start and goal configurations in phase three. Promising results are obtained using this technique for solving shortest-path problem in difficult scenarios. In this technique an explicit representation of the robot's C-space is needed to accurately compute the HFs. A Finite Difference Method (FDM) is employed to solve the Laplace's equation over a uniform mesh of grid points imposed on the robot's C-space.

Since explicit representation of the C-space for robots with many DOF's is not practical, an incremental version of the HFPRM technique has been also developed

which is more general and can be applied to virtually any type of robot. The IHF-PRM technique works as follows: first a roadmap is initialized in robot's C-space by generating a pre-defined number of random nodes (the start and the goal configurations included). The neighboring nodes are then connected via straight lines. The metric used is the Euclidean distance. Then in an iterative scheme the roadmap is enhanced and searched for the shortest path. In each iteration the Laplace's equation, pertinent to potential flow, is solved over the current representation of the C-space using Finite Element Method (FEM) and the velocity of the flow is calculated at each node of the roadmap. The roadmap is then enhanced using a new, yet simple, random sampling scheme based on information obtained about the C-space topology with a *velocity thresholding* technique. This yields a roadmap with better coverage in regions associated with higher velocities (i.e. narrow regions). At the final stage the roadmap is searched by a local planner for the shortest path connecting the start and the goal configurations. The enhancement ends either after a maximum number of iterations has been reached or a collision-free path has been found.

The proposed technique was implemented on a mobile robot equipped with ultrasonic range finders moving in an unknown environment.

1.2 Related Works

In this section we report on literature cited on PF approaches, PRM methods, and some hybrid techniques combining these two to address the motion planning problem. I will also briefly mention the literatures which address the SBMP problem for robots in unknown environments.

1.2.1 Potential Field Approaches

The basic idea of potential field approaches is to construct a potential field with an attractive global minimum at the goal configuration and a repulsive local maximum at the obstacles. Literature pertinent to potential field approaches is vast (e.g., [3, 4, 1]). Potential fields have been introduced as a practical and deterministic approach in robot path planning [3]. The harmonic potential field is an important class of the potential field approach based on HF's, which provides local minima free solutions. Connolly et. al. proposed a robot motion planning method based on harmonic functions. Local minima free paths were obtained in static environments [1]. A fluid dynamic paradigm utilizing an incompressible non-viscous flow was employed in [5] and [6] to generate permissible paths (i.e., streamlines) from the source (i.e., the start configuration) to the sink (i.e., the goal configuration). To include the external forces which influence the robot performance in uneven terrains and also to consider clearance between the robot and obstacles, Louste and Liegeois ([7]) presented a path

planning method using the incompressible but viscous flow equations. Latombe and his group proposed a PF approach that consists of incrementally building a graph connecting the local minima of a PF defined in the robot's C-space and concurrently searching this graph until a goal configuration is attained [4].

The ability to incrementally update the harmonic functions allows one to use them in dynamic as well as stationary environments. Masoud et. al. generalized the harmonic potential approach for tracking a maneuvering target that is moving amidst known stationary obstacles [8]. A recursive motion planning scheme using harmonic functions without complete knowledge of the environment a priori has been proposed in [9]. However, there are some drawbacks in navigating robots based on HFs which limit their usage. For instance, chattering in motion when following a streamline obtained through the harmonic functions may cause deviation from one streamline to another [10]. Besides, finding the shortest streamline needs a global search technique and can be computationally cumbersome. Also Potential Field methods usually work on a regular grid that needs an explicit representation of C-space to accurately compute the potential function ([11]) which is not practical for robots with many DOF's.

Finite Difference and Finite Element Methods are two widely-used techniques to calculate harmonic functions (i.e., solutions to the Laplace's equation) numerically [12]. Some other non conventional approaches such as *Panel Method* has been also

used in the realm of *Computational Aerodynamics* to find the velocity profile of the fluid flow around aerodynamic shapes such as airfoils [13]. Kim and Khosla utilized this method for robot motion planning [14]. Valavanis and his group developed an analytical technique based on the same method for motion planning of robots [15, 16].

Different physical analogies have been employed to adapt the harmonic potential fields for robot path planning. Wang and Chirikjian [17] propose a potential field approach based on a steady-state heat transfer analogy. In their methodology the optimal path is presented as a heat flow with minimal thermal resistance. An electrostatics terminology has been proposed in [18] and [19]. An analogue computation of potential field has been presented in [20]. They proposed a potential field method based on a 2D scalar electric potential which can be implemented with parallel analogue hardware in the form of a resistive grid. In our proposed method we adopt a fluid dynamic paradigm based on an incompressible non-viscous fluid flow to bias the distribution of random nodes in a PRM towards narrow regions.

1.2.2 Probabilistic Roadmap Methods

The PRM method comprises two phases; In the first phase, i.e. *pre-processing* phase, a predefined number of random nodes are generated, and a roadmap is constructed by connecting the neighboring nodes based on an Euclidean metric. In the second phase, i.e. *query* phase, the start and goal configurations are attempted to be

connected to the roadmap obtained in the pre-processing phase, and then the roadmap is searched for the shortest path connecting the start and goal configurations.

This technique has been the method of choice for MP of robots with many degrees of freedom. However, constructing a roadmap that can efficiently capture the connectivity of the free C-space is a challenging issue [21, 22]. The PRM technique can simply fail in environments where narrow passages exist [21]. Many variants of the PRM have been proposed to identify these regions (e.g. [22, 23, 24]). Amato et. al. evaluate various node generation and connection strategies in an Obstacle-Based Probabilistic Roadmap (OBPRM) [22]. Bayazit et. al. developed a technique to enable a human and an automatic planner to cooperatively solve motion planning problems to discover critical configurations using haptic hints [23]. The narrow passage problem was also addressed in [24]. They developed a medial axis approach based on a sampling heuristic as a guideline for choosing configurations in critical regions. Hsu et. al. utilized a free space dilation concept to widen narrow passages to guarantee free space connectivity in those regions [21]. A bridge test strategy has been proposed in [25] which boosts the sampling density inside narrow passages based on simple tests for local geometry. The bridge test strategy reduces the sampling density in wide-open collision-free regions. Berg and Overmars proposed a method which uses the shape of free workspace to guide the sampling in C-space [26]. This technique can only be used when the C-space resembles the workspace. A variant of PRM method based on

a visibility notation has been introduced in [27] and [28] to generate roadmaps with small number of nodes. In their proposed method a random configuration is added to the roadmap if either it connects two connected components of the roadmap or is not "visible" by some so-called guard configurations. To answer different queries, Vallejo et. al. proposed an adaptive approach to select a planner whose strength match to the current query [29]. This arises to some evaluation techniques to characterize the current situation and to choose the best match in a bank of planners. However, this technique is carried out only in the pre-processing phase. It may fail finding narrow passages if not extended to the query phase as well. Some other random sampling approaches have been proposed based on the construction of search trees rather than graph-based roadmaps. Rapidly-exploring Random Trees (RRT) and Ariadne's Clew Algorithm (ACA) are two independent examples under this category [30, 31]. Reliable identification of narrow passages using random planners is an open research area.

1.2.3 Hybrid Techniques

Due to shortcomings of the PF approach and the PRM method in identifying narrow regions some hybrid techniques have been proposed utilizing the advantages of each individual. A hybrid technique based on Harmonic Potential Functions and random sampling of the non regular grid decomposition of the robot's C-space was

presented in [32]. Harmonic functions are utilized to bias the random sampling of the cells, and to explore the C-space. They utilized a hierarchical technique ([33]) to compute the harmonic functions on a non-regular grid decomposition of C-space. The gradient of the potential field is followed to find a sequence of free cells from the start configuration to the goal configuration. Efficiency of these technique in finding collision-free path in presence of narrow passages has not been addressed. An artificial potential based probabilistic roadmap method has been proposed in [34]. In their technique a partial computation of potential field in workspace is utilized to bias the sampling distribution in a PRM approach in C-space. This yields denser sampling of the C-space close to obstacle boundaries, hence a denser coverage in narrow regions. Nonetheless, a high number of samples can be generated in irrelevant narrow regions (i.e., regions with dead-ends) using this method. Furthermore this technique will not generate a uniform coverage on the wide-open regions.

1.2.4 Sensor-Based Motion Planning Techniques

Sensor-based MP for robots has been extensively studied for the past two decades. We mention some of them which are related to our work. *View Planning* problem, i.e. finding the best next configuration where the next scan should be carried out, is one of the main issues in SBMP of robots equipped with on-board sensors.

Renton et. al. presented a sensor-based robotic system, called Plan-N-Scan, using

a wrist mounted laser range finder [35]. In their proposed technique an A^* search is used to select the best view configuration to scan a region (view planning) among all the possible free configurations. The A^* search can be costly in both time and memory as the robot's DOF increases. In their system, to explore the unknown regions of the environment, higher priority is given to the region which is closest to the origin of the workspace, i.e. the initial robot's configuration.

Gupta and his group developed a Sensor-Based Incremental Construction of Probabilistic Roadmaps (SBIC-PRM) approach [36]. In their technique a probabilistic roadmap is incrementally constructed as the robot senses new free spaces in the physical space. The roadmap represents the free C-space connectivity within which the robot carries out its motion to further sense the unknown parts of the environment. They successfully implemented their technique on an Eye-in-Hand system consisting of a 6 DOF's PUMA 560 manipulator equipped with a scan laser range finder mounted on robot's end-effector. To address the view planning problem they proposed an information theoretic approach which maximizes the information gained due to a scan for optimal exploration of the unknown environment [37]. They also integrate the information gain criterion with a *greedy* component to bias the search toward a goal configuration.

