Project Delivery System Selection under Uncertainty:

A Multi-Criteria Multi-Level Decision Aid Model

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Master of Applied Science

in Industrial Systems Engineering

University of Regina

By

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Regina, Saskatchewan, Canada

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Fereshteh Mafakheri, candidate for the degree of Master of Applied Science, has presented a thesis titled, ***Project Delivery System Selection Under Uncertainty: A Multi-Criteria Multi-Level Decision Aid Model***, in an oral examination held on June 28, 2006. The following committee members have found the thesis acceptable in form and content, and that the candidate demonstrated satisfactory knowledge of the subject material.

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ABSTRACT

Selecting a project delivery system is a critical task that project owners should carry out in order to ensure the project's success. This selection involves a complex decision-making process. The complexity arises from uncertain or not well-defined parameters and/or the multiple criteria structure of such decisions.

In this study, a decision aid model using the analytical hierarchy process (AHP) coupled with rough approximation concepts is developed to assist project owners. The selection criteria are determined by studying a number of benchmarks. The decision aid model ranks the alternative delivery systems by considering both the benchmark results and the owners' opinions.

In the proposed model, an optimization procedure is performed via the upper and the lower linear programming models to determine the interval priorities for alternative project delivery systems. In cases where there are incomparable alternatives, which are the most likely case in uncertain decision-making, the model uses rough-set based measures to reduce the number of decision criteria to a subset, which has the capability of fully ranking the alternatives.

To illustrate the applicability and usefulness of this methodology, a real-world case study is performed.

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DEDICATION

This thesis is dedicated to my entire family for their unconditional love, continued prayers and constant encouragement.

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LIST OF ABBREVIATIONS, SYMBOLS and NOMENCLATURE

A	=	The matrix of judgments
A	=	The set of all attributes
AHP	=	Analytic Hierarchy Process
A/E	=	Architect/Engineer
A_α	=	α -level discernibility set of attributes
a_{ij}	=	The strength of object i compared with object j
$\underline{a_{ij}}$	=	The lower bound of a_{ij}
$\overline{a_{ij}}$	=	The upper bound of a_{ij}
$a(x)$	=	The values of attribute a for element x
α	=	The decision maker's degree of optimism
$\alpha_R(X)$	=	The accuracy measure
$\alpha_{xy}^{(k)}$	=	The discernibility ratio of elements x and y with respect to the k -th attribute
B_c	=	The center of interval B
B_k	=	The priority matrix of the k th level, $k = 2, \dots, h$
B_w	=	The half-width of interval B
$BN_R(X)$	=	R -boundary region of X
C	=	The set of condition attributes
$Card(\overline{RX})$	=	The cardinality of upper approximation

$Card(\underline{RX})$	=	The cardinality of lower approximation
$C.I$	=	Consistency Index
CM	=	Construction Manager
CM/GC	=	Construction Management at Risk
CM/PM	=	Construction Management as Program Management
$C.R$	=	Consistency Ratio
c_{ij}	=	The set of all attributes in which objects x_i and x_j are discernible
D	=	The set of decision attributes
DBB	=	Design-Bid-Build
DB	=	Design-Build
DM	=	Decision Maker
$\underline{f}_{wx}$	=	The half-width of the interval-valued information function $f(x, a)$
$\bar{f}_{cx}$	=	The center of the interval-valued information function $f(x, a)$.
GC	=	General Contractor
H	=	A complete hierarchy with h levels
$I(B)$	=	Binary indiscernibility relation
$IND(P)$	=	An indiscernibility relation over P
IVIS	=	Interval-Valued Information System
K	=	knowledge base
$M(S)$	=	A discernibility matrix
MCDM	=	Multi-Criteria Decision Making
$NEG_R(X)$	=	R -negative region of X
$POS_R(X)$	=	R -positive region of X

R	=	An equivalence relation
$\mathbf{R}$	=	The family of equivalence relations
$RED(P)$	=	The family of all reducts of P
RFP	=	Request for Proposal
RFQ	=	Request for Qualifications
$R.I$	=	Random Consistency Index
RST	=	Rough Sets Theory
$\underline{R}X$	=	R -lower approximation of X
$\overline{R}X$	=	R -upper approximation of X
S	=	A knowledge representation system
U	=	Universe of objects
U/R	=	The family of all classifications of U
Va	=	The domain of attribute a
w_i	=	The weight for object i
w_j	=	The weight for object j
W	=	Eigen-vector
Wi	=	The global priority vector for alternative i
w_{ij}	=	The local weight of the i -th alternative with respect to the j -th attribute
λ_{max}	=	The largest Eigen-value

CHAPTER 1 - INTRODUCTION

A project delivery system is a method of procurement that project owners use as a means of providing design and construction services for a project. In each system, there are usually two parties involved: a design entity, which provides design requirements and documents, and a contractor that provides construction services for the project.

All project owners want their projects to be of high quality and completed within the given time and budget. To achieve those goals, besides having efficient project members and successfully executing the project, selecting the optimal project delivery system, the system that would most likely produce the best outcome for the public or private owners, has a high level of importance. This importance is due to the fact that each delivery system has its own approach and level of risk for reaching these goals.

In practice, there are several systems of project delivery available from which to choose. The most commonly used systems are Design-Bid-Build (DBB), Design-Build (DB), Construction Management at Risk (CM/GC), and Construction Management as Program Management (CM/PM).

In real-world applications, in order to select the optimal delivery system for a project, methods such as Low Bid and Best Value Bid have typically been applied (Georgia State Financing and Investment Commission 2003, Ferguson and Keing 2003). During the past few decades, the simplicity of such single-attribute decision-making approaches has made them so popular that there were no efforts either in literature or in practice for designing systematic decision-making structures to facilitate a selection process. Considering the increasing complexities, regulatory mandates and risks nowadays, a

number of researchers have recently proposed systematic and analytical decision making approaches for project delivery system selection (Fong and Choi 2000, Al Khalil 2002, Silva 2002 and Mahdi and Alreshaid 2005).

In order to determine which of these various options is the most appropriate for a particular project, the requirements of the project should be evaluated carefully. There are numerous factors that should be considered in the decision-making process. In this study, an innovative multiple-criteria decision-making structure for project delivery system selection is proposed. The major decision criteria regarding the selection of the delivery system are presented on the basis of different benchmarks and surveys. To appropriately consider these criteria when determining the best-suited project delivery system, an Analytic Hierarchy Process (AHP) model, a multiple criteria decision-making approach, has been applied (Saaty 2001). Choosing AHP as a decision-making tool has the following advantages:

- (a) This method has the ability to incorporate both objective (quantitative) and subjective (qualitative) factors.
- (b) Due to the structure of the hierarchy, the problem can be broken down into smaller parts, which simplifies the decision-making process. Such a structure clarifies the problem and exhibits the contribution of each of the elements to the final decision.

In establishing a framework for decision making, one faces a number of challenges. First, it is necessary to find a suitable way to consider the uncertainties brought about by experts' judgments and imprecise data. Moreover, these embedded uncertainties may cause AHP to result in uncertain ranks for the alternatives (incomparable alternatives). It

is thus desirable that a highly effective approach be developed for dealing with these types of uncertainties. In doing so, this study has undertaken to achieve the following objectives:

- (1) **Recognition of an optimal system:** The main objective of this research is to establish a decision aid model for determining an optimal project delivery system.
- (2) **Treatment of uncertainties in decision-making inputs:** To deal with the uncertainties arising from subjective judgments; in this study, interval assessments are used. An interval-AHP model is established on the basis of Rough Sets Theory (Tanaka 2003). In this model, an optimization procedure is carried out by developing the upper and lower linear programming models. These models will enable the determination of the interval priorities for alternative project delivery systems. These priorities will be compared based on an interval preference relation in order to rank the alternatives.
- (3) **Treatment of uncertainties in decision-making results:** To deal with the possible uncertainties and incomparability in final rankings (model outputs), a multi-level ranking process is designed. In this way, the decision-making process is repeated each time with a new set of criteria in order to finally recognize the ranks of the alternatives.
- (4) **Recognition of a core set of decision criteria:** In order to fully rank the alternatives, in the proposed model the number of criteria are reduced to a set of discernible decision criteria by considering the decision maker's degree of

optimism and proposing a new definition for the interval indiscernibility relation, which is the foundation of criteria reduction.

The remainder of this thesis is structured as follows: Chapter 2 will provide a literature review of project delivery systems and theoretical foundations required to develop the methodology proposed by this research. Chapter 3 will discuss the development of the methodology, which includes the algorithmic model, real world application, summary of assumptions and result analysis. The last chapter will include a summary and appraisal of the proposed methodology, as well as recommendations for future research.

CHAPTER 2 - LITERATURE REVIEW

2.1. Research Background

2.1.1. Project Delivery

In “A Guide to the Project Management Body of Knowledge”, project management is defined as “the application of knowledge, skills, tools, and techniques to project activities in order to meet stakeholder needs and expectations from a project within the given time, budget and scope” (PMBOK 1996).

Project management techniques cover a wide range of knowledge areas, including operations research and artificial intelligence. David B. Leake defines Artificial Intelligence (AI) as “a branch of computer science/engineering that studies the computational requirements for tasks such as perception, reasoning, decision-making and learning, and develops systems to perform those tasks” (Leake 2002). Operation research, on the other hand, is the science of dealing with mathematical or computer models, or other analytical approaches, in order to find better ways of doing things.

One of the areas of decision analysis in project management is determining the most appropriate (optimal) project delivery system with respect to the characteristics of a specific project. In “A Model for Optimizing the Selection of Project Delivery Systems Using AHP” Silva defines a project delivery system as “a contractual structure and compensation arrangement that the project owner uses to acquire a completed facility that meets his/her requirements through the design as well as the construction services of the project” (Silva 2002).

Therefore, an appropriate delivery system is a significant issue in order to ensure the success of a project, and is one of the most important tasks faced by a project owner. No project delivery system is perfect, but based on the unique requirements of each project one particular project delivery system might be better than the others. It is obvious that the decisions made in the early stages of the project's life cycle could have more significant effects on outcome of the project than decisions made in later stages. Proper decisions made early on may help the owner avoid re-bidding and re-designing the construction and therefore may save a considerable amount of cost for the project. Thus, an owner must select an appropriate method for his/her required services after careful evaluation of the project's needs and characteristics.

In this study, four major delivery systems - Design-Bid-Build (DBB), Design-Build (DB), Construction Management at Risk (CM/GC) and Construction Management as Program Management (CM/PM) - are considered. The conventional DBB delivery system has mostly been applied in public systems while private owners have used the DB and CM methods.

Before describing the selection processes and the proposed methodology introduced in this research to find out which delivery system is more fitted to a specific project, it is necessary to understand the characteristics and features of each delivery system in detail. Descriptions of these methods, as well as their advantages and disadvantages, are explained in the next section.

2.1.2. Project Delivery Systems

2.1.2.1. Design-Bid-Build (DBB)

Design-Bid-Build (DBB) is the oldest and most common project delivery method, and can be referred to as the traditional project delivery method. As shown in Figure 2.1, three parties - the owner, the architect/engineer and the general contractor - are involved in the project delivery process. In this method, two separate contracts are required in order to provide design and construction services. One contract is an agreement between a designer and the owner of the project, and deals with the preparation of design documents and specifications. Another contract is made with a contractor, and deals with proper construction of the facility based on documents provided by the design entity. Usually a contractor is selected on the basis of the lowest bid or total contract price.

This method is most appropriate when design documents must be fully developed before the construction phase is started, so that a fixed price for the project is known for the owner. Also, it is appropriate when there is enough time to finish the project sequentially (Schwennsen 2002). This type of delivery system is illustrated in the chart as shown in Figure 2.1 (Osgood 2000).

Figure 2-1. Design-Bid-Build

In order to understand the characteristics of the existing delivery systems that are commonly used in real world applications, the advantages and disadvantages of each delivery system should be taken into consideration. The advantages of Design-Bid-Build (DBB), based on existing investigations, are summarized as follows (Schwennsen 2002, Silva 2002, AIA and AGCA 2004):

- (1) In this method, roles and responsibilities are clear and well documented.
- (2) It is a straightforward process with three sequential phases: design, bid and build.
- (3) Since there is no direct relationship between the design and construction parties, this can result in minimized disputes and conflicts between them.
- (4) Cost is determined based on full design documents and owner is involved in both the design and construction process, so he/she knows what the output will be.

The disadvantages of this method can be summarized as follows (Schwennsen 2002, Silva 2002, AIA and AGCA 2004):

- (1) The price cannot be estimated unless full design documents are available.
- (2) Due to the sequential nature of the process, DBB usually takes more time and is slower than the DB process.
- (3) As design and construction services are provided in two separate contracts, during the design phase there is no input from contractors. Therefore, the contractor's knowledge of construction costs and value engineering is not usefully employed.
- (4) If after the design phase is complete and documents are evaluated by construction experts, re-bidding or redesigning is required, delays and cost overruns might occur.

2.1.2.2. Design-Build (DB)

The Design-Build (DB) system has become popular in both the private and public sectors in recent years. It is an alternative project delivery system that considers both project design and construction under one single contract. For the most part, a DB entity provides design and construction services entirely by its own resources, but in some cases it may require services from outside agencies and subcontractors.

The DB contractor usually is selected either on the basis of qualification or cost. For this purpose, a request for qualifications (RFQ) as well as instructions for bidders and request for proposals (RFP) are developed by the owner and sent to interested contractors. After receiving these documents from them, a short list of qualified DB teams is provided and at the end, an interview is done to select the DB team (Silva 2002, Mahdi and Alreshaid 2005). This type of delivery system can be illustrated in the chart shown in Figure 2.2 (Osgood 2000).

Figure 2-2. Design-Build

The advantages of Design Build based on the research available in the current literature, can be summarized as follows (Dorsey 1997, Schwennsen 2002, Yates 1995):

- (1) Since there is a single party responsible for doing the entire project, it will be easier for the project owner to monitor performance.
- (2) The design and construction processes are integrated in a single contract; therefore, their expertise is combined toward reaching the same goal.
- (3) The possibility of redesign can be minimized because the design has a direct relationship with the construction entity.
- (4) There is a limited ability to change the project definition once the project is started, thus the project is typically done within the defined time and budget.

The disadvantages of this method are summarized as follows (Yates 1995, Schwennsen 2002, Hill 2005):

- (1) Because the owner has little control in the design process, the final product might be different than what he/she expects.
- (2) The DB contractor is generally selected on the basis of qualitative criteria such as experience, qualifications, and best value, which are sometimes difficult to evaluate appropriately.
- (3) Since both design and construction work is awarded at the same time to one single party, the contract is signed before design documents can be fully developed.
- (4) It is possible that there will be conflicts of interest between the design and construction experts, since they are working together.

2.1.2.3. Construction Management at Risk (CM/GC)

Construction Management at Risk is another delivery method in which an Architect/Engineer is selected to provide the design for the project, and a construction manager is selected to act as an advisor and general contractor (GC) on behalf of the project owner. These two are selected separately, as shown in Figure 2.3. Since the risk of constructing the facility is transferred to the construction manager, this method is called construction management at risk. The selection process for this type of project delivery system is similar to the procedure explained in the DB method.

This method is suitable for cases in which multiple prime/general contractors are involved, so the CM is responsible for providing an evaluation of costs and a schedule for these contractors, and also for coordinating the team's efforts and ensuring balances between construction and design in a fast-tracked process, which prevents delay (Mulvey 1998). This type of project delivery is presented in Figure 2.3 (Osgood 2000).

Figure 2-3. Construction Management at Risk

The advantages of Construction Management at Risk are summarized as follows (Silva 2002, Mahdi and Alreshaid 2005):

- (1) It allows for fast tracking, therefore the project duration can be reduced.
- (2) If the fast tracking procedure is done successfully, it is possible to save costs.

- (3) It provides an opportunity to start construction prior to the completion of the design.
- (4) The construction manager provides services like monitoring and coordinating key factors in a project, such as the quality, cost, schedule, etc as an agent of the owner.

The disadvantages of CM/GC are summarized as follows (AAA and APEGGA 2005):

- (1) The roles and responsibilities are not clear and can be confusing.
- (2) The management role of the construction manager can result in added costs for the project.

2.1.2.4. Construction Management as Program Management (CM/PM)

Construction Management as Program Management is a delivery method in which an Architect/Engineer is selected to design the project, a construction manager is selected separately to provide administrative and management services on behalf of the owner, and a general contractor is selected to provide construction services. The construction manager acts like a project manager for the entire process, but does not provide construction services. This type of project delivery is presented in Figure 2.4 (Osgood 2000).