Cheung and Lumelsky proposed a SBMP technique using a whole-sensitive arm

manipulator moving in a three dimensional workspace [38]. In their proposed technique the information obtained in previous scans has little impact on what should be scanned next (i.e., their planner is memoryless). Kutulakos et. al. also proposed a similar algorithm which decomposes the path planning problem in 3 dimensions into two independent subproblems: a planar path planning problem and a three dimensional exploration problem [39]. This technique is applicable for robots with low DOF's (i.e., $n \leq 3$) only.

Mehrandezh and Gupta utilized a variant of the probabilistic roadmap, i.e. Lazy PRM, for the SBMP and exploration of an articulated robot equipped with a large number of proximity sensors mounted on its body [40]. They proposed a novel metric based on the notation of the C-zone map for robot's movement which results in efficient exploration of the environment. This mapping represents the size of the set of configurations associated with each point in the physical space and is calculated offline prior to planning. A path going through those regions of physical space that correspond to higher C-zone volume are given higher priority for the next scan.

In our proposed SBMP technique information obtained about the C-space topology through the FD paradigm is utilized to find the best next scanning strategy. A PRM-based method is employed to efficiently capture the connectivity of the robot's free C-space using information obtained about the topology of the robot's C-space through the FD paradigm.

1.3 Contributions

This thesis presents a new hybrid robot MP technique that can be utilized for model-based and sensor-based scenarios for robots with many DOF's. Computer simulation and real implementation of the proposed technique on a robot with a mobile base have been successfully carried out.

1.3.1 Theoretical Framework

- I present a new hybrid MP technique based on harmonic functions and probabilistic roadmaps. My view is that the combination of them works better than each individual in terms of finding a collision free path in environments where narrow passages exist.
- A fluid dynamic paradigm based on harmonic functions is employed to obtain information about the environment topology.
- I present a new, yet simple, random sampling scheme based on information obtained through harmonic functions to bias the distribution of random nodes in a probabilistic roadmap which results in efficiently capturing the connectivity of the robot's free C-space.
- Finite Difference Method and Finite Element Method (FEM) are utilized to

compute the harmonic functions in the HFPRM and incremental HFPRM technique, respectively.

1.3.2 Planning Algorithm

The crux of our approach is to utilize the information obtained about the environment topology through a FD paradigm based on a harmonic potential field to bias the distribution of nodes in a probabilistic roadmap method towards the critical regions which results in a roadmap with better coverage of the free C-space. We show, through computer simulations and experimental works, that the proposed hybrid technique would perform favorably over the original PRM in finding shortest paths especially in environments where narrow passages exist.

The proposed motion planning algorithm has been successfully implemented on a mobile robot equipped with ultrasonic range finders moving in an unknown environment.

1.3.3 Simulation Software

Different Graphical User Interfaces (GUI) have been developed in MATLAB (v6.5) to implement the original and the incremental version of the proposed HFPRM technique. These will be discussed in more detail in Chapter 3.

1.3.4 The Mobile Robot System

The proposed MP technique was implemented on a wireless mobile robot, called DRK600, manufactured by Dr.Robot Inc. [41]. The system runs on a wheel-based platform with rotary sensors mounted on each wheel to precisely measure and control wheel movement. The mobile platform has a total of 3 DOF, two translational and one rotational. The system uploads all the sensor information to a PC with a 2.4 GHz Intel Celeron processor through a Bluetooth wireless module. All the low-level control commands are accessible in MATLAB environment through the Application User Interface (API) offered in an ActiveX module. The robot is equipped with 4 ultrasonic range finders which retrieve range data from the environment. For more detail one can see Appendix A.

1.4 Thesis Outline

In Chapter 2 the problem formulation is discussed in detail. The proposed HF-PRM technique and the incremental version of it, i.e. IHFPRM, for MBMP with corresponding simulation results are presented in Chapter 3. In Chapter 4 the proposed SBMP framework will be explained. The implementation of the proposed SBMP technique along with the experimental results will be presented in Chapter 5. Chapter 6 represents the conclusions and future works.

Chapter 2

Problem Formulation

In this chapter we present a model-based motion planning approach applicable to path planning of robots with both low and high DOF's in known environments. Extension of this approach to the SBMP framework of robots in environments not known a priori will be also discussed.

2.1 Model-Based Robot Motion Planning Problem

Pertaining to the basic robot motion planning problem (described in Chapter 1) demanding extra features on the sought path such as its length (i.e., known as the shortest-path problem) will increase the complexity of the problem especially in cases where the shortest path has to pass through narrow passages. The natural space where the motion planning can be carried out would be the robot's C-space where

the robot can be represented as an automaton¹. A hybrid motion planning approach based on HF and PRM is proposed for robotic navigation in both low and high dimensional spaces. In this approach a fluid dynamic paradigm based on harmonic functions is employed to obtain information about the robot's C-space topology. These information are then exploited to bias the distribution of random nodes in a PRM that captures the connectivity of the free C-space. The following sub-problems should be investigated in terms of MBMP of robots.

2.1.1 Potential Field Approach Using FD Paradigm

Fluid dynamic paradigm is utilized to generate permissible paths (i.e. streamlines) in the robot's C-space connecting the start and the goal configurations [5, 6]. These configurations are considered as the virtual flow source and sink. Obstacles are also considered as impenetrable objects. In particular, the notion of potential flow, an inviscid, incompressible, and irrotational fluid flow, is employed for computing the streamlines. I will describe how this analogy would help us to find the narrow regions (i.e., regions through which are hard to navigate the robot) in an environment cluttered with obstacles, later on.

¹A point robot.

A potential flow can be mathematically represented by:

$$\begin{cases} \nabla \cdot \vec{v} = 0 & \text{zero compressibility,} \\ \nabla \times \vec{v} = 0 & \text{zero vorticity,} \end{cases}$$

where $\vec{v}$ is the velocity vector of the flow [13]. Zero vorticity implies that the velocity of the potential flow can be stated as the gradient of a scalar potential ϕ :

$$\vec{v} = -\nabla\phi, \quad (2.1)$$

Zero incompressibility of the potential flow combined with Equation 2.1 yields the Laplace's equation:

$$\nabla^2\phi = 0, \quad (2.2)$$

In an n -dimensional space the Laplace's equation can be represented by:

$$\sum_{i=1}^n \frac{\partial^2 \phi}{\partial x_i^2} = 0, \quad (2.3)$$

Solution of Equation 2.3 is in the form of harmonic Functions.

Harmonic Functions (HF) can be used to generate collision-free streamlines without local minima. However, saddle points (aka stagnation points) can occur. Due to Bernoulli's principle for inviscid incompressible fluid flow, higher velocities will appear in narrow regions given that the environment is bounded. Bernoulli's principle is mathematically represented as follow:

$$P + \frac{1}{2}\rho v^2 + \rho gh = \text{constant}, \quad (2.4)$$

Figure 2.1: (a): Potential contours and (b): Velocity vectors pertaining to a potential flow in a sample simulation.

where P denotes the static pressure (pressure induced due to random motions of molecules within the flow), ρ denotes the density, which is constant due to incompressibility of potential flow, v denotes the velocity of the flow and h denotes the height of fluid particles with respect to a fixed (i.e., zero-potential) surface. Assuming constant potential energy, i.e. ρgh is constant, the Bernoulli's equation then reduces to a simple relation between the velocity and the static pressure of the flow which states that: as the speed of a moving fluid increases the static pressure within the fluid decreases and vice versa. Hence due to lower static pressure of flow in narrow regions, higher velocities are observed. One should note that no vortex will be generated due to the assumption that the flow is inviscid.

Figure 2.1(a) and Figure 2.1(b) show the potential contours (i.e., iso-potential lines) and velocity vectors pertaining to a potential flow in a sample simulation,

respectively. The magnitudes of vectors shown in Figure 2.1(b) are proportional to the velocity of the fluid particles. Higher velocity is observed in the narrow passage between the c-shape obstacle and the environment boundary.

Harmonic functions constitute a powerful mean to generate local minima free smooth paths. However, possible deviation from one streamline to another when navigating a robot may cause chattering in motion [10]. Besides, searching through the calculated streamlines for finding the shortest path connecting the start and goal configurations will be computationally cumbersome. These necessitate to use a rudimentary motion planning technique to overcome these shortcomings. A variation of the PRM method has been chosen for this purpose. Sub-section 2.1.2 describes this method in details.

2.1.2 PRM Construction Biased Toward Narrow Regions

It will be shown that a rudimentary motion planning technique based on PRM can generate a better coverage of the C-space topology with the help of the information available to it through HFs. This will empower the PRM-based approach in finding the shortest path through narrow passages. It will be also shown that our proposed hybrid approach would perform favorably over the original PRM in finding shortest paths through narrow regions.

Figure 2.2(a) and Figure 2.2(b) show the roadmaps obtained for a simple, yet

Figure 2.2: (a) and (b): Generated roadmaps using original PRM method and HFPRM technique, respectively.

general scenario, using the original PRM method and our proposed hybrid technique, respectively. As can be seen the roadmap shown in Figure 2.2(b) captures the topology of the confined space between two rectangular-shape black obstacles more efficiently.

In our proposed approach the random sampling scheme utilized for constructing the roadmap is modified based on information obtained on the flow velocity through HF's to bias the distribution of random nodes towards critical regions (i.e., regions inducing high velocity). This guarantees a better coverage of the C-space ($\mathcal{C}_{free}$) than that using the original PRM.

Two different schemes for constructing the PRM are described below:

1. One shot PRM construction: this technique is suitable for MBMP of robots with low DOF's where the explicit representation of the robot's C-space is available

or can be easily computed. In this method the PRM is constructed in one shot based on the information obtained using HF's. This scheme is utilized in the HFPRM technique and will be explained in more details in section 3.1.2 of Chapter 3.

2. Incremental construction of the PRM: this method is most suitable for MP of robots (model-based and/or sensor-based) with high DOF's where the explicit computation of the robot's C-space is not practical. Using this scheme the PRM is incrementally constructed and interactively updated based on the information obtained through HF's at each iteration. We utilize this scheme in our IHFPRM technique to capture the connectivity of the free C-space. The proposed incremental construction of the PRM is explained in more details in section 3.2.3 of Chapter 3.

Extension of this hybrid MP technique to SBMP will be described in the next section.

2.2 Sensor-Based Robot Motion Planning Problem

The main idea of the proposed sensor-based planning approach is to utilize a fluid dynamic paradigm based on potential flows (as explained in section 2.1.1) to identify and prioritize critical regions, i.e. narrow passages and hard-to-navigate regions. The proposed PRM that captures the connectivity of the free space is incrementally expanded as the robot senses the physical environment.

SBMP problem can be decomposed into two main sub-problems: (1) *View Planning* that determines which region should be scanned next and vantage point search that determines from where the scan should be carried out, and (2) *C-space Update/Expansion* that updates/expands the $\mathcal{C}_{free}$ as the robot senses the physical environment. In the following sub-sections we explain these sub-problems in more details.

2.2.1 View Planning and Vantage Point Search

The view planning and vantage point search problem comprises two steps: (1) identifying the best region that should be scanned next, and (2) finding the best permissible robot's configuration from which the region found in step (1) can be scanned. Pertaining to the first step different criteria have been utilized (e.g., [35, 37, 40]). In our proposed planning framework we address this problem from a FD analogy perspective. As it was explained in section 2.1.1, the velocity profile obtained through the FD paradigm based on HF's can be utilized to obtain information about the topology of the environment. These information can be further exploited to prioritize regions of the environment to be scanned next. The main idea of our SBMP approach is to scan the regions associated with higher flow velocity first. Having these regions selected, the best view configuration (i.e., vantage point) among all permissible and reachable free configurations of the robot is then selected in the second step. This procedure is repeated till a collision-free path connecting the start

and goal configuration is found. The view planning and vantage point search will be described in more details in section 4.3 of Chapter 4.

2.2.2 C-space Update/Expansion

The C-space update/expansion problem refers to updating and expanding the free C-space ($\mathcal{C}_{free}$) as the robot senses the physical environment. A variation of the PRM approach that outperforms the original PRM is utilized for better coverage of the connected components of the robot's C-space at each iteration. To avoid the degeneration, i.e. adding new random nodes in the same region over and over when constructing a roadmap, a novel technique based on Fast Fourier Transform (FFT) and convolution theorem has been utilized which will be described in more details in section 4.2.1.

Chapter 3

Model-Based Motion Planning: Solution Methodology

In this chapter the proposed solution methodology to implement the HFPRM and the incremental version of the HFPRM technique for model-based robot motion planning will be explained. First the HFPRM solution methodology proposed for motion planning of robots with low DOF's will be described. The Incremental version of HFPRM solution methodology as an extension of the HFPRM technique is then explained for motion planning of robots with high DOF's.

3.1 The HFPRM Technique: Solution Methodology

The overall solution methodology pertaining to the HFPRM technique is explained in Figure 3.1.

Figure 3.1: The HFPRM solution methodology flowchart.

The proposed HFPRM technique comprises three phases: in phase one the Laplace's equation, pertinent to potential flow, is solved in the computed robot's C-space. In phase two a probabilistic roadmap with a novel sampling scheme is constructed based on information obtained about the environment topology through the HF technique developed in phase one. The roadmap is then searched for the shortest path in phase three. More details are given in the following sub-sections.

3.1.1 Using FD Paradigm to Identify Narrow Regions

As it was explained earlier in section 2.1.1, a fluid dynamic analogy based on potential flows is used to solve the motion planning problem. Considering the properties of the potential flow, the potential field must satisfy the Laplace's equation as in (2.2). Gradient of this potential field constitutes the velocity field of the flow. Solution to the Laplace's equation is categorized as harmonic functions.

Numerical solution of the Laplace's equation can be obtained using Finite Difference Method (FDM). FDM imposes a mesh of uniformly distributed grid points in a region. The Laplace's equation at each grid point is then approximated using finite difference equations. Using the *Taylor series* expansion the second order central difference approximation of the velocity potential ϕ , in a $2D$ Cartesian space, can be represented as follows:

$$\frac{\partial^2 \phi(x, y)}{\partial x^2} = \frac{\phi(x + h, y) - 2\phi(x, y) + \phi(x - h, y)}{h^2} + O(h^2), \quad (3.1)$$

$$\frac{\partial^2 \phi(x, y)}{\partial y^2} = \frac{\phi(x, y + h) - 2\phi(x, y) + \phi(x, y - h)}{h^2} + O(h^2), \quad (3.2)$$

where h is the grid step size¹ and $O(h^2)$ is the truncation error which is of order h^2 . Assuming unit step size h as $h = 1$, and neglecting the truncation errors the discrete form of Laplace's equation (known as Laplacian difference equation) in a two dimensional Cartesian space can be rewritten as follows:

$$\phi(x + 1, y) + \phi(x - 1, y) + \phi(x, y + 1) + \phi(x, y - 1) - 4\phi(x, y) \simeq 0 \quad (3.3)$$

¹For simplicity here it is assumed that the grid size in x and y directions are the same.

This equation can be easily extended to higher dimensions as well.

Expressing the actual physical phenomenon, the Laplace's equation has to be complemented with boundary conditions. These are normally transposed into the conditions on the potential field ϕ . The boundary conditions (BC) have to be satisfied on the obstacle's boundaries, on the boundary of the environment, and at the source and the sink of the flow.

There are two main boundary conditions, namely *Dirichlet* and *Neumann*. The Dirichlet boundary condition can be mathematically stated as $(\phi|_{\partial\Omega} = c)$, where $\partial\Omega$ is the boundary of the domain (or region) of interest, Ω , and c is a constant. In this case the boundary is held at a constant potential and the flow must be along the outward normal of the obstacle surfaces [10]. The Neumann boundary condition can be mathematically described as $(\frac{\partial\phi}{\partial\vec{n}}|_{\partial\Omega} = c)$, where $\vec{n}$ is the vector normal to the boundary of the domain defined as $\partial\Omega$. The Neumann boundary condition constrains the gradient of ϕ to accept a constant value on the boundary. In our case the flow has to move tangential to the obstacle's boundaries, thus c must be set to zero. In some cases the Mixed or Robin boundary condition is employed as a linear combination of Dirichlet and Neumann boundary conditions. This can be mathematically represented as $(\alpha\phi|_{\partial\Omega} + \beta\frac{\partial\phi}{\partial\vec{n}}|_{\partial\Omega} = c)$, where α, β and c are constants [10].

In the proposed fluid dynamic paradigm the source and the sink are fixed at high (i.e. $\phi = 1$) and low (i.e. $\phi = 0$) constant potential values, i.e. Dirichlet BC. The

boundary of the environment and the obstacles inside the environment are considered impenetrable , i.e. Neumann BC, $\frac{\partial \phi}{\partial n}|_{\partial \Omega} = 0$. For all points in the environment other than those on the environment and obstacles boundaries, source and the sink a randomly selected potential value ϕ where $\phi \in [0, 1]$ is assumed as the initial guess. These values are interactively updated till a solution is converged.

The potential field equations (3.3), the boundary conditions and the initial values of the potential field give a complete description of the potential flow which can be mathematically represented in the form of a system of linear equations. *Iterative* schemes are normally utilized to solve these systems. The iterative or indirect methods employ an initial guess and generate a sequence of approximations to the solution of the system. The iterations are then repeated until the accumulated change in the potential values assigned to grid points is within a threshold from one iteration to the next (a variety of other stopping criteria can be utilized as well). *Jacobi*, *Gauss-Seidel*, *Successive Overrelaxation point iterations*, and *Tri-Diagonal Matrix Algorithm* (TDMA) are most commonly used methods for solving systems of linear equations interactively. The TDMA with a better convergence rate than others was employed in the computer simulations.

The TDMA is a technique for rapidly solving tridiagonal systems of linear equations [42]. TDMA utilizes *forward elimination* and *backward substitution* schemes to

Figure 3.2: Velocity profile in narrow region with corresponding velocity colormap

calculate the exact solutions of the system with a finite number of algebraic operations. TDMA can be extended to higher dimensions by applying it interactively in a line-by-line fashion in each dimension [42]. More details on TDMA can be found in the Appendix B.

Once the potential values have been computed the velocity of flow can be calculated using the gradient of ϕ at each grid point. The velocity profile will be helpful to extract some information regarding the environment topology, i.e. identifying narrow regions. As it was explained in section 2.1.1 higher velocities are observed in narrow passages due to Bernoulli law. The concept developed in our proposed method can be better understood through the computer simulation shown in Figure 3.2. Arrows in Figure 3.2 represent the velocity of the flow. The magnitude of the arrows is proportional to the value of velocity at each point. Higher velocities can be noticed in the narrow region between two rectangular shape obstacles.

It should be noted that the narrow passages in the environment can be divided

Figure 3.3: Narrow regions categories; Type I: high velocity narrow regions, Type II: narrow regions with dead-ends, Type III: low velocity narrow regions

into different categories. Three different narrow passages may exist in a typical environment (see Figure 3.3).

In type I region high velocity exists meaning that the possibility of the shortest path going through this region is very high. In the type II region the velocity would be at the minimal level leading to the fact that these narrow passages are dead-ends. One should note that due to inviscid property of the potential flow no vortex will be generated in these dead end regions therefore low velocities will be observed. In type III regions velocity values are low meaning that flow has a low tendency to pass through them. This indicates that the possibility of the shortest path going through these regions is low. These information can be used in the next phase of the proposed motion planning technique, where utilizing a PRM-based approach to achieve a more uniform representation of the free C-space (i.e. $\mathcal{C}_{free}$).