Figure 2-4. Construction Management as Program Management

The advantages of CM/PM can be summarized as follows (Mahdi and Alreshaid 2005):

- (1) The construction manager can provide assistance to the owner during the design phase in order to manage the cost, schedule and constructability.
- (2) By this method it is possible to enjoy the benefits of the fast-tracking approach.
- (3) The owner of the project is responsible for holding all trade contracts, and CM acts as an administrator on behalf of the owner in order to monitor the performance of the work.

The disadvantages of CM/PM are summarized as the followings (Mahdi and Alreshaid 2005):

- (1) Because there are multiple prime contractors – the A/E, CM and GC - involved in this method, the paperwork and administrative time might be increased; in addition, there is potential for disputes and conflicts among these people.
- (2) This method can bring about some confusion regarding roles and responsibilities, being that there is no single point of responsibility.

2.2. Theoretical Foundations

2.2.1. Hierarchical Decision Analysis

Making the proper decision has always been an essential part of human's live. In the science world, modeling decision-making problems has been getting plenty of attention from researchers in the decision analysis area. In the 19th century, Pareto, Von Neumann, Morgenstern and many other researchers began modeling the decision-making problems mathematically. At first, single criterion decision-making problems were studied. It was in 1951 that two research teams, Koopmans and Kuhn & Tucker, introduced the multi-criteria problem (Koopmans 1951, Kuhn & Tucker 1951). In this kind of decision-making problem, the purpose is to find the most appropriate alternative among several alternatives while multiple conflicting criteria are considered (Mateu 2002).

One of the techniques for solving a multi-criteria problem is the Analytic Hierarchy Process (AHP). AHP is a Multi-Criteria Decision-Making (MCDM) technique that allows for the consideration of both objective and subjective factors in selecting the best alternative.

Because in this research AHP is applied in the decision-making procedure, it is necessary to understand its foundations and the steps required in order to obtain the desired results. Therefore, in the next section this technique is discussed in detail.

2.2.1.1. The Analytic Hierarchy Process (AHP)

The Analytic Hierarchy process (AHP) was introduced by Saaty in 1980. The purpose of AHP is to develop a theory and provide a methodology for modeling unstructured problems in different areas. Below, the philosophy behind the components of AHP is described briefly (Bruce et al 1989):

Analytic- AHP is a method in which numbers are used to make decisions. By using mathematics, it is easier to understand or describe better choices to others. Therefore, all methods of describing a decision are analytic, since they must use mathematical/logical reasoning.

Hierarchy- The AHP structures the decision-making problem in levels based on the decision maker's understanding of the situation. By breaking down the problem into levels - goals, criteria, sub-criteria, and alternatives - the decision maker can focus on smaller sets of decisions.

Process- People need time to think about a decision, gather new information, and so on. Therefore, any real decision-making problem involves a process of learning, debating and revising priorities.

Usually, there are three principles in AHP. These principles can be summarized as follows: the first principle is decomposition; decomposition is about breaking down the decision problem into smaller parts in order to simplify the decision-making process. The second principle involves constructing the pair-wise comparison matrices of the elements within a given level of the hierarchical structure with respect to the elements in the next-highest level. The third principle provides a global set of priorities for the elements at the lowest level of the hierarchy, usually alternatives, by synthesizing the local priorities at the various levels of the hierarchy.

The simplest form of AHP is a hierarchy consisting of three levels. Usually, the ultimate goal of the decision is at level one, and is followed by criteria in the second level and alternatives at the third level, in cases where there are three levels; in other cases,

alternatives are at the last level of the hierarchy. If more clarification is required for each criterion, sub-criteria can be added at level 3. A typical AHP hierarchical structure is presented in Figure 2.5 (Silva 2002).

Figure 2-5. A typical AHP hierarchical structure

The number of levels and criteria are determined based on the decision maker's experience and knowledge. The required steps to solve a problem with AHP can be summarized as follows (Saaty and Vargas 1994 and 2001):

- (1) Build the hierarchical structure of the problem by breaking down the general problem into a set of simple criteria.
- (2) Carry out pair-wise comparisons. For this purpose, compare the elements of a level pair-wise according to their strength of influence with the element of the next highest level, and so on for other elements. These comparisons construct a pair-wise comparison matrix. A predetermined scale of relative importance proposed by Saaty is presented in Table 2-1.

Table 2-1. The pair-wise comparison scale

Intensity of importance	Definition	Explanation
1	Equal importance	Two activities contribute equally to the objective
3	Moderate importance	Experience and judgment slightly favour one activity over another
5	Strong importance	Experience and judgment strongly favour one activity over another
7	Very strong or demonstrated importance	An activity is favored very strongly over another; its dominance is demonstrated in practice
9	Extreme importance	The evidence favoring one activity over another is of the highest possible order of affirmation
2, 4, 6, 8	Intermediate values between adjacent scale values	When compromise is needed

The pair-wise comparisons generate a matrix of relative rankings for each level of the hierarchy. In each level, the number indicating the strength of object i compared with object j is denoted by a_{ij} . A then denotes the matrix of these numbers: $A = [a_{ij}]$. A is a reciprocal matrix if $a_{ij} = \frac{1}{a_{ji}}$. If our judgment is perfect in all comparisons, then:

$$a_{ik} = a_{ij} \cdot a_{jk} \quad \text{for all } i, j, k \quad (2.1)$$

Matrix A is considered to be consistent. In a consistent matrix we have:

$$a_{ij} = \frac{w_i}{w_j} \quad \forall i, j = 1, \dots, n \quad (2.2)$$

where w_i , and w_j are the weights for objects i and j .

By placing the Equation (2.2) in Equation (2.1), we have,

$$a_{ij} \cdot a_{jk} = \frac{w_i}{w_j} \cdot \frac{w_j}{w_k} = \frac{w_i}{w_k} = a_{ik} \quad (2.3)$$

Also,

$$a_{ji} = \frac{w_j}{w_i} = \frac{1}{w_i/w_j} = \frac{1}{a_{ij}} \quad (2.4)$$

The number of matrices in an AHP problem depends on the number of elements at each level of the hierarchical structure. To illustrate this step more clearly, let us look at an example. Suppose we have three types of electronic products, A, B and C. We can develop a priority scale of relative reliability for them. Comparisons are made by asking questions such as how significantly more reliable product B is than product C. The answer is placed in position (B, C) in the matrix. The values assigned to fill all of the required spaces are presented in Table 2-2.

Table 2-2. An example of pair-wise comparison

Reliability	A	B	C
A	1	4	3
B	1/4	1	2
C	1/3	1/2	1

For instance, the value (A, B), 4, is formed because A is moderately to strongly more reliable than B. However, for the value of (B, A) the opposite judgment is 1/4. To fill a matrix such as the one above, we need to know three values. The main diagonal is always filled with the value 1, and half of the matrix is filled with the reciprocal values.

After determining pair-wise comparisons for one particular criterion, the values are organized in the form of a matrix. The pair-wise comparison matrix for this example is presented below.

$$\begin{bmatrix} 1 & 4 & 3 \\ 1/4 & 1 & 2 \\ 1/3 & 1/2 & 1 \end{bmatrix}$$

(3) Now that all of the matrices are developed and all the pair-wise comparisons are obtained, Eigen-vectors, i.e. the relative weights of criteria are calculated. There are several ways to find these weights; some of them are summarized as follows (Saaty 1990):

- a) Sum the elements in each row and normalize by dividing each sum by the total of all the sums; the results add up to unity. The first entry of the resulting vector is the priority of the first activity; the second is the priority of the second, and so on. In the example of electronic products mentioned before, the sum of the rows is a column vector; however, here it is written as a row, (8.00, 3.25, 1.833). The total sum of the matrix is 13.083. By dividing each component of the vector by this number we obtain the column vector of priorities (0.611, 0.248, 0.140) for the relative reliability of products - A, B and C - respectively.
- b) Take the sum of the elements in each column and form the reciprocals of these sums. Then, normalize the reciprocals so that these numbers add to unity. By applying this

method in the above example, the sum of the columns of the matrix is a row vector (1.583, 5.50, 6.00). The reciprocals of these sums are (0.632, 0.182, 0.167) and by normalizing them, we have (0.644, 0.185, 0.170).

- c) Divide the elements of each column by the sum of that column and then add the elements in each resulting row. Divide this sum by the number of elements in the row. By applying this method to the above example, the following matrix can be obtained.

$$\begin{bmatrix} 0.632 & 0.727 & 0.5 \\ 0.158 & 0.182 & 0.333 \\ 0.211 & 0.091 & 0.167 \end{bmatrix}$$

The sum of the rows is the column vector; written in row (1.859, 0.673, 0.469) which, when averaged by the sample size of 3 columns, yields the column vector of priorities (0.620, 0.224, 0.156).

- d) Multiply the n elements in each row and then take the n th root. Normalize the resulting numbers. In the above example, applying this method gives (0.630, 0.218, 0.151) as the vector of priorities.

Now, consider this matrix equation:

$$A.x = y \quad (2.5)$$

where $x = (x_1, \dots, x_n)$ and $y = (y_1, \dots, y_n)$ is a shortened notation for the set of equations

$$\sum_{j=1}^n a_{ij}x_j = y_i \quad i = 1, \dots, n \quad (2.6)$$

From Equation (2.2) we have,

$$a_{ij} \frac{w_j}{w_i} = 1 \quad i, j = 1, \dots, n \quad (2.7)$$

$$\therefore \sum_{j=1}^n a_{ij} w_j \frac{1}{w_i} = n \quad i = 1, \dots, n \quad (2.8)$$

$$\text{Or } \sum_{j=1}^n a_{ij} w_j = n w_i \quad i = 1, \dots, n \quad (2.9)$$

which is equal to

$$Aw = nw \quad (2.10)$$

However, it happens only when a_{ij} s are based on exact measurements. In the real world it is not always possible to have precise values of w_i/w_j for Equation (2.7); therefore, the following equation should be considered (Saaty 2001):

$$Aw = \lambda_{\max} w \quad (2.11)$$

where A is the matrix of judgments, w is an Eigen-vector or the relative weights found in Step 3, and $\lambda_{\max}$ is the largest Eigen-value. The Eigen-value is used to quantify the variation of Equation (2.11) from (2.10). Therefore, A is consistent if and only if $\lambda_{\max} = n$. To find $\lambda_{\max}$, consider the following equation:

$$\lambda_{\max}^{(i)} = \sum_{j=1}^n a_{ij} w_j / w_i \quad i = 1, 2, \dots, n \quad (2.12)$$

where by taking the average of all values of $\lambda_{\max}^{(i)}$, the overall $\lambda_{\max}$ is obtained.

To measure the reliability of judgments through the pair-wise comparisons, a consistency index (*C.I.*) should be taken into consideration, as follows (' n ' is the size of matrix):

$$C.I. = (\lambda_{\max} - n)/(n - 1) \quad (2.13)$$

In order to compare the *C.I.* value of a particular matrix with a similar-sized matrix, a Consistency Ratio must be developed, as follows:

$$C.R. = C.I./R.I. \quad (2.14)$$

where *R.I.* is a known Random Consistency Index obtained from a large number of simulation runs, and is different for different size of matrices. Table 2-3 shows the value of the Random Consistency Index (*R.I.*) for matrices of order 1 to 10 obtained by approximating Random Indices using a sample size of 500 different problems (Saaty 2001).

Table 2-3. Average random index (*R.I.*) based on matrix size

Size of matrix (n)	1	2	3	4	5	6	7	8	9	10
Random consistency index	0	0	0.52	0.89	1.11	1.25	1.35	1.40	1.45	1.49

As mentioned, the acceptable *C.R.* range varies according to the size of matrix; for example, it is 0.05 for a 3 by 3 matrix, 0.08 for a 4 by 4 matrix, and 0.1 for all larger matrices, where $n \geq 5$. If the value of *C.R.* is equal to or less than that value, it indicates a good level of consistency in the comparative judgments represented in that matrix.

Inconsistency can be interpreted as required adjustments to improve the consistency of the comparisons, but these adjustments should have reasonable values, neither as large as the judgments themselves, nor so small that using them does not have any meaningful impact on improving the consistency. Therefore, on a scale of zero to one, inconsistency should be one order of magnitude smaller, around 10%. In contrast, if the *C.R.* is more than the acceptable value, an inconsistency of judgments within that matrix has occurred

and the evaluation process should therefore be reviewed, reconsidered, and improved. An acceptable consistency property helps to ensure reliability for the decision maker in determining the priorities of a set of criteria (Saaty and Vargas 1994).

For the above example, consistency calculations are performed as follows. First, we must multiply the original matrix by the relative weights (found using the approach explained in part (a) of Step 3).

$$\begin{bmatrix} 1 & 4 & 3 \\ 1/4 & 1 & 2 \\ 1/3 & 1/2 & 1 \end{bmatrix} \times \begin{bmatrix} 0.611 \\ 0.248 \\ 0.140 \end{bmatrix} = \begin{bmatrix} 2.023 \\ 0.681 \\ 0.468 \end{bmatrix}$$

Then, applying Equation (2.12), we have:

$$\lambda_{\max}^{(i)} = \begin{bmatrix} 3.311 \\ 2.746 \\ 3.343 \end{bmatrix}$$

Next, by taking the average of these values, the value of $\lambda_{\max}$ is determined ($\lambda_{\max}=3.133$). The values for *C.I.* and *C.R.* are computed (the corresponding *R.I.* for a matrix of order 3 is 0.52), as follows:

$$C.I. = (\lambda_{\max} - n)/(n - 1) = (3.133 - 3)/(3 - 1) = 0.066$$

$$C.R. = C.I. / R.I. = 0.066/0.52 = 0.127$$

The value of the *C.R.* is close to the required value of 0.1, thus we are satisfied with the judgments and there is no need for reassessing them.

- (4) To obtain the overall rating for each alternative, the last step involves combining (synthesizing) the judgments. In doing so, the local weights of each level of the

hierarchy will be multiplied from the last level to the first level. Suppose H is a complete hierarchy with h levels, and B_k is the priority matrix of the k th level, $k = 2, \dots, h$. If W' is considered as the priority vector of the p th level compared with some element z in the $(p-1)$ st level, then the priority vector W of the q th level ($p < q$) with respect to z is given by the nonlinear form,

$$W = B_q B_{q-1} \dots B_{p+1} W' \quad (2.15)$$

The global priority vector of the lowest level with respect to the goal is given by,

$$W = B_h B_{h-1} \dots B_2 W' \quad (2.16)$$

In general, $W' = 1$.

2.2.1.2. Revising Judgments

In cases where there is a consistency ratio that is larger than the accepted range, judgments should be revised. Some methods of revising judgments can be summarized as follows (Saaty 1990):

- (1) Construct the matrix of the priority ratios (w_i / w_j) and compute the absolute difference value for each element in the matrix, i.e. $|a_{ij} - (w_i / w_j)|$. For the element with the largest absolute difference, replace a_{ij} with w_i / w_j and recalculate the vector of priorities to see whether $C.R.$ will be improved or not. If it is not improved, continue this procedure to get satisfied result.
- (2) Compute the ratios of a_{ij} to (w_i / w_j) and select the largest one. Then, for that element, replace a_{ij} with w_i / w_j and try to recalculate the vector of priorities to

see whether the *C.R.* will be improved or not. Continue this procedure until you are satisfied.

- (3) Try to improve judgments through experimentation and similar cases and see whether the *C.R.* improves or not.

2.2.1.3. Rationality in AHP

In the analytic hierarchy process, rationality can be defined as follows (Saaty and Vargas 1994):

- For developing a complete structure of relations and influences, the decision maker should have enough knowledge about the problem to carry out considerations and judgments.
- Making a comfortable environment in order to have access to others' knowledge and experience in making judgments.
- In cases where there are several decision makers, being able to make the best compromise, considering differences in opinions.

2.2.1.4. Advantages of AHP

To better understand the reason that AHP has been chosen as the multi-criteria decision-making tool in this research, some of the advantages of using AHP are provided as follows (Saaty 2001):

- AHP will break down complex structures into simpler and more understandable levels.
- It can deal with the interdependence of elements of a system.

- It reflects the natural tendency of the mind to sort elements of a system into different levels.
- It provides a scale for measuring intangibles and a method for establishing priorities.
- AHP leads to an overall estimate of the desirability of each alternative.
- It takes into consideration the relative priorities of factors in a system and enables people to select the best alternatives based on their goals.

One may find more about the advantages of AHP, and in particular advantages regarding this research in section 4.1 (Summary of Research and Achievements).