3.1.2 Using PRM to Capture the Connectivity of the Free C-space

In our proposed PRM-based approach the pre-processing phase of the original PRM is modified considering the velocity regions obtained in phase I by a velocity thresholding technique. In this technique, given the number of velocity levels, each grid point according to its velocity value falls into one of these levels. A point falls into level (i) if and only if its velocity is in the following interval: $(\frac{i-1}{L}V_{max} \dots \frac{i}{L}V_{max}]$, where L is the number of velocity levels and V_{max} is the maximum velocity in the velocity profile obtained using HF. Using this scheme the velocity regions are organized in a predefined number of levels.

In the proposed PRM-based approach more random nodes are generated in high velocity regions in order to achieve a better coverage of the free C-space especially in narrow passages. The number of random nodes selected in each velocity region is determined by a function whose value changes with velocity level and the region area as follows:

$$r(i) = \frac{s(i)}{S} N, \quad (3.4)$$

where

$$s(i) = (1 - \omega)n_i + \omega i, \quad (3.5)$$

$$S = \sum_{i=1}^L s(i), \quad (3.6)$$

and n_i denotes the number of free nodes in level i , ω is a normalized factor, i.e. $\omega \in [0, 1]$, and N is the total number of desired random nodes to be generated. A tradeoff between area and the velocity in each level is established by changing the weight factor, ω . The number of random nodes to be selected for each region with a certain velocity level is computed using (3.4). A metric, i.e. Euclidean distance between nodes, is then used for connecting neighboring nodes in the roadmap in the pre-processing phase. Start and goal configurations are then added to the roadmap in the query phase. Dijkstra algorithm has been utilized to find the shortest path connecting the start and the goal configurations.

One should note that in the HFPRM technique the PRM is constructed in one shot given that the robot's free C-space can be explicitly computed. A FFT-based technique for fast computation of the robot's C-space has been utilized for this purpose. More details on this method can be found in the next section.

3.1.3 Simulation Results And Discussion

A Fast Fourier Transform (FFT) based method was employed to compute the robot's C-space. In this method the C-space is represented as the convolution of the robot and the workspace bitmaps [43]. This convolution can be computed efficiently using FFT. The running time of this method depends only on the resolution of discretization used to create the robot and the workspace bitmaps. For further details

Figure 3.4: The main window of GUI developed implementing HFPRM.

one can refer to [43].

A Graphical User Interface (GUI) in MATLAB has been designed to implement the proposed method. The main window of this GUI with some pull-down menus is shown in Figure 3.4.

The following actions can be carried out using this GUI:

- Define geometry of the robot, the environment and obstacles with arbitrary shapes.
- Compute and display the robot's C-space.
- Set the boundary conditions and initial values of field potentials.
- Select the FDM technique to solve the Laplace's equation: Choices are Jacobi,

Gauss-Seidel, Successive Overrelaxation point iterations, and TDMA.

- Plot the potential contours and streamlines in robot's C-space.
- Do the velocity thresholding.
- Select a planner: Choices are the original PRM, the enhanced PRM or the proposed HFPRM.
- Show the roadmap and the shortest path found if it exists.
- Show the motion of the robot in the physical space (if a path was found) through snap shots.

The proposed HFPRM motion planning technique has been simulated for mobile robots with up to 3 degrees of freedom for sake of illustration. 2D and 3D examples are presented in this section.

In the first example the motion planning problem for a 2-DOF mobile robot with only translational mobility has been carried out in an environment where some narrow passages exist. The shortest path must pass through one of them. Figure 3.5(a)-3.5(f) show the simulation results obtained using three different motion planning technique, namely the original PRM, the enhanced PRM and the proposed HFPRM. The robot is shown as a polygon in green color. The roadmaps constructed in C-space based on three motion planning techniques mentioned above are shown in Figure 3.5(a), Figure 3.5(c) and Figure 3.5(e). The number of random nodes generated by these

(a)

(b)

(c)

(d)

Figure 3.5: Simulation results for motion planning of a 2-DOF mobile robot; (a): the roadmap obtained by original PRM, (b): robot's snap shots following the path obtained by original PRM, (c): the roadmap obtained by enhanced PRM, (d): robot's snap shots following the path obtained by enhanced PRM (continued on next page).

Figure 3.5: (continued from last page) Simulation results for motion planning of a 2-DOF mobile robot; (e): the roadmap obtained by HFPRM, (f): robot's snap shots following the path obtained by HFPRM.

three methods are kept at constant. C-obstacles are presented in black. The snap shots of robot's motion obtained by each technique are also shown in Figure 3.5(b), Figure 3.5(d) and Figure 3.5(f). In this example the shortest path connecting the start and goal configurations must pass through the narrow passage between two big rectangular-shape obstacles.

The thick solid line in Figure 3.5(a) connecting the start and goal configurations shows the generated path obtained through the original PRM. As it is shown narrow passages have not been well covered by the roadmap. Thus the original PRM fails to find the shortest path. Figure 3.5(c) and Figure 3.5(d) show the results obtained by enhanced PRM. In enhanced PRM method The random nodes are generated only within free C-space. As it is shown in Figure 3.5(c) the constructed roadmap presents

a better coverage of the $\mathcal{C}_{free}$. However, narrow passages have not been well covered by the constructed roadmap. Thus the enhanced PRM fails to find the shortest path either. However one should note that the path found by the enhanced PRM has an overall shorter length than that when using the original PRM. The simulation results of the proposed HFPRM are depicted in Figure 3.5(e) and Figure 3.5(f). As it can be seen from Figure 3.5(e) the roadmap constructed based on HFPRM yields a good coverage² of the free C-space especially in hard-to-navigate regions. This method could find the shortest path going through the narrow regions between two obstacles efficiently.

Figure 3.6 shows the simulation results for motion planning of a 2-DOF mobile robot with the same mobility as explained in the first example. In this example any collision-free path connecting start and goal configurations has to go through a narrow passage. Figure 3.6(a) depicts the generated roadmap in the C-space using the proposed HFPRM. As it is shown the roadmap reflects the connectivity of the free C-space very efficiently. The snap shots of robot's motion moving from the start configuration towards the goal configuration are shown in Figure 3.6(b). The original and the enhanced PRM methods have difficulty to capture the connectivity of the free C-space. Figure 3.6(c) and Figure 3.6(d) show the constructed roadmaps based on enhanced PRM and the original PRM, respectively. Both methods fail to find a

²One should note that good coverage means that a large portion of the narrow passages is covered by the roadmap.

Figure 3.6: Simulation results for motion planning of a 2-DOF mobile robot; (a): constructed roadmap based on HFPRM, (b): robot's snapshots following the path obtained by HFPRM, (c): the roadmap obtained by modified PRM, (d): the roadmap obtained by original PRM.

path as it can be seen from these figures.

Figure 3.7(a)-3.7(c) show the simulation results for motion planning of a 3-DOF mobile robot obtained using HFPRM technique. This example has been chosen to show the capability of the proposed HFPRM motion planning technique to find a path through hard-to-navigate regions in 3D. As it is shown in Figure 3.7(a) the robot has to pass through a confined space to reach the goal. The proposed HFPRM technique provides a good coverage of this narrow region. Thus it succeeds to find a path from the start to the goal configuration passing through this confined space. Part of this collision-free path in robot's C-space is depicted in Figure 3.7(b). The robot's 3D C-space is also shown in Figure 3.7(c). One should note that the C-obstacle corresponding to the boundary of the C-space is not illustrated. Otherwise the C-obstacle corresponding to the obstacle within the robot's workspace could not have been shown. The original and the enhanced PRM do not provide a good coverage of the C-space especially in the confined space between two obstacles. Thus they fail in connecting the start node to the roadmap in the query phase.

In the next section we present the solution methodology pertaining to the incremental version of the HFPRM technique, i.e. IHFPRM technique, for model-based motion planning of robots with high DOF's.

(a)

(b)

Figure 3.7: Simulation results for motion planning of a 3-DOF mobile robot obtained using HFPRM; (a): robot's snap shots in physical space, (b): constructed roadmap in C-space (continued on next page).

Figure 3.7: (continued from last page) Simulation results for motion planning of a 3-DOF mobile robot obtained using HFPRM; (c): robot's 3D C-space.

3.2 Incremental HFPRM Technique Solution Methodology

Although the FDM technique used to solve the Laplace's equation can be extended to higher dimensions (i.e., $n > 3$) but the explicit representation of the C-space of robots with many DOF's will pose memory storage problems³. As an alternative to a grid representation of the C-space one can use triangulated mesh generated by finite element mesh generators. This will drastically decrease memory needed to represent the robot's C-space. Therefore Finite Element Method (FEM) was utilized to solve HFPRM incrementally which will be described below. The overall IHFPRM solution methodology is depicted in Figure 3.8.

The IHFPRM technique works as follows: first a roadmap is initialized in robot's

³Explicit representation of the C-space of a robot with n -DOF given that each DOF is discretized by m bits would require m^n bits of data.

Figure 3.8: IHFPRM solution methodology.

C-space by generating a pre-defined number of random nodes (the start and the goal configuration included) and connecting them together based on a metric, i.e. maximum edge size represented as an Euclidean distance. Then in an iterative scheme the roadmap is enhanced and searched for the shortest path. In each iteration the Laplace's equation, pertinent to potential flow, is solved over the current representation of the C-space using Finite Element Method (FEM) and the velocity of the flow is calculated at each node of the roadmap. The roadmap is then enhanced using a new, yet simple, random sampling scheme based on information obtained about the C-space topology with a *velocity thresholding* technique. This yields a roadmap with better coverage in regions associated with higher velocities (i.e. narrow regions).

At the final stage the roadmap is searched by a local planner for the shortest path connecting the start and the goal configurations. The enhancement ends either after a maximum number of iterations has reached or a collision-free path has been found.