Finally, a number of reasons for scaling AHP judgments between one and nine can be summarized as follows (Saaty 1990):

- To have meaningful and precise qualitative distinctions in practice, the items being compared need to be of the same order of magnitude or close together with respect to the criterion used in order to make the comparison.
- To judge the relative importance of the elements, usually five linguistic terms - equal, weak, strong, very strong, and absolute - can be good representatives. If greater precision is needed, compromises can be made between adjacent terms. In this case, nine values are required to make the distinctions.

2.2.2. Rough Sets Theory

2.2.2.1. Background

Acquiring knowledge and reasoning under vagueness and incompleteness has recently been attracting much attention from expert system researchers and the machine learning community. Pawlak defines knowledge as “a body of information about some part of reality, which we are interested in” (Pawlak 1991). The reasons for vagueness in knowledge can be the ambiguity of the exact meaning of terms used in the knowledge domain, uncertainty in the data (e.g. due to noise), or uncertainty about the knowledge itself (e.g. due to a doubtful connection between the antecedent and the consequent in an inferred rule). In addition incompleteness can be the result of the uncertainty of data or the incompleteness of human’s knowledge (Ziarko 1991).

There are several mathematical models used for coping with uncertainty. Probability theory is one well-known, conventional approach to handling uncertainties. However, the problem of implementing probability theories is that an estimation of probabilities can be inaccurate when the relative frequencies are unknown. Another approach to dealing with uncertainty is Fuzzy Logic, which was proposed by Zadeh in 1965. In implementing this theory, determining the grade of membership or the value of possibility can be difficult.

The Rough Sets Theory, introduced by Pawlak in 1982, has often proved to be an excellent mathematical tool for the analysis of a vague description of objects. Some advantages of applying this theory when dealing with uncertainty are described as follows (Hu 1995, Cirovic 2001):

- There is no need to have any preliminary or additional information about the data (like probability in statistics, or the grade of membership or value of possibility in Fuzzy logic theory).
- Rough Sets Theory is easy to apply and has simple algorithms.
- It provides a methodology for simplifying and reducing unrelated information in a database.

2.2.2.2. Principles and Concepts of Rough Sets

In this section, the principles and major concepts of Rough Sets Theory are presented in order to help explain their application in the decision aid model proposed by this research. Suppose we are given a finite set $U \neq \phi$, where ϕ is a null set and U is the universe of objects we are interested in. A partition or classification for the universe U is defined as, $X = \{X_1, X_2, \dots, X_n\}$, such that $X_i \subseteq U$, $X_i \neq \phi$, $X_i \cap X_j = \phi$, for $i, j = 1, \dots, n$, $i \neq j$ and $\cup X_i = U$, where $\cup$ and $\cap$ are union and intersection of sets, respectively.

From now on, for the sake of simplicity and because they are mutually interchangeable equivalence relations will often be used instead of classifications. Along with the above assumptions, the basic definition of Rough Sets Theory can be given as the following (Pawlak 1991): suppose R is an equivalence relation, and U/R is the family of all classifications of U . If $\mathbf{R}$ is a family of equivalence relations over U , then $K = (U, \mathbf{R})$ is a knowledge base.

If $P \subseteq R$ and $P \neq \emptyset$, then $\cap P$ (the intersection of all equivalence relations belonging to P), denoted by $IND(P)$, will be called an indiscernibility relation over P .

2.2.2.3. Rough Sets and Approximations

In Rough Sets Theory, to define impreciseness approximately, two concepts, called lower and upper approximations, are employed. The lower approximation consists of all objects which surely belong to the concept; therefore the R -lower approximation of X , $X \subseteq U$, is defined as (Pawlak 1991):

$$\underline{R}X = \cup\{Y \in U / R : Y \subseteq X\}, \text{ where } R \in IND(K) \text{ and } K = (U, R). \quad (2.17)$$

The upper approximation consists of all objects which possibly belong to the concept in question. Therefore the R -upper approximation of X is defined as:

$$\overline{R}X = \cup\{Y \in U / R : Y \cap X \neq \emptyset\} \quad (2.18)$$

The objects which cannot be classified with certainty within the concept belong to the boundary region. The boundary region is the difference between the lower and upper approximations. Therefore, the R -boundary region of X is defined as:

$$BN_R(X) = \overline{R}X - \underline{R}X \quad (2.19)$$

If the boundary region is an empty set, then set X is an exact set and otherwise it is a Rough Set. Furthermore, the positive region is the set of objects which can be classified surely as members of X . The negative region is the set of objects which do not belong to X with certainty. These regions can be presented as follows:

$$R\text{-positive region of } X: POS_R(X) = \underline{R}X \quad (2.20)$$

$$R\text{-negative region of } X: NEG_R(X) = U - \overline{R}X \quad (2.21)$$

It should be pointed out that the greater the boundary region of a set is, the lower the accuracy is. Therefore, in order to find the degree of completeness for our knowledge of set X , the accuracy measure is defined as (Pawlak 1991, Greco et al 2001):

$$\alpha_R(X) = \frac{Card(\underline{RX})}{Card(\overline{RX})}, \text{ where } X \neq \phi \quad (2.22)$$

where ‘*Card*’ denotes the cardinality of the set. It is clear that $0 \leq \alpha_R(X) \leq 1$. If $\alpha_R(X) < 1$, then the R -boundary region of X is non-empty and X is a Rough Set. Likewise, if $\alpha_R(X) = 1$, then the R -boundary region of X is empty, which means the lower and upper approximation is equal; thus, X is an Exact Set.

2.2.2.4. Reduct and Core of Knowledge

Pawlak defines the reduct and core of knowledge in the following way: “A reduct of knowledge is its essential part, which suffices to define all basic concepts occurring in the considered knowledge. The core is its most important part of knowledge, which cannot be eliminated when reducing the knowledge” (Pawlak 1991).

Suppose $\mathbf{R}$ is a family of equivalence relations and $R \in \mathbf{R}$. If $IND(\mathbf{R}) = IND(\mathbf{R} - \{R\})$ then R is dispensable in $\mathbf{R}$; otherwise, R is indispensable in $\mathbf{R}$. Therefore, if each $R \in \mathbf{R}$ is indispensable in $\mathbf{R}$, then the family $\mathbf{R}$ is independent, and otherwise it is dependent. $\mathbf{Q} \subseteq \mathbf{P}$ is a reduct of $\mathbf{P}$ if $\mathbf{Q}$ is independent and $IND(\mathbf{Q}) = IND(\mathbf{P})$, where $\mathbf{P} \subseteq \mathbf{R}$. The set of all indispensable relations in $\mathbf{P}$ will be called the core of $\mathbf{P}$ and can be denoted by $CORE(\mathbf{P})$.

$$CORE(\mathbf{P}) = \cap RED(\mathbf{P}) \quad (2.23)$$

where $RED(\mathbf{P})$ is the family of all reducts of $\mathbf{P}$.

To better understand the reduct and core of knowledge, let us suppose we are given a family $\mathbf{R} = \{P, Q, R\}$ where P , Q , and R are three equivalence relations. Their equivalence classes are presented as the following:

$$U / P = \{\{x_1, x_4\}, \{x_2\}, \{x_3\}, \{x_5, x_6\}\}$$

$$U / Q = \{\{x_1, x_3\}, \{x_6\}, \{x_2, x_4, x_5\}\}$$

$$U / R = \{\{x_1\}, \{x_5, x_6\}, \{x_2\}, \{x_3, x_4\}\}$$

Therefore,

$$U / IND(\mathbf{R}) = \{\{x_1\}, \{x_2\}, \{x_3\}, \{x_4\}, \{x_5\}, \{x_6\}\}$$

$$U / IND(\mathbf{R} - \{P\}) = \{\{x_1\}, \{x_2\}, \{x_3\}, \{x_4\}, \{x_5\}, \{x_6\}\} = U / IND(\mathbf{R})$$

$$U / IND(\mathbf{R} - \{Q\}) = \{\{x_1\}, \{x_2\}, \{x_3\}, \{x_4\}, \{x_5, x_6\}\} \neq U / IND(\mathbf{R})$$

$$U / IND(\mathbf{R} - \{R\}) = \{\{x_1\}, \{x_2\}, \{x_3\}, \{x_4\}, \{x_5\}, \{x_6\}\} = U / IND(\mathbf{R})$$

Thus, only Q is indispensable in $\mathbf{R}$. In order to find the reducts of the family $\mathbf{R}$, we will check if the pairs of relation, P , Q and R , Q , are independent or not. $U / IND(\{P, Q\}) \neq U / IND(P)$ and $U / IND(\{Q, R\}) \neq U / IND(R)$. Therefore, the relations P , Q and R , Q are independent. Consequently, $\{P, Q\}$ and $\{Q, R\}$ are two reducts of $\mathbf{R}$. Moreover, $\{P, Q\} \cap \{Q, R\} = \{Q\}$ is the core of $\mathbf{R}$.

2.2.2.5. Decision Tables

One important class of knowledge representation systems is the decision table. A decision table is a table which determines what decisions should be made once certain

conditions are satisfied. In fact it is a table in which rows are different objects (alternatives) and columns are attributes (factors).

Suppose $S = (U, A)$ is a knowledge representation system, and the attributes of set A are divided into a set of condition attributes ($C \neq \emptyset$) and decision attributes ($D \neq \emptyset$), $C \cup D = A$ and $C \cap D = \emptyset$; such a table is called a decision table, and is denoted by $T = (U, A, C, D)$.

A decision table may also be interpreted as a set of decision rules, because there are functional dependencies between condition and decision attributes. Decision rules are logical statements of the type “if...then...” because they specify that if certain conditions are applied, then certain decisions may be made (Greco et al 2001). A decision table is consistent if all of its decision rules are consistent, i.e., similar conditions should result in the same decision, not different decisions. In other words, $T = (U, A, C, D)$ is consistent if and only if: $C \Rightarrow D$.

2.2.2.6. Simplification of Decision Tables

Simplification and knowledge reduction are done by removing the irrelevant elements of the decision table so that the accuracy of the classification of the table will remain the same. This simplification is particularly important in complex decision making with several evaluation criteria because it provides quality decisions while reducing the number of criteria and even decision alternatives. Therefore the reduct, which is called the relative reduct in the decision table, contains enough information to discern objects in one class from all of the other classes.

To find the reducts in decision tables, three methods are generally used. The first method is the classical data analysis method, or backward eliminations method. In this method, the condition attributes will be removed one by one to see whether there are any changes in the positive region. If there are changes in the positive region, then that attribute cannot be removed.

The second method is the forward selection method, in which condition attributes are added to the core one by one, and the changes to the positive region are observed (Yuguo 2004). Again, if the positive region remains unchanged, then that attribute can be removed.

A third method involves using a Discernibility Matrix to find the reducts. Due to the popularity of this method, it is described in the next section.

2.2.2.7. Discernibility Matrix

One way of representing knowledge is by using discernibility matrix, which was proposed by Skowron and Rauszer in 1992. This form of knowledge representation makes computation of the core, the reduct and so on, much simpler.

Suppose, $S = (U, A)$ is a knowledge representation system with $U = \{x_1, x_2, \dots, x_n\}$. A discernibility matrix is a $n \times n$ matrix denoted by $M(S)$, such that:

$$(c_{ij}) = \{a \in A : a(x_i) \neq a(x_j)\} \quad \forall i, j = 1, 2, \dots, n \quad (2.24)$$

where c_{ij} is the set of all attributes in which the objects x_i and x_j are discernible.

The core can be defined as the set of all single element entries of the discernibility matrix, i.e.,

$$\text{Core}(A) = \{a \in A : c_{ij} = (a), \text{ for some } i, j\} \quad (2.25)$$

The reduct is the minimal subset of attributes that discerns all objects discernible by the whole set of attributes. To illustrate the use of the discernibility matrix, suppose we are given the knowledge representation system shown in Table 2-4.

Table 2-4. Knowledge representation system

U	a	b	c	d
1	1	1	0	2
2	0	2	1	0
3	1	1	1	0
4	2	1	0	2
5	0	1	2	0

The discernibility matrix for this table is presented in Table 2-5:

Table 2-5. Discernibility matrix

	1	2	3	4	5
1	-				
2	<i>a, b, c, d</i>	-			
3	<i>c, d</i>	<i>a, b</i>	-		
4	<i>a</i>	<i>a, c, d</i>	<i>a, c, d</i>	-	
5	<i>a, c, d</i>	<i>b, c</i>	<i>a, c</i>	<i>a, c, d</i>	-

Only half of the elements in the matrix are required, because it is a symmetric matrix.

It is clear from the above table that attribute *a* is the core, and also that $\{a, c\}$ is the only

reduct in the system. Therefore, the minimal discernibility matrix is presented as shown in Table 2-6.

Table 2-6. The minimal discernibility matrix

	1	2	3	4	5
1	-				
2	<i>c</i>	-			
3	<i>c</i>	<i>a</i>	-		
4	<i>a</i>	<i>c</i>	<i>c</i>	-	
5	<i>c</i>	<i>c</i>	<i>c</i>	<i>c</i>	-

From the above table it can be concluded that not all data is necessary in distinguishing objects. Therefore, finding the minimal set of attributes to discern the objects is always of high priority.

2.2.2.8. Rough-Set Based Interval Indiscernibility

As mentioned in previous sections, the philosophy of rough approximations is based on the assumption that for every object in the universe, some information or knowledge exists that can be assigned to that object. Thus, objects that are characterized by the same information are usually indiscernible, i.e., indistinguishable from each other, considering the information available. Furthermore, the patterns or internal structures of a set of condition-decision data records can be easily reduced and extracted as a set of rules without any prior knowledge.

Assume that U and $\mathbf{A}$ are two finite, non-empty sets where U is the universe, and $\mathbf{A}$ is a set of attributes. Every attribute $a \in \mathbf{A}$ is associated with a set of values called the domain of a . In the original approach, given a subset $B \subseteq \mathbf{A}$, a binary (because each object is either discernible or indiscernible) indiscernibility relation $IND(B)$ on U is expressed as follows (Pawlak 1991):

$$IND(B) = \{(x, y) \in U \times U : a(x) = a(y), \forall a \in B\} \quad (2.26)$$

where $a(x)$ and $a(y)$ are the values of the attribute a for elements x and y , respectively.

Previous research has proposed a definition of rough-set based interval indiscernibility in order to find irrelevant criteria (attributes) and remove them from the decision-making process. This definition is described as follows (Alam and Shrabonti 2002): consider the interval set $B = [\underline{b}_1, \overline{b}_1]$. The center and half-width of B can be defined as $B_c = (\overline{b}_1 + \underline{b}_1)/2$ and $B_w = (\overline{b}_1 - \underline{b}_1)/2$, respectively. An Interval Valued Information System (IVIS) is a pair of $S = (U, \mathbf{A})$ where U is a nonempty finite set called the universe, and $\mathbf{A}$ is a finite set of attributes, $\mathbf{A} = \{a\}$, with a domain of $V = \bigcup_{a \in \mathbf{A}} V_a$ where V_a is called the domain of attribute a , and $f : U \times \mathbf{A} \rightarrow V$ is an information function such that the interval $f(x, a) = [\underline{f}_x(a), \overline{f}_x(a)] \in V_a$, for every $a \in \mathbf{A}$ and $x \in U$. Let $S = (U, \mathbf{A})$ be an IVIS, and let $P \subseteq \mathbf{A}$. By $IIND(P)$ (the Interval Indiscernibility Relation), we denote a binary relation defined over U as:

$$(x, y) \in IIND(P) \text{ iff } [\underline{f}_{wx}(a), \overline{f}_{cx}(a)] \cap [\underline{f}_{wy}(a), \overline{f}_{cy}(a)] \neq \emptyset, \forall a \in P \quad (2.27)$$

where $\underline{f}_{wx}$ and $\overline{f}_{cx}$ are, respectively, the half-width and center of the interval-valued information function $f(x, a)$.

It can be seen that the binary relation $IIND(P)$ is an equivalence relation over U . Let $S = (U, A)$ be an IVIS and $P \subseteq A$. An attribute ' a ' is superfluous in P if $IIND(Q)$ is equal to $IIND(P)$, where $Q = P - \{a\}$; otherwise the attribute is indispensable.