First we elaborate on the required C-space representation to compute the HF's using FEM, and then the above mentioned steps will be discussed in more details in the following sub-sections.

3.2.1 C-space Representation

In the proposed technique two different representations of the robot's C-space are maintained: The graph-based representation, i.e. roadmap, and the grid-based representation. The former captures the connectivity of the free C-space in form of a graph structure that is exploited for path planning. The latter captures the topology of the C-space obstacles (C-obstacle) in form of a composite geometry object and is utilized by Finite Element Analyzer (FEA) to compute the harmonic functions over the C-space.

In this representation the start and the goal configurations and each non-free configuration in the C-space are represented by rectangular grids. The corresponding boundary conditions pertinent to the potential flow are assigned to the boundaries of these grids in the next step. At each iteration of the proposed technique a FEM mesh generator builds a triangulated mesh based on the updated C-space representation⁴.

⁴However, one can generate a coarse representation of meshes using rectangular elements with

Figure 3.9: (a): example of grid-based representation of a 2D C-space, (b): the corresponding mesh generated by a FEA mesh generator.

The generated mesh is then utilized by the FEA to solve the Laplace's equation. An example of this representation and the corresponding mesh for a 2D C-space are depicted in Figure 3.9. One should note that this representation can be mathematically extended to higher dimensions simply by extending the Dirichlet/Neumann condition to an n -dimensional space.

3.2.2 Using FD Paradigm to Identify Narrow Regions

Similar to the original HFPRM technique a fluid dynamic analogy based on potential flow is used to obtain information about the environment topology. For a complete discussion on this one can refer to sub-section 3.1.1.

the same level of accuracy.

Numerical solution of Laplace's equation can be obtained using Finite Element Method (FEM) [42]. The FEM discretizes the domain into simple geometry objects, i.e. elements, and the Laplace's equation is approximated by a piecewise linear function over each element.

In the proposed IHFPRM technique a triangulated mesh is generated based on the current grid-based representation of the C-space using a FEM mesh generator. The Laplace's equation, considering the boundary conditions as described earlier in sub-section 3.1.1, is then approximated on each mesh element by piecewise linear functions. This results in a system of linear equations whose solution yields the potential of the flow at each node of the triangulated mesh.

Having the potential values computed, the velocity of flow can be calculated using the gradient of ϕ at each node of the triangulated mesh. The velocity profile will be helpful to extract some information regarding the environment topology, i.e. identifying narrow regions. Higher velocities are observed in narrow passages due to Bernoulli's law [13]. For a complete discussion on this one can refer to sub-section 3.1.1. The information obtained through the velocity profile are used to enhance the PRM at each iteration to achieve a more uniform representation of the free C-space.

3.2.3 Incremental Construction and Update of the PRM

In our proposed IHFPRM technique a coarse roadmap with a small set of randomly generated nodes is initially built in the free C-space. The roadmap is then enhanced in an iterative scheme based on the information obtained about the C-space topology through the FD paradigm. One should note that in the basic PRM the roadmap is constructed in the pre-processing phase without considering the locations of the start and goal configurations. However, in our proposed PRM-like approach the topology of the generated roadmap depends on the locations of the start and goal configurations. This will be shown through computer simulation presented in Section 3.2.4.

Based on the velocity profile computed through the FD paradigm, a velocity thresholding technique is utilized to identify regions with different topologies. At each iteration of the IHFPRM technique, given the number of velocity levels, each node of the roadmap according to the velocity of the potential flow at that node falls into one of these levels⁵. A node falls into level (i) if and only if its velocity would be in the following interval: $(\frac{i-1}{L}V_{max} \dots \frac{i}{L}V_{max}]$, where L is the number of velocity levels and V_{max} is the maximum velocity in the velocity profile. Using this scheme the nodes of the roadmap are categorized in a predefined number of levels according to the corresponding velocity regions. One should note that there may be different number of nodes in each level corresponding to the area associated with different

⁵The velocity of the flow at each node of the roadmap, which does not lie on any node of the mesh, is calculated using linear interpolation.

velocities.

The main idea of the proposed PRM approach is to update and enhance the roadmap in regions associated with higher velocities. Update of the PRM is carried out in an iterative scheme by selecting milestone nodes to enhance the existing PRM. The number of milestone nodes selected at each velocity level is determined based on the average velocity and the number of nodes (area) of the corresponding level. This can be mathematically expressed by:

$$r_i = \frac{s_i}{S} R, \quad (3.7)$$

where

$$s_i = (1 - \omega) \frac{n_i}{N} + \omega \frac{V_{avi}}{V_{max}}, \quad (3.8)$$

$$S = \sum_{i=1}^L s_i, \quad (3.9)$$

r_i : number of milestone nodes selected in level i ,

R : total number of milestone nodes,

ω : weight factor,

n_i : total number of nodes in level i ,

N : total number of nodes in the roadmap,

V_{avi} : average velocity in level i ,

V_{max} : maximum velocity in the roadmap.

A tradeoff between area and the velocity at each level is established by changing the weight factor, ω . Equation 3.8 is the dynamic version of Equation 3.5. The maximum velocity is calculated online in Equation 3.8 while in Equation 3.5 the maximum velocity is considered to be constant. The number of milestone nodes to be randomly selected at each threshold level is computed using (3.7). At each iteration, having the number of milestone nodes computed, the roadmap is enhanced in the following steps:

Step (1): For $i = 1, \dots, L$, at each level i , r_i nodes with the smallest number of adjacent nodes are selected as the milestone nodes. This results in a uniform distribution of nodes at each level i .

Step (2): A pre-defined number of randomly selected configurations is generated within the neighborhood of each milestone node.

Step (3): The new generated nodes are checked by a local planner for the collision. The free configurations are added to the roadmap based on a metric, i.e. Euclidean distance between nodes, and the non-free configurations are kept in a list of C-obstacle nodes which will be used to update the grid-based representation of the C-space in the next iteration, as explained in 3.2.1.

The updated roadmap is then searched for the shortest path. If no path is found the grid-based representation of the C-space is updated by adding the C-obstacle grids corresponding to the C-obstacle nodes found in the previous iteration and HFs are

calculated using FEM based on the updated C-space representation. The roadmap is enhanced based on the new velocity profile found at each iteration, and the procedure continues. The iteration continues until a path is found or a maximum number of enhancements has been reached. In the next section some of the simulation results obtained through the proposed IHFPRM technique are presented.

3.2.4 Simulation Results and Discussion

The proposed IHFPRM technique has been simulated for 2-DOF mobile robots with only translational mobility for sake of illustration. One should note that since no explicit representation of the C-space is required this technique can be extended to robots with many DOF's. In this section we present 2D simulation results for some examples where critical regions, i.e. narrow regions and hard-to-navigate passages, exist. The effect of different parameters, which are set before the planning starts, are also investigated on overall performance of our proposed hybrid motion planning technique.

Figure 3.10(a)-Figure 3.10(d) show the simulation results obtained using the IHFPRM for an environment where the robot can pass through more than one passage to reach the goal configuration. However, the shortest path passes through a narrow region which is between two rectangular shape C-obstacles as depicted in Figure 3.10(a). The explicit representation of the C-space, for sake of illustration, is also

ITERATION#	ROADMAP SEARCH (sec.)	FEM SOLVE (sec.)	ROADMAP EN- HANCE (sec.)
1	0.00	4.01	0.50
2	0.01	5.81	0.88
3	0.02	9.84	1.61
4	0.04	11.73	3.77
5	0.06	14.45	6.01
6	0.12	20.44	8.09
7	0.37	-	-

Table 3.1: Run time statistics pertaining to the example shown in Figure 3.10.

shown. The enhanced roadmap after the second and the fifth iteration are shown in Figure 3.10(b) and Figure 3.10(c), respectively. The C-obstacle grids are presented in black, and the corresponding velocity profile is shown in color in the background. The shortest path has been found at the 7th iteration as depicted in Figure 3.10(d). As it is shown the obtained roadmap yields a good coverage in narrow region.

The CPU time associated with main functions of the IHFPRM technique, using a 2.4 GHz Intel Celeron, is summarized in Table 3.1. Almost 75% of the total running time relates to the solving of the Laplace's equation using the FEM.

Figure 3.11 shows the simulation results for motion planning of a 2-DOF mobile robot with the same mobility as explained in the first example. In this example the shortest path connecting the start and the goal configurations has to go through a hard-to-navigate passage. The shortest path, highlighted in Figure 3.11(b), has been found through the hard-to-navigate region after 9 iterations.

Figure 3.10: (a): Explicit representation of the C-space and the initial roadmap, (b) and (c): the enhanced roadmap after the 2nd and the 5th iteration, (d): the shortest path found along with the roadmap after the 7th iteration.

Figure 3.11: (a): Explicit representation of the C-space and the initial roadmap, (b): the shortest path along with the roadmap after the 9th iteration.

Figure 3.12 shows another simulation results obtained in an environment where any collision-free path connecting the start and the goal configurations has to go through a narrow passage. The obtained path along with the enhanced roadmap after 4 iterations is shown in Figure 3.12(b). As it is depicted the obtained roadmap maintains a good coverage in the narrow passage as well as in the wide-open regions.

As mentioned earlier the performance of the proposed technique will depend on the location of the start and goal configurations. This has been shown through Figure 3.13. Figure 3.13(a) shows the C-space of the robot, which is shown only for sake of illustration (no explicit C-space computation has been carried out in the computer simulations). In Figure 3.13(b) through Figure 3.13(f) the positions of the

Figure 3.12: (a): Explicit representation of the C-space and the initial roadmap, (b): the shortest path along with the roadmap after the 4th enhancement.

start and goal configurations have been changed. As can be seen from these figures the enhancement of the roadmap will be always biased towards regions with higher velocity thresholds. In all cases the planner is capable of finding the shortest path going through narrow passages. In the next section we discuss the effect of some parameters on the performance of the proposed technique through simulation results.