2.2.3. Rough-AHP Model

Traditionally, in the multi-criteria decision-making (MCDM) process, the assumption was that there was no uncertainty in the parameters and required values. However, due to the complexity and uncertainties involved in real world decision-making problems and the inherent subjective nature of human judgments, it is sometimes unrealistic and unfeasible to acquire exact values. Cases in which the components are inherently vague are called "uncertain". This uncertainty can be the result of different situations, as described by Mateu in 2002:

- Unquantifiable information: in real world applications, it is sometimes not possible to make judgments using numbers; in this case, linguistic terms such as good, weak can be used. This type of criteria is qualitative.
- Incomplete information: acquiring exact measurements is not always possible because there may be inaccuracies in the equipment used for the measurement.
- Non-obtainable information: in this case, the methodology proposed for carrying out the measurement is not simple and takes a long time to be applied. Instead, approximations of the value are used.
- Partial ignorance: the experts who make the judgments may not always be aware of all the details of the cases, which may result in imprecise data.

In a variety of studies, a number of techniques have been developed to deal with uncertainty in decision analysis. Among these studies are Saaty and Vargas (1987), Arbel (1989, 1991), Zahir (1991), Arbel and Vargas (1992, 1993), Basak (1993), Salo (1993), Moreno-Jimenez and Vargas (1993), Genest and Rivest (1994), Paulson and Zahir (1995), Lipovetsky and Tishler (1999). Also, several attempts have already been made to use the Rough Sets Theory in dealing with uncertainty in multi-criteria decision making (Slowinski 1993, Pawlak and Slowinski 1994).

In order to introduce judgment uncertainties into multiple-criteria decision analysis problems, a new generation of AHP models using interval data called “interval-AHP” has been developed. The most recent approaches in this area are the models that utilize the Rough Sets Theory concepts of lower and upper approximations, which can be summarized as follows (Tanaka 2003, Sugihara et al 2004).

To introduce a Rough-AHP approach, suppose that in a reciprocal matrix of judgment $[A]$, the pair-wise comparisons a_{ij} s are given as interval scales, i.e. $A_{ij} = [\underline{a_{ij}}, \overline{a_{ij}}]$ where $\underline{a_{ij}}$ and $\overline{a_{ij}}$ are the lower and upper bounds of pair-wise comparisons for the element A_{ij} of matrix $[A]$:

$$[A] = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1n} \\ A_{21} & A_{22} & \cdots & A_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ A_{n1} & A_{n2} & \cdots & A_{nn} \end{bmatrix} \quad (2.28)$$

Given the pair-wise comparisons a_{ij} as interval scales, our problem is to find the interval weights $w_i = [\underline{w_i}, \overline{w_i}]$, which can be an approximation for the given interval

matrix $[A]$, where $\underline{w}_i$ and $\overline{w}_i$ are the lower and upper bounds of the estimated interval weights, respectively. The estimated interval matrix can be defined as follows:

$$\forall i, j (i \neq j) \quad W_{ij} = \left[\frac{\underline{w}_i}{\underline{w}_j}, \frac{\overline{w}_i}{\overline{w}_j} \right] \quad (2.29)$$

This range is the maximal range obtained from the estimated interval weights.

Since the interval data is to be considered in this research, two approximations, the lower and the upper, should be considered. These approximations should satisfy the following conditions:

$$\underline{W}_{ij} \subseteq [A_{ij}] \text{ (Lower Approximation),} \quad (2.30)$$

$$\overline{W}_{ij} \supseteq [A_{ij}] \text{ (Upper Approximation),} \quad (2.31)$$

where $\underline{W}_{ij}$ and $\overline{W}_{ij}$ denote the estimations of the lower and upper intervals of pair wise comparison ratios. Now, considering Equation (2.29), the above conditions can be written as follows:

$$\begin{aligned} \underline{W}_{ij} \subseteq [A_{ij}] &\leftrightarrow \left[\frac{\underline{w}_i}{\underline{w}_j}, \frac{\overline{w}_i}{\overline{w}_j} \right] \subseteq [A_{ij}] \leftrightarrow \underline{a}_{ij} \leq \frac{\underline{w}_i}{\underline{w}_j}, \quad \frac{\overline{w}_i}{\overline{w}_j} \leq \overline{a}_{ij} \\ &\leftrightarrow \underline{a}_{ij} \overline{w}_j \leq \underline{w}_i, \quad \overline{a}_{ij} \underline{w}_j \geq \overline{w}_i \end{aligned} \quad (2.32)$$

$$\begin{aligned} \overline{W}_{ij} \supseteq [A_{ij}] &\leftrightarrow \left[\frac{\underline{w}_i}{\underline{w}_j}, \frac{\overline{w}_i}{\overline{w}_j} \right] \supseteq [A_{ij}] \leftrightarrow \frac{\underline{w}_i}{\underline{w}_j} \leq \underline{a}_{ij}, \quad \overline{a}_{ij} \leq \frac{\overline{w}_i}{\overline{w}_j} \\ &\leftrightarrow \underline{a}_{ij} \overline{w}_j \geq \underline{w}_i, \quad \overline{a}_{ij} \underline{w}_j \leq \overline{w}_i \end{aligned} \quad (2.33)$$

Besides the above conditions, the normalization conditions should be satisfied as well. An interval weight vector $(w_1, \dots, w_n)$ is called a normalized vector if and only if the following conditions are satisfied:

$$\sum_i \overline{w_i} - \max_j (\overline{w_j} - \underline{w_j}) \geq 1 \quad (2.34)$$

$$\sum_i \underline{w_i} + \max_j (\overline{w_j} - \underline{w_j}) \leq 1 \quad (2.35)$$

These conditions can be rewritten as follows:

$$\forall j \quad \sum_{i \in \Omega - \{j\}} \overline{w_i} + \underline{w_j} \geq 1 \quad (2.36)$$

$$\forall j \quad \sum_{i \in \Omega - \{j\}} \underline{w_i} + \overline{w_j} \leq 1 \quad (2.37)$$

Now, considering the above conditions and using the concepts of the ‘Greatest Lower Bound’ and ‘Least Upper Bound’, the problem of finding interval weights can be solved using two linear programming models, the lower and upper models. In the lower model, the objective is to maximize the sum of the widths of W_{ij} subject to the conditions shown in the inequality (2.32), normalization and non-negativity conditions. In the upper model, the objective is to minimize the sum of the widths of W_{ij} subject to the conditions shown in the inequality (2.33), normalization and non-negativity conditions. In this sense, the lower and upper models can be summarized as follows:

<Lower Model>

$$\begin{aligned}
\text{Max} \quad & \sum_i (\overline{w_i} - \underline{w_i}) \\
\text{S.t.} \quad & \forall i, j (i \neq j) \quad \overline{a_{ij}} \overline{w_j} \geq \overline{w_i} \\
& \forall i, j (i \neq j) \quad \underline{a_{ij}} \underline{w_j} \leq \underline{w_i} \\
& \forall j \quad \underline{w_j} \geq 1 - \sum_{i \in \Omega - \{j\}} \overline{w_i} \\
& \forall j \quad \overline{w_j} \leq 1 - \sum_{i \in \Omega - \{j\}} \underline{w_i} \\
& \forall i \quad \underline{w_i} \leq \overline{w_i} \\
& \forall i \quad \underline{w_i}, \overline{w_i} \geq 0
\end{aligned} \tag{2.38}$$

<Upper Model>

$$\begin{aligned}
\text{Min} \quad & \sum_i (\overline{w_i} - \underline{w_i}) \\
\text{S.t.} \quad & \forall i, j (i \neq j) \quad \overline{a_{ij}} \underline{w_j} \leq \overline{w_i} \\
& \forall i, j (i \neq j) \quad \underline{a_{ij}} \overline{w_j} \geq \underline{w_i} \\
& \forall j \quad \underline{w_j} \geq 1 - \sum_{i \in \Omega - \{j\}} \overline{w_i} \\
& \forall j \quad \overline{w_j} \leq 1 - \sum_{i \in \Omega - \{j\}} \underline{w_i} \\
& \forall i \quad \underline{w_i} \leq \overline{w_i} \\
& \forall i \quad \underline{w_i}, \overline{w_i} \geq 0
\end{aligned} \tag{2.39}$$

($\Omega = \{1, 2, \dots, n\}$ is the set of considered alternatives)

CHAPTER 3 - A MULTI-CRITERIA MULTI-LEVEL DECISION AID MODEL FOR PROJECT DELIVERY SYSTEM SELECTION UNDER UNCERTAINTY

3.1. Mathematical Model

Assume that $i = 1, 2, \dots, m$ delivery options are available for a given project. In this decision-making problem, $j = 1, 2, \dots, n$ different evaluation criteria are considered for determining which of these delivery choices is the best-suited one for the project (i.e. the one that satisfies these criteria more than the others). Keeping in mind the previously mentioned definitions and theoretical foundations (AHP and Rough-AHP), here an algorithmic decision aid model for selecting this optimal delivery system is proposed. For the sake of clarity, the model is described in sequential steps below.

Step 1 – The alternative project delivery systems should be compared with respect to each individual decision criterion. This will lead to finding the reciprocal pair-wise comparison matrix A_j , which represents the comparison of delivery alternatives with respect to criterion 'j':

$$A_j = [a_j^{ik}] \quad i, k = 1, 2, \dots, m \quad (3.1)$$

where a_j^{ik} represents the pair-wise comparison of the alternatives 'i' and 'k' with respect to decision criterion 'j', which is defined as follows:

$$a_j^{ik} = \frac{\text{Importance of criterion 'j' in delivery system 'i'}}{\text{Importance of criterion 'j' in delivery system 'k'}} \quad (3.2)$$

(Note that $a_j^{ii} = 1$ and $a_j^{ki} = \frac{1}{a_j^{ik}}$)

The above importance values are defined via benchmarks or else on the basis of an expert's judgments. Due to the uncertainties arising from these subjective judgments or variations in the benchmark results, we must introduce a_j^{ik} values to the model via interval numbers. In this sense we have:

$$a_j^{ik} = [\underline{a_j^{ik}}, \overline{a_j^{ik}}] \quad (3.3)$$

Step 2 – The local weights of the project delivery systems with respect to decision criteria should be defined. These interval weights are determined via a modified version of Equations (2.38) and (2.39):

$$\text{Max} \quad \sum_{i=1}^m (U_i^{(j)} - L_i^{(j)}) \quad (3.4)$$

Subject to:

$$\overline{a_j^{ik}} L_k^{(j)} \geq U_i^{(j)} \quad (i, k = 1, 2, \dots, m, i \neq k)$$

$$\underline{a_j^{ik}} U_k^{(j)} \leq L_i^{(j)} \quad (i, k = 1, 2, \dots, m, i \neq k)$$

$$L_k^{(j)} \geq 1 - \sum_{\substack{i=1 \\ i \neq k}}^m U_i^{(j)} \quad (k = 1, 2, \dots, m)$$

$$U_k^{(j)} \leq 1 - \sum_{\substack{i=1 \\ i \neq k}}^m L_i^{(j)} \quad (k = 1, 2, \dots, m)$$

$$L_i^{(j)} \leq U_i^{(j)}$$

$$L_i^{(j)}, U_i^{(j)} \geq 0$$

and

$$\text{Min} \quad \sum_i (U_i^{(j)} - L_i^{(j)}) \quad (3.5)$$

Subject to:

$$\overline{a_j^{ik}} L_k^{(j)} \leq U_i^{(j)} \quad (i, k = 1, 2, \dots, m, i \neq k)$$

$$\underline{a_j^{ik}} U_k^{(j)} \geq L_i^{(j)} \quad (i, k = 1, 2, \dots, m, i \neq k)$$

$$L_k^{(j)} \geq 1 - \sum_{\substack{i=1 \\ i \neq k}}^m U_i^{(j)} \quad (k = 1, 2, \dots, m)$$

$$U_k^{(j)} \leq 1 - \sum_{\substack{i=1 \\ i \neq k}}^m L_i^{(j)} \quad (k = 1, 2, \dots, m)$$

$$L_i^{(j)} \leq U_i^{(j)}$$

$$L_i^{(j)}, U_i^{(j)} \geq 0$$

where $L_i^{(j)}$ and $U_i^{(j)}$ are the lower and upper approximations for the local weight of alternative 'i' with respect to decision criterion 'j', respectively.

On the basis of Equations (3.4) and (3.5), two sets of approximations, $\{(L_i^{(j)}, U_i^{(j)}) | i = 1, 2, \dots, m\}_{\text{lower model}}$ and $\{(L_i^{(j)}, U_i^{(j)}) | i = 1, 2, \dots, m\}_{\text{upper model}}$, are calculated that will form approximations of the local weights with respect to alternatives, $\{w_i^{(j)}\}_{\text{lower model}}$ and $\{w_i^{(j)}\}_{\text{upper model}}$:

$$w_i^{(j)} = [L_i^{(j)}, U_i^{(j)}] \quad (3.6)$$

Step 3 – Now the decision criteria should be compared with respect to the goal of system, which is the success of project. In doing so, a pair-wise comparison matrix will be defined, the elements of which are defined based on the owner's priorities in the decision criteria:

$$X = [x_{jh}] \quad (3.7)$$

where x_{jh} represents the pair-wise comparison of decision criteria 'j' and 'h' with respect to the project success as defined by the project owner and interpreted by the scaling system of Table 2-1 (note that $x_{jj} = 1$ and $x_{hj} = \frac{1}{x_{jh}}$).

Step 4 – The next step is to define the local weights of the decision criteria. These weights are determined through an additive weighting system, as follows:

$$w_j = \frac{\sum_{h=1}^n x_{jh}}{\sum_{j=1}^n \sum_{h=1}^n x_{jh}} \quad (3.8)$$

where w_j is the local weight of decision criterion j .

Step 5 – the global weights of the project delivery options are then calculated by synthesizing the local weights of the delivery options and decision criteria, as follows:

$$W_i = [\underline{W}_i, \overline{W}_i] \quad (i = 1, 2, \dots, m) \quad (3.9)$$

where:

$$\underline{W}_i = \sum_{j=1}^n w_j L_i^{(j)} \quad (3.10)$$

$$\overline{W}_i = \sum_{j=1}^n w_j U_i^{(j)} \quad (3.11)$$

Step 6 – Following these steps, the alternative project delivery systems can be ranked using their global weights to determine the optimal choice. This information can be obtained by using the following definition to compare the global interval weights of the alternatives:

The interval weight $W_i = [\underline{w}_i, \overline{w}_i]$ is preferred to $W_j = [\underline{w}_j, \overline{w}_j]$, i.e. $W_i > W_j$, if and only if:

$$\underline{w}_i \geq \underline{w}_j \quad \text{and} \quad \overline{w}_i \geq \overline{w}_j \quad (3.12)$$

(If $W_i > W_j \Rightarrow$ *Alternative i* is preferred to *Alternative j*)

If the project delivery systems are fully ranked using this definition, the process stops and the alternative with the highest rank will be the optimal system with which to proceed. If the alternatives cannot be fully compared, then Step 7 will be performed.

Step 7 – In this step, the set of decision criteria is reduced to one of its subsets, which has the capability of fully ranking the alternatives. The motivation behind using such a process is the fact that in case there are many condition attributes, it may be impossible to rank the alternatives because the intervals behave differently for different attributes. If we can appropriately remove the superfluous attributes, we may simplify and in some sense “clean” the model. After this process, the construction of the ranking will potentially be easier.

In carrying out this process, as discussed previously, we can use Equation (2.27), that is, a rough-set based interval indiscernibility definition. Based on this definition, two intervals with equal differences between their upper and lower bounds will be placed in the same interval indiscernibility class. For instance, consider the [1,3] and [7,9]

intervals, which have quite different meaning, as far as the importance degree is concerned. The first interval considers a criterion that is not important, and the second one considers the criterion to be important; however based on Equation (2.27), they are considered to be in the same class and are assumed to have intersection. Therefore, this process can result in removing this specific criterion from the decision-making process, which should be avoided.

Considering the above problem, the modification of rough-set based interval indiscernibility (2.27) to select a set of decision criteria capable of fully comparing the alternatives in the decision-making process, by accepting some degrees of optimism, will now be proposed.

Assume as before that U and A are two finite, non-empty sets where U is the universe, and A is a set of $k=1,2,\dots,m$ attributes, now corresponding to the interval values. Consider a set of real-valued intervals in the form of $Int^{(k)} = [a^{(k)}, b^{(k)}]$, which represents the domain of the k -th attribute. Given a subset $B \subseteq A$, a binary indiscernibility relation $I(B)$ on U is expressed for every k -th attribute belonging to B , as follows (Mafakheri et al 2005):

$$x I(B) y \quad \text{if and only if} \quad \alpha_{xy}^{(k)} \leq \alpha \quad (3.13)$$

where $\alpha_{xy}^{(k)}$ is the discernibility ratio of the elements x and y with respect to the k -th attribute, defined as:

$$\alpha_{xy}^{(k)} = \text{Max}(|a_x^{(k)} - a_y^{(k)}|, |b_x^{(k)} - b_y^{(k)}|) / (\bar{b} - \underline{a}) \quad (3.14)$$

and

$$\bar{b} = \underset{k=1,2,\dots,m}{Max} (b^{(k)}) \quad (3.15)$$

$$\underline{a} = \underset{k=1,2,\dots,m}{Min} (a^{(k)}) \quad (3.16)$$

In this sense, $\alpha_{xy}^{(k)}$ shows how much an attribute k is capable of ranking x and y elements. Basically, it shows to what extent the associated values of elements x and y , defined via interval numbers, are discernable based on attribute k . Parameter $\alpha \in [0,1]$ is the degree of optimism that the decision maker (and consequently the owner) takes for this attribute reduction. The larger the value of α is, the more risk-loving (optimistic) the decision maker (DM) is, which means he/she is ignoring bigger differences (between intervals). The extreme cases are $\alpha=0$, where the DM is completely risk-averse (pessimistic) and $\alpha=1$, where the DM is completely optimistic.

The k -th attribute is superfluous in $\mathbf{A}$ for ranking the alternatives if inequality (3.13) holds for every pair of elements (alternatives). A so-called α -level discernibility set of attributes is then defined as:

$$\mathbf{A}_\alpha = \mathbf{A} - \{\alpha\text{-level superfluous attributes}\} \quad (3.17)$$

Once the new set of attributes is defined, the above procedure is repeated, beginning with Step 3. This process will continue (by increasing the value of α) until the alternative ranks are fully determined (stop at Step 6). The decision flow chart of the developed decision aid model is shown in Figure 3.1.

The above algorithmic model has the following advantages:

- This method provides an opportunity to evaluate different project delivery alternatives on the basis of several criteria. These criteria are obtained based on different benchmarks.
- Moreover, it provides a systematic approach to treating uncertainties (arising both from subjective judgments and variations in experts' opinions) by using interval values in the decision-making process. The interval inputs of the model may cause uncertainties in the outputs of the model (alternative ranks), which are defined by comparing the interval global weights of the alternatives. To assist the decision maker in dealing with the incomparability of the alternatives, a complementary procedure is designed to increase the precision of the comparisons by reducing the number of decision criteria to a set of attributes, which can fully compare the alternatives.
- This model also provides an opportunity to recognize the most influential decision criterion for each individual project delivery system selection problem. This innovative attribute recognition simplifies the implemented multi-criteria decision analysis model by reducing the complexity of a problem by reducing the number of criteria, especially in real world case studies involving several decision attributes.
- To define the core set of attributes, a new definition for interval indiscernibility relation was proposed. This definition has an innovative property of linking the attribute recognition procedure to the decision maker's degree of optimism. It replaced the previously proposed definition in literature, which, as mentioned before, has a number of disadvantages, including difficulties in the comparison of intervals with the same length but with no intersection.

- Last, but not least, is the fact that this is the first time that project delivery system selection has been modeled via an algorithmic decision making process which uses an innovative Rough-AHP structure.

Figure 3-1. Project delivery system selection decision aid model

3.2. Real-World Application and Applicability of the Model

In order to fully comprehend the development of the model established, and to demonstrate the applicability of the methodology, a real-world case study of the implementation of the model is necessary. This case study is also used to exhibit the important points meriting attention, as well as the proposed steps needing to be followed when implementing the model developed.

In this study, the construction of two residence buildings (Figure 3.2) at the University of Regina in Saskatchewan, Canada is considered. The number of rooms and other facilities in these buildings are briefly described below. This information has been obtained from a University of Regina request for proposal report in 2001:

The first residence is to have 350 beds in dormitory rooms, with shared washrooms between rooms. Also, a cafeteria with a capacity of 400 people is to be included. Some parts of the residences, such as floors one, two, and parts of a floor below ground will be used for communal and academic uses.

The second residence is to include 350 beds, 200 of which are in four-bedroom suites and 150 are dormitory rooms with shared washrooms between them. Also, a cafeteria with a capacity of 250 people in two different parts is to be included. Just as in the first building, floors one, two and parts of a floor below ground can be used for communal and academic uses.

To select the contractors for this project at the University of Regina, a selection committee will examine each proposal sent by the architects interested in the project. The evaluation of the proposals will be done by the selection committee on the basis of a number of criteria, including the following:

Figure 3-2. The University of Regina residence buildings (Photo courtesy of the University of Regina)

- Availability of the consultant and key personnel
- Consultant's understanding of the project 's scope and objectives
- Consultant's proposed methodology or approach, time estimates, and resource allocation
- Consultant's qualifications and related experience, particularly in the areas of institutional and residence buildings
- Consultant's performance in past projects
- Consultant's references

After the evaluation, the selection committee will provide a short-list of qualified candidates, and these candidates will be asked to attend an interview. The selection committee will announce the final decision, and the selection of the remainder of the design team will start immediately after the decision. Some of architect's responsibilities can be summarized as follows:

- Providing design documents for both buildings
- Controlling the key factors such as cost, schedule, quality and value management
- Assisting in selecting a team that will initially include a Construction Manager, Structural, Mechanical, and Electrical engineers, and members of other professions. The Construction Manager provides costing expertise and construct-ability insights at an early stage.
- Providing contract documents on the basis of the final design documents

The selection method used for the construction of the residence buildings in University of Regina has some weaknesses, including the following:

- There is a bias toward the Construction Management methods as the preferred project delivery system, which may lead to the wrong choice for delivering the project.
- There is reliance on human judgment in assessing the criteria affecting the selection process without treating the uncertainties involved in such decision-making.

Different benchmarks and surveys, introduced by Stillman and Tomlinson in 1998 and Osgood in 2000, have shown that there are numerous factors to be considered in the selection of a delivery system. This research classifies the major factors as follows:

- **Cost:** Achieving the lowest cost is a major concern in project delivery contracts.
- **Schedule:** Time is an important factor for project success. There may need to use a fast-tracking method, i.e. overlapping design and construction phases.
- **Quality:** The desire to maximize the control of aesthetic and physical quality is a major factor to be considered.
- **Complexity:** A project may have a standard, repetitive, or complex and unique design.
- **Scope Change:** Sometimes it is necessary to have some changes in the scope definition of the project.
- **Experience:** To decide which project delivery form of contract is recommended; the availability of the Client Agency staff resources and experience is a major consideration.
- **Value Engineering:** The owner may have additional requirements such as maximizing the operational ROI and/or facility life-cycle value and consequently must decide which of the organization types will best meet those requirements.
- **Financial Guarantee:** The availability of enough funding at an early stage of a project is an important.
- **Risk Management:** Managing to reduce possible risks is also an important factor.
- **Uniqueness:** This addresses non-prototypical project requirements.
- **External Approvals:** Getting the earliest possible approval is something that must be considered.

- **Project Size:** Although this is not one of the major drivers in the selection process, it may still have some significance.
- **Culture:** For instance, bias toward any specific project delivery method can have an impact on the selection process.

For this case study, the criteria that may have an influence on selecting the project delivery system (DBB, DB, CM/GC or CM/PM) are listed in Table 3-1. There are two major benchmarks available in literature for ranking effectiveness of these criteria in project success (Table 3-1).

Table 3-1. Benchmarks for project delivery systems

	Criteria													
	Schedule	Quality	Complexity	Value engineering	External Approvals	Risk	Uniqueness	Culture	Cost	Financial Guarantee	Experience	Scope change	Project Size	Decision
Benchmark 1	2	3	5	6	11	8	12	13	1	9	10	7	4	DBB
	1	3	5	10	11	6	12	13	2	8	7	4	9	DB
	2	3	4	8	11	7	12	13	1	10	6	5	9	CM/GC
	2	1	4	6	11	7	12	13	3	9	8	5	10	CM/PM
Benchmark 2	2	3	4	6	10	8	9	12	1	11	7	5	13	DBB
	1	2	5	4	10	11	6	12	3	8	7	9	13	DB
	2	3	4	7	9	11	10	13	1	6	8	5	12	CM/GC
	4	1	6	8	9	10	7	13	5	11	2	3	12	CM/PM

To perform the decision-making process based on these benchmarks, the importance of each criterion for different alternative delivery systems is interpreted by an interval value (Table 3-2) to be prepared for a three-level interval-AHP model (Figure 3-3). In doing so, the [1,13] interval of ranks is translated to a [1,9] interval of scores (AHP scale) by the following conversion:

$$1 \rightarrow 9$$

$$2 \rightarrow 9 - \frac{9-1}{13-1} = 9 - \frac{8}{12} = \frac{25}{3}$$

$$3 \rightarrow \frac{25}{3} - \frac{8}{12} = \frac{23}{3}$$

$\vdots$

$$13 \rightarrow 1$$

These interval values will form the lowest level of the AHP structure, which is the attainment of the decision criteria by the alternatives.

The pair-wise comparison matrices must now be constructed. At first, the alternative delivery systems should be compared with respect to the attributes. These pair-wise comparison matrices are presented in Tables 3-3 to 3-15. Note that the pair-wise comparison of the $[a, b]$ and $[c, d]$ interval values are also interval numbers $([\frac{a}{d}, \frac{b}{c}]$ for comparing $[a, b]$ with $[c, d]$, and $[\frac{c}{b}, \frac{d}{a}]$ for comparing $[c, d]$ with $[a, b]$).

Figure 3-3. The hierarchical structure for selecting a project delivery system

Table 3-2. Interval scales for the importance of selection criteria for each delivery system

Criteria	DBB	DB	CM/GC	CM/PM
Schedule	[25/3,25/3]	[9,9]	[25/3,25/3]	[7,25/3]
Quality	[23/3,23/3]	[23/3,25/3]	[23/3,23/3]	[9,9]
Complexity	[19/3,7]	[19/3,19/3]	[7,7]	[17/3,7]
Value Engineering	[17/3,17/3]	[3,7]	[13/3,5]	[13/3,17/3]
External Approval	[7/3,3]	[7/3,3]	[7/3,11/3]	[7/3,11/3]
Risk	[13/3,13/3]	[7/3,17/3]	[7/3,5]	[3,5]
Uniqueness	[5/3,11/3]	[5/3,17/3]	[5/3,3]	[5/3,5]
Culture	[1,5/3]	[1,5/3]	[1,1]	[1,1]
Cost	[9,9]	[23/3,25/3]	[9,9]	[19/3,23/3]
Financial Guarantee	[7/3,11/3]	[13/3,13/3]	[3,17/3]	[7/3,11/3]
Experience	[3,5]	[5,5]	[13/3,17/3]	[13/3,25/3]
Scope Change	[5,19/3]	[11/3,7]	[19/3,19/3]	[19/3,23/3]
Size of Project	[1,7]	[1,11/3]	[5/3,11/3]	[5/3,3]

Table 3-3. Pair-wise comparison matrix of alternatives with respect to ‘Schedule’

Alts	DBB	DB	CM/GC	CM/PM
DBB	[1, 1]	[25/27,25/27]	[1,1]	[1,25/21]
DB	[27/25,27/25]	[1, 1]	[27/25,27/25]	[27/25, 9/7]
CM/GC	[1,1]	[25/27,25/27]	[1,1]	[1,25/21]
CM/PM	[21/25,1]	[7/9, 25/27]	[21/25,1]	[1, 1]

Table 3-4. Pair-wise comparison matrix of alternatives with respect to ‘Quality’

Alts	DBB	DB	CM/GC	CM/PM
DBB	[1, 1]	[23/25,1]	[1,1]	[23/27, 23/27]
DB	[1,25/23]	[1, 1]	[1,25/23]	[23/27, 25/27]
CM/GC	[1,1]	[23/25,1]	[1,1]	[23/27, 23/27]
CM/PM	[27/23,27/23]	[27/25, 27/23]	[27/23, 27/23]	[1, 1]

Table 3-5. Pair-wise comparison matrix of alternatives with respect to ‘Complexity’

Alts	DBB	DB	CM/GC	CM/PM
DBB	[1, 1]	[1, 21/19]	[19/21, 1]	[19/21, 21/17]
DB	[19/21, 1]	[1, 1]	[19/21, 19/21]	[19/21,19/17]
CM/GC	[1, 21/19]	[21/19, 21/19]	[1,1]	[1, 21/17]
CM/PM	[17/21, 21/19]	[17/19, 21/19]	[17/21,1]	[1, 1]

Table 3-6. Pair-wise comparison matrix of alternatives with respect to ‘Value Engineering’

Alts	DBB	DB	CM/GC	CM/PM
DBB	[1, 1]	[17/21, 17/9]	[17/15, 17/13]	[1,17/13]
DB	[9/17, 21/17]	[1, 1]	[3/5, 21/13]	[9/17, 21/13]
CM/GC	[13/17,15/17]	[13/21,5/3]	[1,1]	[13/17, 15/13]
CM/PM	[13/17,1]	[13/21, 17/9]	[13/15, 17/13]	[1, 1]

Table 3-7. Pair-wise comparison matrix of alternatives with respect to ‘External Approvals’

Alts	DBB	DB	CM/GC	CM/PM
DBB	[1, 1]	[7/9,9/7]	[7/11, 9/7]	[7/11, 9/7]
DB	[7/9, 9/7]	[1,1]	[7/11, 9/7]	[7/11, 9/7]
CM/GC	[7/9, 11/7]	[7/9, 11/7]	[1, 1]	[7/11,11/7]
CM/PM	[7/9, 11/7]	[7/9, 11/7]	[7/11,11/7]	[1, 1]

Table 3-8. Pair-wise comparison matrix of alternatives with respect to ‘Risk’

Alts	DBB	DB	CM/GC	CM/PM
DBB	[1, 1]	[13/17,13/7]	[13/15, 13/7]	[13/15, 13/9]
DB	[7/13,17/13]	[1, 1]	[7/15, 17/7]	[7/15, 17/9]
CM/GC	[7/13, 15/13]	[7/17, 15/7]	[1,1]	[7/15, 15/9]
CM/PM	[9/13, 15/13]	[9/17, 15/7]	[9/15, 15/7]	[1, 1]

Table 3-9. Pair-wise comparison matrix of alternatives with respect to ‘Uniqueness’

Alts	DBB	DB	CM/GC	CM/PM
DBB	[1, 1]	[5/17, 11/5]	[5/9, 11/5]	[5/15, 11/5]
DB	[5/11, 17/5]	[1, 1]	[5/9, 17/5]	[5/15, 17/5]
CM/GC	[5/11, 9/5]	[5/17, 9/5]	[1, 1]	[5/15, 9/5]
CM/PM	[5/11, 15/5]	[5/17, 15/5]	[5/9, 15/5]	[1, 1]

Table 3-10. Pair-wise comparison matrix of alternatives with respect to ‘Culture’

Alts	DBB	DB	CM/GC	CM/PM
DBB	[1, 1]	[3/5, 5/3]	[1, 5/3]	[1, 5/3]
DB	[3/5, 5/3]	[1, 1]	[1, 5/3]	[1, 5/3]
CM/GC	[3/5, 1]	[3/5, 1/1]	[1, 1]	[1, 1]
CM/PM	[3/5, 1]	[3/5, 1]	[1, 1]	[1, 1]

Table 3-11. Pair-wise comparison matrix of alternatives with respect to ‘Cost’

Alts	DBB	DB	CM/GC	CM/PM
DBB	[1, 1]	[27/25, 27/23]	[1, 1]	[27/23, 27/19]
DB	[23/27, 25/27]	[1, 1]	[23/27, 25/27]	[1, 25/19]
CM/GC	[1, 1]	[27/25, 27/23]	[1, 1]	[27/23, 27/19]
CM/PM	[19/27, 23/27]	[19/25, 1]	[19/27, 23/27]	[1, 1]

Table 3-12. Pair-wise comparison matrix of alternatives with respect to ‘Financial Guarantee’

Alts	DBB	DB	CM/GC	CM/PM
DBB	[1, 1]	[7/13,11/13]	[7/17,11/9]	[7/11,11/7]
DB	[13/11,13/7]	[1, 1]	[13/17,13/9]	[13/11,13/7]
CM/GC	[9/11, 17/7]	[9/13, 17/13]	[1,1]	[9/11, 17/7]
CM/PM	[7/11, 11/7]	[7/13, 11/13]	[7/17, 11/9]	[1, 1]

Table 3-13. Pair-wise comparison matrix of alternatives with respect to ‘Experience’

Alts	DBB	DB	CM/GC	CM/PM
DBB	[1, 1]	[3/5, 5/5]	[9/17, 15/13]	[9/25, 15/13]
DB	[5/5,5/3]	[1, 1]	[15/17, 15/13]	[15/25, 15/13]
CM/GC	[13/15, 17/9]	[13/15, 17/15]	[1,1]	[13/25,17/13]
CM/PM	[13/15, 25/9]	[13/15, 25/15]	[13/17, 25/13]	[1, 1]

Table 3-14. Pair-wise comparison matrix of alternatives with respect to ‘Scope Change’

Alts	DBB	DB	CM/GC	CM/PM
DBB	[1, 1]	[5/7, 19/11]	[15/19,1]	[15/23, 1]
DB	[11/19, 7/5]	[1, 1]	[11/19, 21/19]	[11/23, 21/19]
CM/GC	[1, 19/15]	[19/21,19/11]	[1,1]	[19/23,1]
CM/PM	[1, 23/15]	[19/21,23/11]	[1,23/19]	[1, 1]

Table 3-15. Pair-wise comparison matrix of alternatives with respect to ‘Size of Project’

Alts	DBB	DB	CM/GC	CM/PM
DBB	[1, 1]	[3/11, 7]	[3/11, 21/5]	[1/3, 21/5]
DB	[1/7, 11/3]	[1, 1]	[3/11, 11/5]	[1/3, 11/5]
CM/GC	[5/21, 11/3]	[5/11, 11/3]	[1, 1]	[5/9, 11/5]
CM/PM	[5/21, 3]	[5/11, 3]	[5/11, 9/5]	[1, 1]

Following Step Two, the local interval weights for all of the alternatives are obtained in Table 3-16 by solving the Lower (LM) and Upper (UM) linear programming models 3.4 and 3.5 using LINDO software (LINDO 1999). The reason for using this software is that it has some advantages over other linear programming soft wares, including the following:

- It has a higher speed compared to others (Excel, GAMS, GLPK and CPLEX).
- It is easy to use and suitable for small-scale problems having less than 200 variables/ constraints.