3.2.5 Sensitivity Analysis on Motion Planning Parameters

We examine the effect of three important parameters on the overall performance of the proposed technique in finding the shortest path: (1) L the number of velocity threshold level, (2) ω the velocity weight factor, and (3) the maximum edge size in

(a)

(b)

(c)

(d)

Figure 3.13: (a): Explicit representation of the C-space, (b)-(f): the shortest path along with the roadmap pertaining to different positions of the start and the goal configurations in the same environment (continued on next page).

Figure 3.13: (continued from last page).

the roadmap. We discuss the effects of these parameters on the performance of the IHFPRM technique for a 2-DOF mobile robot moving in the environment shown in Figure 3.13(a) with the position of the start and the goal configuration being set as in Figure 3.13(d). One should note that in each of the following examples the roadmap is generated from the same seed of random numbers.

1. Effect of the number of velocity threshold levels, L

The number of the velocity threshold levels, L plays an important role in finding paths going through narrow regions. The contribution of this parameter can be better understood through some representative simulation results given in Figure 3.14(a) and Figure 3.14(b). The simulation results are based on scenario given in Figure

Figure 3.14: The roadmap along with the corresponding shortest path with (a): 5 threshold levels, (b): 15 threshold levels.

3.13(d). The shortest path going through the narrow region between two rectangular-shape obstacles was found after 4 iterations with the total number of random nodes at 472 with $L = 5$ in Figure 3.14(a). Figure 3.14(b) shows the results for the case where $L = 15$. The shortest path is found after 4 iterations but with the total number of random nodes at 572.

2. Effect of the velocity weight factor, ω

The velocity weight factor is utilized to make a tradeoff between the average velocity of the flow at a specific level with the number of previously generated nodes in that level. The contribution of this factor on the performance of the proposed technique is illustrated through representative simulations. Again the simulation

Figure 3.15: The roadmap along with the corresponding shortest path with velocity weight factor equal to (a): 0.20 and (b): 0.50.

results are based on the scenario shown in Figure 3.13(d). Weight factors of $\omega = 0.2$ and $\omega = 0.5$ are chosen in two simulations whose results are presented in Figure 3.15(a) and Figure 3.15(b), respectively. As can be seen in Figure 3.15(a), more random nodes are generated in wide-open regions than that in Figure 3.15(b). In short, high velocity weight factor will cause generating less random nodes in wide-open regions far from the obstacles. Therefore finding the shortest path with lower number of nodes can be achieved by choosing higher weight factor. One should also note that a part of the shortest path could go through wide-open regions. A fine coverage of these regions would be needed to make sure the shortest path found in the roadmap closely represents the actual shortest path.

Figure 3.16: The roadmap along with the corresponding shortest path with the maximum edge size of (a): 10 and (b): 15.

3. Effect of the maximum edge size

Choosing bigger maximum edge size to connect the nodes in the roadmap results in less planning time and coarser roadmap which consequently yields a longer path.

Figure 3.16 illustrates the effect of the edge size on the obtained roadmap.

Figure 3.16(a) and Figure 3.16(b) show the obtained roadmap after 3 and 2 iterations with maximum edge size of 10 and 15, respectively, for the same scenario as shown in Figure 3.13(d). As it is shown with bigger maximum edge size the planner needs less enhancement to find the path, however, the obtained shortest path will be most probably very longer than the actual shortest path.

Chapter 4

Sensor-Based Motion Planning: Solution Methodology

4.1 Definitions and Notations

Let $\mathcal{A}$ denote a cylindrical-shape robot with 3 DOF (x - y translations and θ rotation) moving in a 2D workspace $\mathcal{W}$. Considering the circular cross-section of the robot, the robot's C-space $\mathcal{C}$ can be introduced as a 2D space where each point in this space represents the x - y coordinates of the robot's center of rotation represented in a fixed coordinate frame attached to the robot's workspace.

The robot is equipped with ultrasonic range finders. The $\mathcal{W}$ is incrementally updated as the robot scans the physical workspace and correspondingly the $\mathcal{C}$ representation will be also updated. The free part of the C-space ΔC_{free} , which is made

known free after each scan can be computed comparing the updated $\mathcal{C}$ representations obtained through the last two scans. A probabilistic roadmap $\mathcal{R}$ is constructed within the $\Delta\mathcal{C}_{free}$ after each scan to capture the connectivity of the $\mathcal{C}_{free}$. The robot is only allowed to move within the $\mathcal{C}_{free}$ represented by the $\mathcal{R}$.

4.2 Algorithm

The overall algorithm of the sensor-based motion planning framework is given as the following pseudo-code:

```

SB-IHFPRM
{
     $\mathcal{W} \leftarrow \text{Workspace.Initialize};$ 

     $[\mathcal{C}, \Delta\mathcal{C}_{free}, \mathcal{C}_{unkbnd}] \leftarrow \text{Cspace.Initialize}(\mathcal{W});$ 

     $\mathcal{R} \leftarrow \text{Roadmap.Initialize}(\Delta\mathcal{C}_{free});$ 

     $q_{cur} \leftarrow q_s;$ 

     $it \leftarrow 0;$ 

    repeat

         $it \leftarrow it+1;$ 

         $path \leftarrow \text{Roadmap.Search}(q_{cur}, q_g, \mathcal{R});$ 

        if  $path \neq \text{NULL}$ 

             $q_{cur} \leftarrow \text{Robot.Move}(path);$ 

```

```

        Exit;

    end;

     $\mathcal{V} \leftarrow \text{Fem.Solve}(q_s, q_g, \mathcal{C});$ 

     $q_v \leftarrow \text{Planner.NextBestView}(\mathcal{V}, \mathcal{C}_{unkbnd}, \mathcal{R}, \mathcal{W});$ 

     $path \leftarrow \text{Roadmap.Search}(q_{cur}, q_v, \mathcal{R});$ 

     $q_{cur} \leftarrow \text{Robot.Move}(path);$ 

     $\Delta\mathcal{W} \leftarrow \text{Robot.Scan};$ 

     $\mathcal{W} \leftarrow \text{Workspace.Update}(\Delta\mathcal{W}, \mathcal{W});$ 

     $[\mathcal{C}, \Delta\mathcal{C}_{free}, \mathcal{C}_{unkbnd}] \leftarrow \text{Cspace.Update}(\mathcal{C}, \mathcal{W});$ 

     $\mathcal{R} \leftarrow \text{Roadmap.Expand}(\Delta\mathcal{C}_{free}, \mathcal{R});$ 

    until  $it \leq \text{MAXITNUM}$ 
}

```

The proposed algorithm works as follow: first the workspace and correspondingly the robot's C-space representations are initialized¹. A probabilistic roadmap $\mathcal{R}$ is then initialized in the small free region surrounding the robot's start configuration q_s . Then in an iterative "Scan and Go" fashion the roadmap is incrementally enhanced and expanded as the robot senses the workspace after each scan. At each iteration the roadmap is searched for the shortest path between the robot's current configuration q_{cur} and the goal configuration q_g . If such a path exists the robot moves to the

¹Initially the C-space includes a free region surrounding the robot's start configuration, and the boundary set for the C-space to be considered as C-obstacles. The remaining part of the C-space is assumed to be unknown.

goal configuration following the obtained path. Otherwise the robot plans for the next scan. The Laplace's equation, pertinent to the potential flow, is solved over the current representation of the C-space $\mathcal{C}$ using a Finite Element Analyzer (FEA) package and subsequently the flow's velocity profile $\mathcal{V}$ is computed. Planner then finds the next best view configuration q_v where the next scan should be carried out using the information obtained through this velocity profile. This is done through the `Planner.NextBestView()` class which will be explained in more details in Section 4.3. The roadmap is then searched for the shortest path from q_{cur} to q_v and the robot moves to the configuration q_v to do the next scan. Based on the new information obtained about the workspace through a scan, i.e. $\Delta\mathcal{W}$, the workspace and correspondingly the C-space representations are updated and the new free part of the C-space $\Delta\mathcal{C}_{free}$ is calculated. This procedure is particularly further explained in Section 4.2.1. The roadmap is enhanced and expanded within the $\Delta\mathcal{C}_{free}$ found in the previous scan. More details on roadmap expansion is given in Section 4.2.1. The updated roadmap is then searched for the shortest path between q_{cur} and q_g . If a path is found the robot moves to the goal configuration and the planner exits, otherwise the robot plans for the next scan and repeats all the above steps. The iterations continue until either a maximum iteration number, i.e. MAXITNUM, is reached or a collision-free path is found. The main parts of the proposed algorithm is explained in more details in the following sub-sections.

4.2.1 Workspace and C-space Representations

We utilize a Fast Fourier Transform (FFT)-based method using convolution theorem to compute the $\Delta\mathcal{C}_{free}$ explicitly². Using this technique the robot's C-space can be represented as the convolution of the robot and workspace representations. In this section we describe this technique in details.

Consider a physical workspace $\mathcal{W} = [a, b] \times [c, d] \subset \mathbb{R}^2$. $\mathcal{W}$ can be discretized into an $N \times M$ array W . Each grid w_{ij} of $\mathcal{W}$ is defined as:

$$w_{ij} = [a + i\frac{(b-a)}{N}, a + (i+1)\frac{(b-a)}{N}] \times [c + j\frac{(d-c)}{M}, c + (j+1)\frac{(d-c)}{M}], \quad (4.1)$$

where $i = 1, \dots, N$, and $j = 1, \dots, M$. Initially we assume that there is no obstacle in the workspace and all the cells of array W are assigned 0 (i.e., free) other than the boundaries of the workspace which are assigned 1 (i.e., obstacle). Array W is incrementally updated as the robot senses the physical space. If an obstacle is found anywhere in a grid w_{ij} of the workspace we let $W(i, j) = 1$, otherwise 0. Considering that the robot has a circular cross-section, the robot's C-space, $\mathcal{C}$, can be represented as a set of (x, y) coordinates of the robot's center of rotation. For a planar robot in a 2D workspace the C-space bitmap array C can be explicitly obtained as the convolution of the workspace bitmap array W and the mirror of the robot's bitmap

²In general there is no need for explicit representation of the $\Delta\mathcal{C}_{free}$ using the PRM. However, explicit representation of $\Delta\mathcal{C}_{free}$ can drastically decrease the CPU-time needed for constructing the roadmap. One should note that $\Delta\mathcal{C}_{free}$ can be computed explicitly only for robots with low DOF (i.e., $n \leq 3$).

array, A' [43]:

$$C = W * A', \quad (4.2)$$

where $A'(i, j) = A(-i, -j)$ ³. The above convolution can be computed using Fast Fourier Transform (FFT). The running time of this calculation depends only on the resolution of the discretization. For more details on this one can refer to [43].

We also maintain a bitmap array W_{unk} which represents the unknown part of the workspace. Each cell of W_{unk} is set to 1 if the corresponding grid in the workspace is unknown, otherwise 0. Initially we assume that all the cells in the workspace other than the small region around the robot at the start configuration are unknown and the corresponding cells in W_{unk} are set to 1. W_{unk} is incrementally updated as the robot scans the workspace and makes some part of it known. Correspondingly C_{unk} represents the bitmap array pertinent to the unknown C-space C_{unk} , and is computed using the convolution method explained earlier as follow:

$$C_{unk} = W_{unk} * A'. \quad (4.3)$$

The set of configurations which constitute the boundary of the unknown C-space, i.e. C_{unkbnd} the cross-section of the free C-space C_{free} and the unknown C-space C_{unk} , can be computed using this technique interactively. In the view planning stage of the proposed technique the next region which should be scanned next is selected based on information obtained through the FD paradigm. This is explained in more details

³ A denotes the robot's bitmap array.

in Section 4.3.

The bitmap array C_{free} corresponding to the free C-space at each iteration is computed as follow:

$$C_{free}(i, j) = \begin{cases} 1 & \text{if } C_{unk}(i, j) = 0 \text{ \& } C(i, j) = 0 \\ 0 & \text{otherwise,} \end{cases}$$

where $i = 1, \dots, N$, $j = 1, \dots, M$, and each cell of the C_{free} which is assigned 1 shows that the corresponding configuration is free. The C_{free} is utilized to calculate the new free part of the C-space ΔC_{free} which is made known after each scan to expand the roadmap. ΔC_{free} can be calculated as: $\Delta C_{free} = C_{free}^i - C_{free}^{i-1}$, where C_{free}^n denotes the C_{free} calculated at the n^{th} iteration. Having this computed, the roadmap is then expanded within this region by generating a pre-defined number of random nodes and connecting them together based on a predefined maximum edge size of the roadmap. The explicit computation of the ΔC_{free} prevents the roadmap to be degenerated in the same region.

4.3 View Planning Using FD paradigm

The view planning problem is solved in two steps: in the first step a region of the workspace which should be scanned next is selected based on information obtained through the FD paradigm, and in the second step a configuration that can be reached by the robot is selected to scan the region found in the first step from. In the following

sub-sections first we explain how FD paradigm can be used to identify critical regions in the environment, and then we will describe our proposed technique to address the steps mentioned above.

4.3.1 Using FD Paradigm to Identify Critical Regions

The same fluid dynamic analogy explained in Section 3.2.2 is used to solve the view planning problem. In the proposed SBMP technique FEM is utilized to solve the Laplace’s equation over the grid-based representation of the C-space. A triangulated mesh is generated based on the current grid-based representation of the C-space using a FEM mesh generator. The Laplace’s equation is then approximated on each mesh element by piecewise linear functions. This results in a system of linear equations whose solution yields the potential of the flow at each node of the triangulated mesh.

After computing the potential values the velocity of flow can be calculated directly using the gradient of ϕ at each node of the triangulated mesh. This velocity profile is used to extract some information regarding the environment topology, i.e. identifying narrow regions. Higher velocities are observed in narrow passages due to Bernoulli’s law [13]. For a complete discussion on this one can refer to our previous paper [44]. The information obtained through the velocity profile is used to prioritize different regions of the environment when it comes to the scan planning stage of our proposed sensor-based motion planning technique, i.e. *Which Region to Scan*. This will be

discussed in further details in Section 4.3.2.

4.3.2 Which Region to Scan

The crux of our planning approach is to expand the roadmap into the regions of the C-space associated with higher velocities pertinent to the potential flow. Therefore in our view planning approach higher priority is given to the high velocity regions (configurations). The boundary of the unknown C-space, $\mathcal{C}_{unkbnd}$ is calculated and updated using the convolution method explained in sub-section 4.2.1. The velocity of the flow is calculated at each configuration of the $\mathcal{C}_{unkbnd}$ using a linear interpolation. The configuration q_x with highest velocity among all the configurations in $\mathcal{C}_{unkbnd}$ is selected as the central configuration. The region represented by $\mathcal{A}(q_x)$ (i.e., the region occupied by the robot when being at configuration q_x) is scanned next. At this stage it remains to find the best feasible configuration on the roadmap that can be reached by the robot to do the next scan from, hence the name i.e. *From Where to Scan*.

4.3.3 From Where to Scan

To find the best view configuration to scan the region represented by $\mathcal{A}(q_x)$ obtained in the previous step, all the nodes (configurations) of the connected component of the existing roadmap containing the start configuration q_s are examined and the best view configuration, q_v is selected. The q_v is chosen in a way that if the robot is

placed at that configuration the distance between the center of the sensor's field of view $\mathcal{V}_s$ and the center of workspace grid corresponding to the center of $\mathcal{A}(q_x)$ would be minimum without having any known obstacle obstructing the sensor's view.

Chapter 5

Implementation And Experimental Results

The proposed motion planning technique was implemented on a wireless mobile robot, called DRK6000, manufactured by Dr.Robot Inc. [41]. The robot is equipped with an array of ultrasonic range finders which retrieve range data from the physical space. The sensors' field of view is 30 degrees with a range limited to 100cm. For more details on the robot see Appendix A. The robot's motion is limited within a $375cm \times 300cm$ rectangular-shape arena (see Figure 5.1). Small cylindrical objects are used to construct different shapes of obstacles in this workspace. The best size for these cylindrical shape objects were selected after conducting a series of tests. The Aluminum cans with 5cm of diameter showed the least sonar shadow. Thus they were selected in experiments.

As it was explained earlier in Section 4.2.1 a grid is used to represent the robot's

Figure 5.1: The robot's workspace (a rectangular-shape arena).

workspace. In our experiments we discretize the robot's workspace into a 60×75 grid. This workspace is simulated in MATLAB. Furthermore the simulation updates the grid-based representation of the workspace, the robot's C-space, the roadmap, and the robot's location as it moves. The workspace and correspondingly the robot's C-space representations are updated as the robot scans the environment. Figure 5.2 shows the simulator initialized for a sample experimental setup. What is represented through this simulator is snapshots of the real robot moving in real environment. More explanation on the simulator will follow.

A large number of experiments have been carried out. However, only two representative experiment runs are presented in this section. Figure 5.3 shows the workspace

Figure 5.2: Sensor-based simulator designed in MATLAB.

and robot's C-space for 14 scans (i.e., iterations). The internal model of the robot's workspace and the path segments that it follows at each iteration are shown in the first column of Figure 5.3. The second column shows the corresponding robot's C-space along with the roadmap at each iteration. Figure 5.3(a) shows the initialized workspace along with the robot (represented as a green circle) at its initial configuration. A small region (a $65\text{cm} \times 65\text{cm}$ rectangular area) around the robot, shown in white color, is initially assumed to be free to accommodate the first movements of the robot. The grids corresponding to the unknown free regions of the workspace are shown in gray and those corresponding to the unknown obstacles are shown in darker color¹. The boundary of the workspace is assumed to be known and it is shown in

¹One should note that the obstacles are plotted for illustration only. The positions of obstacles are not known a priori.

black. The initial robot's C-space is shown in Figure 5.3(b). The grids corresponding to the unknown configurations are shown in gray. The black region shown in robot's C-space denotes C-obstacles. The roadmap initially generated in the small free part of the C-space corresponding to the small free region around the robot at its initial configuration is shown in green. The grids with yellow colors in the C-space represent the boundary of the unknown C-space, $\mathcal{C}_{unkbnd}$ calculated using FFT.

The robot's task in this experiment is to start from the initial configuration and to scan the workspace interactively till it finds a feasible path (i.e., a collision-free path) to reach the goal configuration. Each time the robot takes a new scan the workspace $\mathcal{W}$, free C-space, $\mathcal{C}_{free}$, and the roadmap $\mathcal{R}$ are updated. In the next step the planner finds the next view configuration q_v to scan the area defined by $\mathcal{A}(q_x)^2$. The grids corresponding to the q_v and q_x at iteration i are denoted by v_i and x_i and are displayed in the workspace model, respectively. Figure 5.3(p) shows the final roadmap obtained at the 14th iteration where goal has been reached. Collision-free path connecting the start and the goal configurations is shown as a thick line on this roadmap as well (see Figure 5.3(p)). The robot at the goal configuration along with the path traversed by the robot in the workspace are shown in Figure 5.3(o). As it is shown the planner finds the shortest path passing through the narrow region between the two obstacles successfully.

² $\mathcal{A}(q)$ denotes the physical space occupied by the robot when the robot is at configuration q .

(a)

(b)

(c)

(d)

Figure 5.3: Robot's snapshots obtained using our sensor-based technique implemented on a mobile robot with 3 DOF. Left column: the workspace models from iteration 0 to 14. Right column: the robot's C-space along with the roadmap obtained from iteration 0 to 14.