To understand how these models are developed, as an example, a lower model with respect to ‘size’ is explained below (For the sake of clarity and convenience, the rest of the models and the results corresponding to each criterion are shown in Appendix, and can be used by any interested parties).

Considering the linear programming model in (3.4), the objective function is $Max \sum_{i=1}^m (U_i^{(j)} - L_i^{(j)})$; therefore, since four delivery systems are considered in the application, we have:

$$Max U1+U2+U3+U4-L1-L2-L3-L4$$

To develop the first set of constraints, which are greater than or equal to zero,

$\overline{a_j^{ik}} L_k^{(j)} - U_i^{(j)} \geq 0$ ($i, k = 1, 2, \dots, m$, $i \neq k$), the upper bounds of Table 3-15 must be

replaced as coefficients. Therefore, for each row of this table, three constraints are written:

$$7L2 - U1 \geq 0$$

$$21L3 - 5U1 \geq 0$$

$$21L4 - 5U1 \geq 0$$

and for the second row,

$$11L1 - 3U2 \geq 0$$

$$11L3 - 5U2 \geq 0$$

$$11L4 - 5U2 \geq 0$$

and for the third row,

$$11L1 - 3U3 \geq 0$$

$$11L2 - 3U3 \geq 0$$

$$11L4 - 5U3 \geq 0$$

and finally for the last row,

$$3L1 - U4 \geq 0$$

$$3L2 - U4 \geq 0$$

$$9L3 - 5U4 \geq 0$$

The second groups of constraints are the ones which are less than or equal to zero,

$\underline{a_j^{ik}} U_k^{(j)} - L_i^{(j)} \leq 0$ ($i, k = 1, 2, \dots, m$, $i \neq k$). For these constraints, the coefficients are

obtained from the lower bounds of Table 3-15. Therefore for each row of this table, three constraints are written:

$$3U2 - 11L1 \leq 0$$

$$3U3 - 11L1 \leq 0$$

$$U4-3L1 \leq 0$$

for the second row,

$$U1-7L2 \leq 0$$

$$3U3-11L2 \leq 0$$

$$1U4-3L2 \leq 0$$

for the third row,

$$5U1-21L3 \leq 0$$

$$5U2-11L3 \leq 0$$

$$5U4-9L3 \leq 0$$

and for the last row,

$$5U1-21L4 \leq 0$$

$$5U2-11L4 \leq 0$$

$$5U3-11L4 \leq 0$$

The normalization constraints, $L_k^{(j)} \geq 1 - \sum_{i=1}^m U_i^{(j)}$ ($i, k = 1, 2, \dots, m, i \neq k$),

$U_k^{(j)} \leq 1 - \sum_{i=1}^m L_i^{(j)}$ ($i, k = 1, 2, \dots, m, i \neq k$), are written as follows:

$$L1+U2+U3+U4 \geq 1$$

$$L2+U1+U3+U4 \geq 1$$

$$L3+U1+U2+U4 \geq 1$$

$$L4+U1+U2+U3 \geq 1$$

$$U1+L2+L3+L4 \leq 1$$

$$U2+L1+L3+L4 \leq 1$$

$$U3+L1+L2+L4 \leq 1$$

$$U4+L1+L2+L3 \leq 1$$

It also should be emphasized that the lower bound of weights are always smaller than or equal to the upper bounds ($L_i^{(j)} \leq U_i^{(j)}$).

$$L1-U1 \leq 0$$

$$L2-U2 \leq 0$$

$$L3-U3 \leq 0$$

$$L4-U4 \leq 0$$

When using LINDO software it should be noted that for all of the constraints, parameters should be transferred to the left side and numbers to the right side. Moreover, the non-negativity condition does not need to be written because it is the default condition. In this way, the lower model of size is written as follows:

$$\text{Max } U1+U2+U3+U4-L1-L2-L3-L4$$

S.T. $7L2-U1 \geq 0$	$5U1-21L3 \leq 0$
$21L3-5U1 \geq 0$	$5U2-11L3 \leq 0$
$21L4-5U1 \geq 0$	$5U4-9L3 \leq 0$
$11L1-3U2 \geq 0$	$5U1-21L4 \leq 0$
$11L3-5U2 \geq 0$	$5U2-11L4 \leq 0$
$11L4-5U2 \geq 0$	$5U3-11L4 \leq 0$
$11L1-3U3 \geq 0$	$L1+U2+U3+U4 \geq 1$
$11L2-3U3 \geq 0$	$L2+U1+U3+U4 \geq 1$
$11L4-5U3 \geq 0$	$L3+U1+U2+U4 \geq 1$
$3L1-U4 \geq 0$	$L4+U1+U2+U3 \geq 1$
$3L2-U4 \geq 0$	$U1+L2+L3+L4 \leq 1$
$9L3-5U4 \geq 0$	$U2+L1+L3+L4 \leq 1$
$3U2-11L1 \leq 0$	$U3+L1+L2+L4 \leq 1$
$3U3-11L1 \leq 0$	$U4+L1+L2+L3 \leq 1$
$U4-3L1 \leq 0$	$L1-U1 \leq 0$
$U1-7L2 \leq 0$	$L2-U2 \leq 0$
$3U3-11L2 \leq 0$	$L3-U3 \leq 0$
$1U4-3L2 \leq 0$	$L4-U4 \leq 0$

Table 3-16. Local interval weights of alternatives using the LM and UM models

Criteria	Interval weights	LM	UM
S	W_{DBB}	[0.255, 0.255]	[0.245, 0.255]
	W_{DB}	[0.275, 0.275]	[0.265, 0.276]
	$W_{CM/GC}$	[0.255, 0.255]	[0.245, 0.255]
	$W_{CM/PM}$	[0.214, 0.214]	[0.214, 0.245]
Q	W_{DBB}	[0.240, 0.240]	[0.235, 0.240]
	W_{DB}	[0.240, 0.240]	[0.240, 0.255]
	$W_{CM/GC}$	[0.240, 0.240]	[0.235, 0.240]
	$W_{CM/PM}$	[0.281, 0.281]	[0.275, 0.281]
Com	W_{DBB}	[0.244, 0.269]	[0.237, 0.269]
	W_{DB}	[0.244, 0.244]	[0.237, 0.244]
	$W_{CM/GC}$	[0.269, 0.269]	[0.262, 0.269]
	$W_{CM/PM}$	[0.218, 0.244]	[0.218, 0.262]
V	W_{DBB}	[0.293, 0.293]	[0.266, 0.293]
	W_{DB}	[0.155, 0.259]	[0.155, 0.328]
	$W_{CM/GC}$	[0.224, 0.259]	[0.203, 0.259]
	$W_{CM/PM}$	[0.224, 0.293]	[0.203, 0.293]
Ex	W_{DBB}	[0.219, 0.281]	[0.194, 0.250]
	W_{DB}	[0.219, 0.281]	[0.194, 0.250]
	$W_{CM/GC}$	[0.219, 0.344]	[0.194, 0.306]
	$W_{CM/PM}$	[0.219, 0.344]	[0.194, 0.306]

Table 3-16. Local interval weights of alternatives using the LM and UM models
(Continued...)

Criteria	Interval weights	LM	UM
R	W_{DBB}	[0.283, 0.283]	[0.260, 0.260]
	W_{DB}	[0.152, 0.370]	[0.140, 0.340]
	$W_{CM/GC}$	[0.152, 0.326]	[0.140, 0.300]
	$W_{CM/PM}$	[0.196, 0.326]	[0.180, 0.300]
U	W_{DBB}	[0.167, 0.367]	[0.114, 0.275]
	W_{DB}	[0.167, 0.500]	[0.125, 0.386]
	$W_{CM/GC}$	[0.167, 0.300]	[0.114, 0.225]
	$W_{CM/PM}$	[0.167, 0.500]	[0.114, 0.375]
Cu	W_{DBB}	[0.214, 0.357]	[0.214, 0.357]
	W_{DB}	[0.214, 0.357]	[0.214, 0.357]
	$W_{CM/GC}$	[0.214, 0.214]	[0.214, 0.214]
	$W_{CM/PM}$	[0.214, 0.214]	[0.214, 0.214]
C	W_{DBB}	[0.275, 0.275]	[0.270, 0.275]
	W_{DB}	[0.235, 0.255]	[0.230, 0.255]
	$W_{CM/GC}$	[0.275, 0.275]	[0.270, 0.275]
	$W_{CM/PM}$	[0.194, 0.214]	[0.194, 0.230]
FG	W_{DBB}	[0.159, 0.250]	[0.159, 0.250]
	W_{DB}	[0.295, 0.295]	[0.295, 0.295]
	$W_{CM/GC}$	[0.204, 0.386]	[0.205, 0.386]
	$W_{CM/PM}$	[0.159, 0.250]	[0.159, 0.250]

Table 3-16. Local interval weights of alternatives using the LM and UM models

(Continued...)

Criteria	Interval weights	LM	UM
E	W_{DBB}	[0.150, 0.250]	[0.145, 0.250]
	W_{DB}	[0.250, 0.250]	[0.242, 0.250]
	$W_{CM/GC}$	[0.217, 0.283]	[0.210, 0.283]
	$W_{CM/PM}$	[0.217, 0.383]	[0.217, 0.403]
SC	W_{DBB}	[0.208, 0.264]	[0.203, 0.264]
	W_{DB}	[0.153, 0.264]	[0.153, 0.284]
	$W_{CM/GC}$	[0.264, 0.264]	[0.257, 0.264]
	$W_{CM/PM}$	[0.264, 0.319]	[0.257, 0.319]
Siz	W_{DBB}	[0.125, 0.458]	[0.088, 0.360]
	W_{DB}	[0.125, 0.458]	[0.051, 0.323]
	$W_{CM/GC}$	[0.208, 0.458]	[0.086, 0.323]
	$W_{CM/PM}$	[0.208, 0.375]	[0.086, 0.265]

Next, the matrix of pair-wise comparison of the attributes with respect to the top goal, which is the selection of an optimal delivery system, is provided in Table 3-17. To determine the attribute weights, the elements of each row of the pair-wise comparison matrix of attributes are added and then normalized by dividing the sum of the row to the total sum. For this AHP model, the consistency ratio, C.R., is less than 0.1; therefore, we are satisfied with the owner's judgments and there is no need to revise the judgments.

Table 3-17. Pair-wise comparison matrix of attributes with respect to the top goal
(Project success)

Criteria	S	Q	Com	V	Ex	R	U	Cu	C	FG	E	SC	Siz	Weight
S	1	7/5	7/5	7/5	7/3	7/5	7/3	1	1	7/9	7/9	7/5	7/9	0.089
Q	5/7	1	1	1	5/3	1	5/3	5/7	5/7	5/9	5/9	1	5/9	0.063
Com	5/7	1	1	1	5/3	1	5/3	5/7	5/7	5/9	5/9	1	5/9	0.063
V	5/7	1	1	1	5/3	1	5/3	5/7	5/7	5/9	5/9	1	5/9	0.063
Ex	3/7	3/5	3/5	3/5	1	3/5	1	3/7	3/7	3/9	3/9	3/5	3/9	0.038
R	5/7	1	1	1	5/3	1	5/3	5/7	5/7	5/9	5/9	1	5/9	0.063
U	3/7	3/5	3/5	3/5	1	3/5	1	3/7	3/7	3/9	3/9	3/5	3/9	0.038
Cu	1	7/5	7/5	7/5	7/3	7/5	7/3	1	1	7/9	7/9	7/5	7/9	0.089
C	1	7/5	7/5	7/5	7/3	7/5	7/3	1	1	7/9	7/9	7/5	7/9	0.089
FG	9/7	9/5	9/5	9/5	3	9/5	3	9/7	9/7	1	1	9/5	1	0.114
E	9/7	9/5	9/5	9/5	3	9/5	3	9/7	9/7	1	1	9/5	1	0.114
SC	5/7	1	1	1	5/3	1	5/3	5/7	5/7	5/9	5/9	1	5/9	0.063
Siz	9/7	9/5	9/5	9/5	3	9/5	3	9/7	9/7	1	1	9/5	1	0.114

$$\lambda_{\max} = 13.00, \text{ C.I.} = 0, \text{ C.R.} = 0 < 0.1$$

Considering Equations (3.10) and (3.11), the global weights of the delivery system alternatives can now be calculated. For instance, for the delivery system DBB, the lower model weights are calculated as follows:

$$[0.255,0.255]*0.089+[0.240,0.240]*0.063+[0.244,0.269]*0.063+[0.293,0.293]*0.063+[0.219,0.281]*0.038+[0.283,0.283]*0.063+[0.167,0.367]*0.038+[0.214,0.357]*0.089+[0.275,0.275]*0.089+[0.159,0.250]*0.114+[0.150,0.250]*0.114+[0.208,0.264]*0.063+[0.125,0.458]*0.114= [0.210,0.298]$$

Similarly, the global weights of the other delivery systems are obtained in Table 3-18, under the ' $\alpha=0.00$ ' section. From these interval weights, and by following Step 6 (3.12) of the model, the ranking of the alternatives in the lower and upper models can be determined (see Table 3-19, under the ' $\alpha=0.00$ ' section).

As seen in Table 3-19, the final ranking shows that alternatives 2 and 3, and also 1 and 4, are incomparable. It only shows that alternatives 2 and 3 are better than 1 and 4. For further determination of the alternative ranks, Step 7 of the model will be carried out in order to find the discernible set of attributes on the basis of the decision-maker's degree of optimism. The results of this process are also presented in Tables 3-18 and 3.19. For example, for $\alpha=0.10$, the two attributes 'external approval' (Ex) and 'culture' (Cu) are recognized as α -level superfluous attributes. From Table 3-2, we have:

$$I_1^{Ex} = [2.33,3], I_2^{Ex} = [2.33,3], I_3^{Ex} = [2.33,3.67], I_4^{Ex} = [2.33,3.67], I_1^{Cu} = [1,1.67], I_2^{Cu} = [1,1.67], I_3^{Cu} = [1,1], I_4^{Cu} = [1,1]$$

Table 3-18. Interval weights of alternatives (W_i) defined using the LM and UM models

α	Alternatives		LM	UM
0.00	1	DBB	[0.210,0.298]	[0.197,0.280]
	2	DB	[0.215,0.310]	[0.201,0.294]
	3	CM/GC	[0.225,0.305]	[0.203,0.284]
	4	CM/PM	[0.212,0.296]	[0.192,0.283]
0.05	1	DBB	[0.210,0.298]	[0.197,0.280]
	2	DB	[0.215,0.310]	[0.201,0.294]
	3	CM/GC	[0.225,0.305]	[0.203,0.284]
	4	CM/PM	[0.212,0.296]	[0.192,0.283]
0.10	1	DBB	[0.209,0.292]	[0.195,0.274]
	2	DB	[0.215,0.306]	[0.200,0.289]
	3	CM/GC	[0.226,0.312]	[0.202,0.290]
	4	CM/PM	[0.211,0.303]	[0.189,0.290]
0.20	1	DBB	[0.204,0.299]	[0.188,0.277]
	2	DB	[0.210,0.317]	[0.193,0.296]
	3	CM/GC	[0.222,0.322]	[0.194,0.296]
	4	CM/PM	[0.204,0.310]	[0.179,0.293]
0.30	1	DBB	[0.195,0.321]	[0.176,0.291]
	2	DB	[0.186,0.323]	[0.162,0.294]
	3	CM/GC	[0.229,0.319]	[0.189,0.281]
	4	CM/PM	[0.214,0.339]	[0.177,0.311]

Table 3-19. Alternative ranks using the lower and upper models for different α values

α	0.00		0.05		0.10		0.20		0.30	
Models	LM	UM	LM	UM	LM	UM	LM	UM	LM	UM
Superfluous Attributes	-		-		Ex, Cu		Ex, Cu, Q, Com		Ex, Cu, Q, Com, S, R, FG	
Proved orders	3,2 >1,4	3,2 >1,4	3,2 >1,4	3,2 >1,4	3 > 2 >4 > 1	3 > 2 >1,4	3 > 2 >4 > 1	3 > 2 >1,4	4 > 1	4 > 1
Proved Ranks	3,2 > 1,4		3,2 > 1,4		3 > 2 > 1,4		3 > 2 > 1,4		3 > 2 > 4 > 1	

If we consider that $\bar{b} - \underline{a} = 9 - 1 = 8$:

$$\alpha_{12}^{Ex} = 0.00, \alpha_{13}^{Ex} = \frac{0.67}{8} = 0.084, \alpha_{14}^{Ex} = 0.084, \alpha_{23}^{Ex} = 0.084, \alpha_{24}^{Ex} = 0.084, \alpha_{34}^{Ex} = 0.00$$

$$\alpha_{12}^{Cu} = 0.00, \alpha_{13}^{Cu} = 0.084, \alpha_{14}^{Cu} = 0.084, \alpha_{23}^{Cu} = 0.084, \alpha_{24}^{Cu} = 0.084, \alpha_{34}^{Cu} = 0.00$$

All of these quantities are less than $\alpha=0.10$. Therefore, for the ‘external approval’ and ‘culture’ attributes, inequality (3.13) is satisfied for all pairs of alternatives – the attributes are recognized as superfluous criteria. In this way, by increasing the degree of optimism to $\alpha=0.10$ (10%), the priority of alternative 3 (CM/GC) in comparison with alternative 2 (DB) is recognized, and, finally, at $\alpha=0.30$ (30%), we can fully distinguish and rank all of the alternatives (the priority of alternative 4 in comparison to alternative 1 is recognized).

3.3. Summary of Assumptions and Conditions

The assumptions and conditions used in this research can be classified into two main categories: assumptions in the model and assumptions in the case study. The assumptions used to develop the decision aid model can be summarized as follows:

- The parameters used in the upper and lower models are non-negative since they represent the weights of the alternatives.
- The decision-making process allows for the decision maker to accept a certain degree of risk.
- The data used in the lower and upper models are intervals; in case there is crisp data, it should be represented by an interval scale with the same upper and lower bounds.
- For the sake of simplicity, criteria weights are assumed to be crisp values; the process of using them (in synthesizing - Step 5) does not change even when there are interval values for them.
- The hierarchical structure of the problem consists of three levels; the overall goal, criteria and alternatives. It should be noted that each decision criterion could have a number of sub-criteria, which may result in having more levels in the hierarchical structure.
- Since the criteria involved in the decision-making process have trade offs, judgments are provided on the basis of pair-wise comparisons instead of there being absolute judgments.
- The overall weights of the alternatives are obtained indirectly through synthesizing the local weights, and not only by direct judgments because generally the

knowledge about the weights of the decision criteria and alternatives comes from separate and independent sources (for example the owners and the benchmarks).

- There is no bias towards any specific alternative (project delivery system), so that the decision maker has the opportunity to decide based on his/her own optimism.

In addition, the assumptions made in the real-world application can be presented as follows:

- There are four major delivery systems from which to choose.
- The relative importance of the criteria for each delivery system is based on the major available benchmarks.

3.4. Result Analysis

The results of the decision aid model that has been established for the selection of a project delivery system can be analyzed from three points of view:

- Methodological logic:

The results of the proposed decision aid model consist of three major findings: the local weights of the criteria, the local weights of the alternatives, and the reduction of the criteria. The local weights of the criteria are obtained based on a conventional AHP process. On the other hand, the local weights of the alternatives are defined on the basis of a Rough-AHP model, which is the implementation of Rough Sets Theory concepts in the analytic hierarchy process. The more important part for this discussion is the reduction process, which should be confirmed from a practical point of view:

The proposed reduction process works on the basis of the fact that if a decision attribute has the same degree of importance for all decision alternatives, then it does not have a significant role in the ranking of them. In Step 7 of the proposed decision aid model, a mathematical rule has been proposed to represent this fact. Using the decision-maker degree of optimism provides us with an opportunity to determine how much of a difference in criteria importance values can be neglected. Having said that, by increasing the decision-maker degree of optimism, more attributes are recognized as superfluous attributes because the decision maker is accepting more risk in recognition of the superfluous attributes, and neglecting the differences of the criteria importance values. In this sense, at, for instance, 10% degree of optimism, the two attributes, “external approval” and “culture”, are recognized as superfluous attributes. As can be seen in Table 3-2, these two attributes have very close importance values for different alternatives, which show that their differences are considered negligible (based on 3.13) at a 10% degree of optimism.

- Practical logic:

The final ranks obtained from the proposed decision aid model can be confirmed by taking into account the characteristics of the studied project. The construction of the new residence buildings at the University of Regina was restricted in terms of completion deadlines, because the buildings should have been prepared for 2005 Canada Summer Games. Considering this fact, this project is categorized as a fast-tracking project. Experience has shown (see chapter 2, “research background”) that the CM/GC and DB systems are the most appropriate choice of project delivery for these types of projects. However, as stated by the owner of the project, cost was

playing a very important role in decisions regarding the choice of project delivery system. This cost-based decision-making approach will give a higher priority to the CM/GC project delivery system in comparison with the DB system.

- Competitiveness

In addition to the above aspects, it should be mentioned that the proposed methodology is a quite comparable model in the general area of decision analysis. To reveal the competitive advantages of this model, Table 3-20 provides a comparison between the proposed model and the multiple-criteria decision analysis techniques that are more popular in real-world practical decision-making (for more details regarding these methodologies see Hwang and Chen 1992 and Figueira et al 2004):

In order to have a quantitative comparison, the selection of the optimal project delivery system for the presented case study will be double-checked with one of the above-mentioned multiple-criteria decision analysis techniques. For this purpose, the Linear Assignment method is adapted to our case study because it does not require any additional assumption for implementation in the case study, such as target values, and it does not represent biased decision makers. Because the Linear Assignment method works with absolute attainments of alternatives (weights of alternatives with respect to the decision criteria), we still need Steps 1 and 2 of the proposed methodology to obtain the local weights of alternatives from the pair-wise judgements (Table 3-16). The relative weights of the criteria should also be calculated from a pair-wise comparison matrix of decision criteria (Table 3-17). The alternatives are then compared with respect to each criteria based on definition (3.12). The results of this comparison (on the basis of Table 3-16) are provided in Table 3-21. A

criterion weight is assigned to an alternative if that alternative has the first rank with respect to that criterion. In this way, from Table 3-21 we find:

For the lower model:

$$\text{Score (DBB)} = 0.063 (V) + 0.089 (Cu) + 0.089 (C) = 0.241$$

$$\text{Score (DB)} = 0.089 (S) + 0.038 (U) + 0.089 (Cu) = 0.216$$

$$\text{Score (CM/GC)} = 0.063 (Com) + 0.038 (Ex) + 0.089 (C) + 0.114 (Siz) = 0.304$$

$$\text{Score (CM/PM)} = 0.063 (Q) + 0.038 (Ex) + 0.038 (U) + 0.063 (SC) = 0.202$$

For the upper model:

$$\text{Score (DBB)} = 0.089 (Cu) + 0.089 (C) + 0.114 (Siz) = 0.292$$

$$\text{Score (DB)} = 0.089 (S) + 0.038 (U) + 0.089 (Cu) = 0.216$$

$$\text{Score (CM/GC)} = 0.063 (Com) + 0.038 (Ex) + 0.089 (C) = 0.190$$

$$\text{Score (CM/PM)} = 0.063 (Q) + 0.038 (Ex) + 0.063 (SC) = 0.164$$

It can be seen that by applying this method, the highest score for the lower model is assigned to CM/GC and for the upper model is assigned to DBB. Therefore, we cannot get a clear rank for project delivery systems. And, the linear assignment method does not provide any mechanism to treat these uncertainties and does not support any risk-based decisions. Also, in this method if the decision maker needs the ranking of all of the alternatives in order to fully understand his/her choices, this process should be repeated after omitting the best alternative in order to investigate the remaining ones. On the other hand, the proposed decision aid model in this study needs to repeat the process only when there are uncertainties in the final rankings. Moreover, it is able to fully rank the alternatives.

Table 3-20. Competitive advantages and disadvantages of the proposed Rough-AHP based model

Methods	Description	
Simple additive weighting (SAW)	Process	Alternatives are ranked based on the weighted average of their criteria attainments.
	Advantages	- Easy to understand
	Disadvantages	<ul style="list-style-type: none"> - It requires the absolute attainments of alternatives and is not capable of working directly with pair-wise judgements. - In cases where there is interval data, there is a great deal of calculations. - There is no consideration for the decision-maker's risk attitude.
Maxmin	Process	The alternatives are ranked on the basis of the highest minimum criteria attainment.
	Advantages	<ul style="list-style-type: none"> - Easy to understand - Computationally simple.
	Disadvantages	<ul style="list-style-type: none"> - This is a non-compensatory method, which is applicable if there is not any compensation between the criteria attainments or the compensation is not important. - This method is biased, in that it provides a pessimistic decision on the basis of minimum values. - Most real-world decision making based on this method will not consider the global weights of the criteria.

Table 3-20. Competitive advantages and disadvantages of the proposed Rough-AHP based model (continued...)

Methods	Description	
Maxmax	Process	The alternatives are ranked on the basis of the highest maximum criteria attainment.
	Advantages	<ul style="list-style-type: none"> - Easy to understand. - Computationally simple.
	Disadvantages	<ul style="list-style-type: none"> - This is a non-compensatory method, which is applicable if there is not any compensation between the criteria attainments or else if the compensation is not important. - This method is biased, in that it provides an optimistic decision on the basis of maximum values. - Most real-world decision making based on this method will not consider the global weights of the criteria.
Distance to target	Process	The alternatives are ranked on the basis of their weighted distance from a hypothetical target (or ideal) option.
	Advantages	<ul style="list-style-type: none"> - It provides an opportunity for the decision maker to make decisions on the basis of his/her ideals (and his/her risk attitudes)
	Disadvantages	<ul style="list-style-type: none"> - Considering a target is not always possible; in some cases, a target that is proposed might be a controversial issue with a great deal of uncertainty involved.

Table 3-20. Competitive advantages and disadvantages of the proposed Rough-AHP based model (continued...)

Methods	Description	
Linear assignment	Process	The alternatives are ranked on the basis of their criteria-based ranks. In this way, the global weight of a criterion will be assigned to the alternative that gets the highest associated rank. The sum of these assigned weights will be the ranking score of the alternatives.
	Advantages	- Computationally simple for small- scale problems
	Disadvantages	<ul style="list-style-type: none"> - It requires the absolute attainments of the alternatives and is not capable of working directly with pair-wise judgements. - In case where there is interval data or several criteria/ alternatives, the ranking of the alternatives might become a very confusing process for the decision maker. - There is no consideration for the decision maker's risk attitude. - This method is generally used to find the best alternative rather than ranking all of the alternatives

Table 3-21. Linear assignment method

Criterion	Criterion weight	Interval weights	LM	UM
S	0.089	DBB	2	2
		DB	1	1
		CM/GC	2	2
		CM/PM	3	3
Q	0.063	DBB	2	3
		DB	2	2
		CM/GC	2	3
		CM/PM	1	1
Com	0.063	DBB	2	2
		DB	3	3,4
		CM/GC	1	1
		CM/PM	4	3,4
V	0.063	DBB	1	1,2
		DB	4	1,2,3,4
		CM/GC	3	3,4
		CM/PM	2	2,3
Ex	0.038	DBB	2	2
		DB	2	2
		CM/GC	1	1
		CM/PM	1	1

Table 3-21. Linear assignment method (Continued...)

Criterion	Criterion weight	Interval weights	LM	UM
R	0.063	DBB	3,4	3,4
		DB	1,2	1,2
		CM/GC	3,4	3,4
		CM/PM	1,2	1,2
U	0.038	DBB	2	3
		DB	1	1
		CM/GC	3	4
		CM/PM	1	2
Cu	0.089	DBB	1	1
		DB	1	1
		CM/GC	2	2
		CM/PM	2	2
C	0.089	DBB	1	1
		DB	2	2
		CM/GC	1	1
		CM/PM	3	3
FG	0.114	DBB	3	3
		DB	1,2	1,2
		CM/GC	1,2	1,2
		CM/PM	3	3

Table 3-21. Linear assignment method (Continued...)

Criterion	Criterion weight	Interval weights	LM	UM
E	0.114	DBB	4	4
		DB	1,2,3	1,2,3
		CM/GC	2,3	2,3
		CM/PM	1,2	1,2
SC	0.063	DBB	3	3,4
		DB	4	2,3,4
		CM/GC	2	2,3
		CM/PM	1	1
Siz	0.114	DBB	2,3	1
		DB	2,3	3,4
		CM/GC	1	2
		CM/PM	2,3,4	3,4

CHAPTER 4 - CONCLUSIONS

4.1. Summary of Research and Achievements

This research has proposed a decision aid model that will help evaluate different project delivery systems. The proposed model consists of a number of components. It considers the relevant factors involved in making the decision, factors which were determined from a number of benchmarks. The analytic hierarchy process was used as a decision-making tool to analyze the alternatives using a hierarchical structure, with decision criteria at the second level and “project success” at the upper most level.

The AHP hierarchical design and evaluation allows us to readily determine the relative contribution of each of the factors to the final decision. To use the model, the user (owner) needs to assess the relative weights he/she wishes to assign to each factor. Thus, the decision reflects the owner’s needs and preferences.

To deal with uncertainties in judgment, interval values were applied. To obtain the interval weights based on these values, two approximation models, the upper and lower linear programming models, were used.

The pair-wise comparison procedure allowed for the capturing of the relative judgments of two elements at one time in a trustworthy manner, and ensured the consistency of those values (judgments).

The application of this model in a real-world situation has also been provided. The data used in this case study was obtained from a number of benchmarks. The proposed methodology is more comprehensive than the previous work in the field, being that it is able to work with imprecise information. Moreover, the method allows us to increase the

comparability of the alternatives by providing a multi-level decision-making structure by accepting different levels of optimism (i.e. risk-based decision making). At each level, based on the optimism degree, a number of decision attributes were discovered to be superfluous attributes after applying the rough-set based criteria of discernibility between the alternatives.

It can be concluded that this model is capable of effectively facilitating the decision-making process. The findings of this research also provide an understanding of a set of importance factors influencing the selection of a delivery system. Briefly, the major achievements of this study can be listed as follows:

- (1) A Multi-Criteria Multi-Level Decision Support system has been designed for selecting the optimal project delivery system for a given project.
- (2) A set of important factors influencing the selection of a delivery system has been determined.
- (3) A new definition has been provided for interval indiscernibility relation, which will reduce a number of irrelevant criteria by accepting certain levels of risk
- (4) The existing incomparability between the alternatives has been dealt with using the above definition and by introducing an optimism degree in the decision-making process.
- (5) Uncertainties in the decision-making process has been dealt with by providing an algorithm that will use interval values

4.2. Recommendations for Future Research

Opportunities for future studies in developing decision aid models for the selection of the optimal project delivery system might include the following:

- (1) The proposed decision aid model could be utilized as a base for designing an application-oriented web-based decision support system. It could provide an interface for model users and facilitate the calculation procedure.
- (2) The proposed model could be equipped with other uncertainty management tools that are capable of dealing with other forms of uncertainty such as linguistic judgments or possibility measures.
- (3) After the selection of the optimal project delivery system, the proposed model could be used to define the best set of contractors in terms of compatibility with the previously selected system.
- (4) Using the proposed model in real-world case studies will help the decision makers to design rule-based decision-making systems for their own case studies. This will facilitate the evaluation process, because they will only need to run the model once in order to provide their required inference.
- (5) Application in the general area of project delivery. Considering the general concept of a project, which is a temporary endeavor undertaken to create a unique product or service within defined start and end dates, this methodology can be used wherever a decision is needed for outsourcing and delivering project tasks.
- (6) The general outline of such a model can be used with some modifications wherever there is a need for decision-making that will rank several alternative courses of action according to a number of influential criteria. This model is very

helpful in cases where there is interval values derived from expert judgments about the importance and/or attainments of the alternatives.