(e)

(f)

(g)

(h)

Figure 5.3: (continued from last page) Robot's snapshots obtained using our sensor-based technique implemented on a mobile robot with 3 DOF (continued on next page).

(i)

(j)

(k)

(1)

Figure 5.3: (continued from last page) Robot's snapshots obtained using our sensor-based technique implemented on a mobile robot with 3 DOF (continued on next page).

(m)

(n)

(o)

(p)

Figure 5.3: (continued from last page) Robot's snapshots obtained using our sensor-based technique implemented on a mobile robot with 3 DOF.

Figure 5.4 shows the robot's snapshots obtained using the proposed motion planning technique for a different scenario. The shortest path in this experiment must pass through a hard-to-navigate passage. As it can be seen through Figure 5.4(a)-5.4(h) the planner finds the path connecting the start and goal configurations going through the narrow region between two obstacles³ after 11 iterations.

It is noteworthy that the robot starts wandering around at a couple of points but it finds its path through the narrow region eventually. One should also note that due to the odometry errors the actual position of the robot in the real workspace will be different from the position calculated based on odometry data. This results in: (1) the map of the environment obtained based on range data through the planning process does not exactly correspond to the real workspace (This can be better understood by comparing the workspace model shown in Figure 5.4(a) and the actual model of the workspace shown in Figure 5.4(g)), and (2) the actual position of the robot when the goal is reached would be different from the final position of the robot calculated based on odometry data (This can be better visualized by looking at Figure 5.4(g) where the position of the robot at its goal configuration is slightly different from the position that the robot is supposed to reach.). In average an error of 10 to 20cm in position and 5 to 10 degrees in orientation are observed throughout the experiments.

³One should note that any other path not going through this narrow region will be longer than the path going through this region in average.

(a)

(b)

(c)

(d)

Figure 5.4: Robot's snapshots obtained using our sensor-based technique implemented on a mobile robot with 3 DOF. (a), (c), (e), and (g): the workspace model at iteration 0, 1, 6, and 11, respectively, (c), (d), (f), and (h): the robot's C-space along with the roadmap obtained at iteration 0, 1, 6, and 11 (continued on next page).

(e)

(f)

(g)

(h)

Figure 5.4: (continued from last page) Robot's snapshots obtained using our sensor-based technique implemented on a mobile robot with 3 DOF.

These representative experimental results along with a number of other experiments carried out off-line reflect on the credibility of the proposed robot motion planning technique.

Chapter 6

Conclusions and Future Works

A hybrid motion planning technique based on harmonic functions and probabilistic roadmaps was proposed. It was shown through computer simulations and experimental results that this technique can solve MBMP and SBMP problems efficiently. The main idea of the proposed approach is to identify critical regions, i.e. narrow passages and/or hard-to-navigate regions, using information obtained about the environment topology through a fluid dynamic paradigm. Furthermore, these information will be passed to a PRM-based method to build a roadmap which efficiently captures the connectivity of the free C-Space.

To implement the proposed approach, two techniques were presented: (1) the Harmonic Function-based Probabilistic Roadmap (HFPRM) that is applicable to motion planning of robots with low DOF's, and (2) the Incremental HFPRM (IHFPRM) technique which is applicable to motion planning of robots with high DOF's. The

proposed HFPRM approach was implemented on a mobile robot with 3 DOF moving in an unknown environment cluttered with unknown obstacles. The robot's task is to gain information about the unknown environment incrementally through iterative scans using its on-board sensors and to find a collision-free path to reach a pre-defined goal configuration. The information obtained through a fluid dynamic paradigm is utilized to prioritize different regions in the workspace at the scan planning stage. A PRM-like method is utilized to capture the free portion of the robot's C-space in form of a graph. This graph is then searched for the shortest path that connects the robot to the goal configuration.

Performance of the proposed motion planning technique was fully investigated through computer simulations and experimental results. It was shown that the proposed technique is capable of solving the target-seeking problem in complex environments not known a priori.

This work can be extended in the following areas:

- extension of this work to the sensor-based motion planning of robotic arms with many DOF, and
- sensor-based motion planning in environments with dynamic obstacles.

Furthermore, a more comprehensive study on the error analysis of the system will add to the robustness of the proposed technique.

Appendix A

The Mobile Robot System

The proposed SBMP approach was implemented on a mobile robot system, called DRK6000, manufactured by DrRobot Inc. [41] (see Figure A.1). The system is equipped with a Multimedia Controller, a Sensing-Motion Controller, and various peripheral sensors. The system uploads all the sensor information to a PC with a 2.4 GHz Intel Celeron processor through a Bluetooth wireless module. All the low-level control commands are accessible in MATLAB environment through the API offered in an ActiveX module which accesses the sensor information, sends control commands, and configures the system settings.

The mobile platform has a total of 3 DOF, two translational and one rotational. The system runs on a wheel-based platform equipped with two wheels at the left and right sides of the chassis and two passive castor at the front and rear sides. Each wheel is setup with a rotary sensor mounted on the motor's output shaft for

Figure A.1: Mobile robot system (DRK6000) used in our experiments.

precision control of the wheel movements. The robot's movement when traversing a path segment in our experiments comprises two actions: go forward, and rotate left/right. A Proportional Integral Derivative (PID) velocity control is utilized to move the robot on straight lines or to rotate it around its center of rotation with high precision. The Ziegler-Nichols tuning method [45] was utilized to tune the gain of the PID controller.

The robot is equipped with ultrasonic range finders which retrieve range data from the physical space. The range finders scan in the range of $4cm$ - $255cm$. The sensors' field of view is about 30 degrees. In our experiments only the front sensor was employed. At each scan the robot rotates about 50 degrees with 10-degrees

increments.

Appendix B

Using TDMA method to Solve Laplace's equation

We utilized the TDMA method in the original HFPRM technique (section 3.1) for solving the Laplace's equation in 2D and 3D problems. The mathematical base of TDMA method has been presented in [42]. The TDMA method is basically used for solving the Tri-diagonal systems. Solving the Laplace's equation in one-dimensional problems rises to such systems. The general form of these equations is represented as follow:

$$-\beta_j\phi_{j-1} + D_n\phi_j - \alpha_j\phi_{j+1} = C_j \quad \text{for } j = 2, \dots, n \quad (\text{B.1})$$

where ϕ_{j-1} , ϕ_j and ϕ_{j+1} represent the potential values of three successive points in a one-dimensional grid. The values of β_j , D_n and α_j depend on the boundary conditions. In (B.1) the ϕ_1 and ϕ_{n+1} are assumed to be known as boundary conditions. Equation

(B.1) can be solved interactively by *forward elimination* and *back-substitution*. The forward elimination solves each equation for ϕ_j in terms of ϕ_{j+1} and substitutes this value in next equation. This process continues until ϕ_n is calculated in terms of ϕ_{n+1} . The general form of this equation as it has been presented in [42] is as follow:

$$\phi_j = A_j \phi_{j+1} + C'_j, \quad (\text{B.2})$$

where:

$$A_j = \frac{\alpha_j}{D_j - \beta_j A_{j-1}} \quad (\text{B.3})$$

$$C'_j = \frac{\beta_j C'_{j-1} + C_j}{D_j - \beta_j A_{j-1}} \quad (\text{B.4})$$

In back-substitution, the general form of recurrence relationship (B.2) is used to find the value of ϕ_j for $j = 2, \dots, n$. For points $j = 1$ and $j = n + 1$ the value of A_j and C'_j are substituted as follow:

$$A_1 = 0 \quad (\text{B.5})$$

$$C'_1 = \phi_1 \quad (\text{B.6})$$

$$A_{n+1} = 0 \quad (\text{B.7})$$

$$C'_{n+1} = \phi_{n+1} \quad (\text{B.8})$$

The values of A_j and C'_j are subsequently calculated for $j = 2$ to n using (B.3) and (B.4). Since the value of ϕ_{n+1} is known the value for ϕ_j can be obtained in reverse order using (B.2).

The TDMA method can be used interactively to solve the two-dimensional problems. Considering a two-dimensional discretized plane (in rows and columns), a typical equation obtained solving the two dimensional problem will be in the following general form:

$$a_P\phi_P = a_W\phi_W + a_E\phi_E + a_S\phi_S + a_N\phi_N + C, \quad (\text{B.9})$$

where ϕ_W , ϕ_E , ϕ_S and ϕ_N represent the potential value of four grid points around central point P . The values of a_P , a_W , a_E , a_S , a_N and C are calculated based on the problem definition and boundary conditions. For example the Laplacian difference equation (2.2) for an interior point is stated as follow:

$$4\phi_P = \phi_W + \phi_E + \phi_S + \phi_N \quad (\text{B.10})$$

To solve the system of linear equations obtained in a two-dimensional problem using TDMA one needs to convert (B.9) to a tri-diagonal form, i.e. (B.1). Assuming temporarily known values for ϕ_W and ϕ_E , (B.9) can be rewritten as follow:

$$-a_S\phi_S + a_P\phi_P - a_N\phi_N = a_W\phi_W + a_E\phi_E + C, \quad (\text{B.11})$$

where the right hand side of (B.11) is assumed to be temporarily known. (B.11) is in the form of (B.1) and the same scheme can be used to find the potential values. Iterations will continue until the stopping criteria is satisfied.

For three-dimensional problems, the TDMA method can be applied line by line in a selected plane and then the calculation can be moved to the next plane and the

domain is scanned plane by plane. The converted discretized equation in this case is stated as follow:

$$-a_S\phi_S + a_P\phi_P - a_N\phi_N = a_W\phi_W + a_E\phi_E + a_T\phi_T + a_B\phi_B + C, \quad (\text{B.12})$$

where the right hand side of the above equation is assumed to be temporarily known.

Using this scheme the TDMA can be extended to higher dimensions as well.

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