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APPENDIX - THE LOWER AND UPPER MODELS WITH RESPECT TO EACH OF THE CRITERIA

Appendix A consists of the lower and upper models for all 13 criteria described in this thesis. Each of the models corresponding to a criterion is shown as follows:

<Lower Model with respect to Schedule>

Max $U1+U2+U3+U4-L1-L2-L3-L4$

S.T.

$25L2-27U1 \geq 0$

$L3-U1 \geq 0$

$25L4-21U1 \geq 0$

$27L1-25U2 \geq 0$

$27L3-25U2 \geq 0$

$9L4-7U2 \geq 0$

$L1-U3 \geq 0$

$25L2-27U3 \geq 0$

$25L4-21U3 \geq 0$

$L1-U4 \geq 0$

$25L2-27U4 \geq 0$

$L3-U4 \geq 0$

$25U2-27L1 \leq 0$

$U3-L1 \leq 0$

$U4-L1 \leq 0$

$27U1-25L2 \leq 0$

$27U3-25L2 \leq 0$

$27U4-25L2 \leq 0$

$U1-L3 \leq 0$

$25U2-27L3 \leq 0$

$U4-L3 \leq 0$

$21U1-25L4 \leq 0$

$7U2-9L4 \leq 0$

$21U3-25L4 \leq 0$

$L1+U2+U3+U4 \geq 1$

$L2+U1+U3+U4 \geq 1$

$L3+U1+U2+U4 \geq 1$

$L4+U1+U2+U3 \geq 1$

$U1+L2+L3+L4 \leq 1$

$U2+L1+L3+L4 \leq 1$

$U3+L1+L2+L4 \leq 1$

$U4+L1+L2+L3 \leq 1$

$L1-U1 \leq 0$

$L2-U2 \leq 0$

$L3-U3 \leq 0$

$L4-U4 \leq 0$

END

LP OPTIMUM FOUND AT STEP 17

OBJECTIVE FUNCTION VALUE

1) 0.5551115E-16

VARIABLE	VALUE
U1	0.255102
U2	0.275510
U3	0.255102
U4	0.214286
L1	0.255102
L2	0.275510
L3	0.255102
L4	0.214286

<Upper Model with respect to Schedule>

Min $U1+U2+U3+U4-L1-L2-L3-L4$

S.T.

$25L2-27U1 \leq 0$

$L3-U1 \leq 0$

$25L4-21U1 \leq 0$

$27L1-25U2 \leq 0$

$27L3-25U2 \leq 0$

$9L4-7U2 \leq 0$

$L1-U3 \leq 0$

$25L2-27U3 \leq 0$

$25L4-21U3 \leq 0$

$L1-U4 \leq 0$

$25L2-27U4 \leq 0$

$L3-U4 \leq 0$

$25U2-27L1 \geq 0$

$U3-L1 \geq 0$

$U4-L1 \geq 0$

$27U1-25L2 \geq 0$

$27U3-25L2 \geq 0$

$27U4-25L2 \geq 0$

$U1-L3 \geq 0$

$25U2-27L3 \geq 0$

$U4-L3 \geq 0$

$21U1-25L4 \geq 0$

$7U2-9L4 \geq 0$

$21U3-25L4 \geq 0$

$L1+U2+U3+U4 \geq 1$

$L2+U1+U3+U4 \geq 1$

$L3+U1+U2+U4 \geq 1$

$L4+U1+U2+U3 \geq 1$

$U1+L2+L3+L4 \leq 1$

$U2+L1+L3+L4 \leq 1$

$U3+L1+L2+L4 \leq 1$

$U4+L1+L2+L3 \leq 1$

$L1-U1 \leq 0$

$L2-U2 \leq 0$

$L3-U3 \leq 0$

$L4-U4 \leq 0$

END

LP OPTIMUM FOUND AT STEP 16

OBJECTIVE FUNCTION VALUE

1) 0.6162465E-01

VARIABLE	VALUE
U1	0.255102
U2	0.275510
U3	0.255102
U4	0.245098
L1	0.245098
L2	0.264706
L3	0.245098
L4	0.214286

<Lower Model with respect to Quality>

Max $U1+U2+U3+U4-L1-L2-L3-L4$

S.T.

$L2-U1 \geq 0$

$L3-U1 \geq 0$

$23L4-27U1 \geq 0$

$25L1-23U2 \geq 0$

$25L3-23U2 \geq 0$

$25L4-27U2 \geq 0$

$L1-U3 \geq 0$

$L2-U3 \geq 0$

$23L4-27U3 \geq 0$

$27L1-23U4 \geq 0$

$27L2-23U4 \geq 0$

$27L3-23U4 \geq 0$

$23U2-25L1 \leq 0$

$U3-L1 \leq 0$

$23U4-27L1 \leq 0$

$U1-L2 \leq 0$

$U3-L2 \leq 0$

$23U4-27L2 \leq 0$

$U1-L3 \leq 0$

$23U2-25L3 \leq 0$

$23U4-27L3 \leq 0$

$27U1-23L4 \leq 0$

$27U2-25L4 \leq 0$

$27U3-23L4 \leq 0$

$L1+U2+U3+U4 \geq 1$

$L2+U1+U3+U4 \geq 1$

$L3+U1+U2+U4 \geq 1$

$L4+U1+U2+U3 \geq 1$

$U1+L2+L3+L4 \leq 1$

$U2+L1+L3+L4 \leq 1$

$U3+L1+L2+L4 \leq 1$

$U4+L1+L2+L3 \leq 1$

$L1-U1 \leq 0$

$L2-U2 \leq 0$

$L3-U3 \leq 0$

$L4-U4 \leq 0$

END

LP OPTIMUM FOUND AT STEP 12

OBJECTIVE FUNCTION VALUE

1) -0.5551115E-16

VARIABLE	VALUE
U1	0.239583
U2	0.239583
U3	0.239583
U4	0.281250
L1	0.239583
L2	0.239583
L3	0.239583
L4	0.281250

<Upper Model with respect to Quality>

Min $U1+U2+U3+U4-L1-L2-L3-L4$

S.T.

$L2-U1 \leq 0$

$L3-U1 \leq 0$

$23L4-27U1 \leq 0$

$25L1-23U2 \leq 0$

$25L3-23U2 \leq 0$

$25L4-27U2 \leq 0$

$L1-U3 \leq 0$

$L2-U3 \leq 0$

$23L4-27U3 \leq 0$

$27L1-23U4 \leq 0$

$27L2-23U4 \leq 0$

$27L3-23U4 \leq 0$

$23U2-25L1 \geq 0$

$U3-L1 \geq 0$

$23U4-27L1 \geq 0$

$U1-L2 \geq 0$

$U3-L2 \geq 0$

$23U4-27L2 \geq 0$

$U1-L3 \geq 0$

$23U2-25L3 \geq 0$

$23U4-27L3 \geq 0$

$27U1-23L4 \geq 0$

$27U2-25L4 \geq 0$

$27U3-23L4 \geq 0$

$L1+U2+U3+U4 \geq 1$

$L2+U1+U3+U4 \geq 1$

$L3+U1+U2+U4 \geq 1$

$L4+U1+U2+U3 \geq 1$

$U1+L2+L3+L4 \leq 1$

$U2+L1+L3+L4 \leq 1$

$U3+L1+L2+L4 \leq 1$

$U4+L1+L2+L3 \leq 1$

$L1-U1 \leq 0$

$L2-U2 \leq 0$

$L3-U3 \leq 0$

$L4-U4 \leq 0$

END

LP OPTIMUM FOUND AT STEP 16

OBJECTIVE FUNCTION VALUE

1) 0.3103741E-01

VARIABLE	VALUE
U1	0.239583
U2	0.255102
U3	0.239583
U4	0.281250
L1	0.234694
L2	0.239583
L3	0.234694
L4	0.275510

<Lower Model with respect to Complexity>
Max $U1+U2+U3+U4-L1-L2-L3-L4$

S.T.

$21L2-19U1 \geq 0$

$L3-U1 \geq 0$

$21L4-17U1 \geq 0$

$L1-U2 \geq 0$

$19L3-21U2 \geq 0$

$19L4-17U2 \geq 0$

$21L1-19U3 \geq 0$

$21L2-19U3 \geq 0$

$21L4-17U3 \geq 0$

$21L1-19U4 \geq 0$

$21L2-19U4 \geq 0$

$L3-U4 \geq 0$

$U2-L1 \leq 0$

$19U3-21L1 \leq 0$

$19U4-21L1 \leq 0$

$19U1-21L2 \leq 0$

$19U3-21L2 \leq 0$

$19U4-21L2 \leq 0$

$U1-L3 \leq 0$

$21U2-19L3 \leq 0$

$U4-L3 \leq 0$

$17U1-21L4 \leq 0$

$17U2-19L4 \leq 0$

$17U3-21L4 \leq 0$

$L1+U2+U3+U4 \geq 1$

$L2+U1+U3+U4 \geq 1$

$L3+U1+U2+U4 \geq 1$

$L4+U1+U2+U3 \geq 1$

$U1+L2+L3+L4 \leq 1$

$U2+L1+L3+L4 \leq 1$

$U3+L1+L2+L4 \leq 1$

$U4+L1+L2+L3 \leq 1$

$L1-U1 \leq 0$

$L2-U2 \leq 0$

$L3-U3 \leq 0$

$L4-U4 \leq 0$

END

LP OPTIMUM FOUND AT STEP 22

OBJECTIVE FUNCTION VALUE

1) 0.5128205E-01

VARIABLE	VALUE
U1	0.269231
U2	0.243590
U3	0.269231
U4	0.243590
L1	0.243590
L2	0.243590
L3	0.269231
L4	0.217949

<Upper Model with respect to Complexity>
Min $U1+U2+U3+U4-L1-L2-L3-L4$

S.T.

$21L2-19U1 \leq 0$
 $L3-U1 \leq 0$
 $21L4-17U1 \leq 0$
 $L1-U2 \leq 0$
 $19L3-21U2 \leq 0$
 $19L4-17U2 \leq 0$
 $21L1-19U3 \leq 0$
 $21L2-19U3 \leq 0$
 $21L4-17U3 \leq 0$
 $21L1-19U4 \leq 0$
 $21L2-19U4 \leq 0$
 $L3-U4 \leq 0$
 $U2-L1 \geq 0$
 $19U3-21L1 \geq 0$
 $19U4-21L1 \geq 0$
 $19U1-21L2 \geq 0$
 $19U3-21L2 \geq 0$
 $19U4-21L2 \geq 0$
 $U1-L3 \geq 0$
 $21U2-19L3 \geq 0$
 $U4-L3 \geq 0$
 $17U1-21L4 \geq 0$
 $17U2-19L4 \geq 0$
 $17U3-21L4 \geq 0$
 $L1+U2+U3+U4 \geq 1$
 $L2+U1+U3+U4 \geq 1$
 $L3+U1+U2+U4 \geq 1$
 $L4+U1+U2+U3 \geq 1$
 $U1+L2+L3+L4 \leq 1$
 $U2+L1+L3+L4 \leq 1$
 $U3+L1+L2+L4 \leq 1$
 $U4+L1+L2+L3 \leq 1$
 $L1-U1 \leq 0$
 $L2-U2 \leq 0$
 $L3-U3 \leq 0$
 $L4-U4 \leq 0$
END

LP OPTIMUM FOUND AT STEP 8

OBJECTIVE FUNCTION VALUE

1) 0.8910257E-01

VARIABLE	VALUE
U1	0.269231
U2	0.243590
U3	0.269231
U4	0.262500
L1	0.237500
L2	0.237500
L3	0.262500
L4	0.217949

<Lower Model with respect to Value Engineering>

Max $U1+U2+U3+U4-L1-L2-L3-L4$

S.T.

$17L2-9U1 \geq 0$

$17L3-13U1 \geq 0$

$17L4-13U1 \geq 0$

$21L1-17U2 \geq 0$

$21L3-13U2 \geq 0$

$21L4-13U2 \geq 0$

$15L1-17U3 \geq 0$

$5L2-3U3 \geq 0$

$15L4-13U3 \geq 0$

$L1-U4 \geq 0$

$17L2-9U4 \geq 0$

$17L3-13U4 \geq 0$

$17U2-21L1 \leq 0$

$17U3-15L1 \leq 0$

$U4-L1 \leq 0$

$9U1-17L2 \leq 0$

$3U3-5L2 \leq 0$

$9U4-17L2 \leq 0$

$13U1-17L3 \leq 0$

$13U2-21L3 \leq 0$

$13U4-17L3 \leq 0$

$13U1-17L4 \leq 0$

$13U2-21L4 \leq 0$

$13U3-15L4 \leq 0$

$L1+U2+U3+U4 \geq 1$

$L2+U1+U3+U4 \geq 1$

$L3+U1+U2+U4 \geq 1$

$L4+U1+U2+U3 \geq 1$

$U1+L2+L3+L4 \leq 1$

$U2+L1+L3+L4 \leq 1$

$U3+L1+L2+L4 \leq 1$

$U4+L1+L2+L3 \leq 1$

$L1-U1 \leq 0$

$L2-U2 \leq 0$

$L3-U3 \leq 0$

$L4-U4 \leq 0$

END

LP OPTIMUM FOUND AT STEP 14

OBJECTIVE FUNCTION VALUE

1) 0.2068966

VARIABLE	VALUE
U1	0.293103
U2	0.258621
U3	0.258621
U4	0.293103
L1	0.293103
L2	0.155172
L3	0.224138
L4	0.224138

<Upper Model with respect to Value Engineering>

Min $U1+U2+U3+U4-L1-L2-L3-L4$

S.T.

$17L2-9U1 \leq 0$

$17L3-13U1 \leq 0$

$17L4-13U1 \leq 0$

$21L1-17U2 \leq 0$

$21L3-13U2 \leq 0$

$21L4-13U2 \leq 0$

$15L1-17U3 \leq 0$

$5L2-3U3 \leq 0$

$15L4-13U3 \leq 0$

$L1-U4 \leq 0$

$17L2-9U4 \leq 0$

$17L3-13U4 \leq 0$

$17U2-21L1 \geq 0$

$17U3-15L1 \geq 0$

$U4-L1 \geq 0$

$9U1-17L2 \geq 0$

$3U3-5L2 \geq 0$

$9U4-17L2 \geq 0$

$13U1-17L3 \geq 0$

$13U2-21L3 \geq 0$

$13U4-17L3 \geq 0$

$13U1-17L4 \geq 0$

$13U2-21L4 \geq 0$

$13U3-15L4 \geq 0$

$L1+U2+U3+U4 \geq 1$

$L2+U1+U3+U4 \geq 1$

$L3+U1+U2+U4 \geq 1$

$L4+U1+U2+U3 \geq 1$

$U1+L2+L3+L4 \leq 1$

$U2+L1+L3+L4 \leq 1$

$U3+L1+L2+L4 \leq 1$

$U4+L1+L2+L3 \leq 1$

$L1-U1 \leq 0$

$L2-U2 \leq 0$

$L3-U3 \leq 0$

$L4-U4 \leq 0$

END

LP OPTIMUM FOUND AT STEP 20

OBJECTIVE FUNCTION VALUE

1) 0.3459052

VARIABLE	VALUE
U1	0.293103
U2	0.328125
U3	0.258621
U4	0.293103
L1	0.265625
L2	0.155172
L3	0.203125
L4	0.203125

<Lower Model with respect to External Approval>

Max $U1+U2+U3+U4-L1-L2-L3-L4$

S.T.

$9L2-7U1 \geq 0$

$9L3-7U1 \geq 0$

$9L4-7U1 \geq 0$

$9L1-7U2 \geq 0$

$9L3-7U2 \geq 0$

$9L4-7U2 \geq 0$

$11L1-7U3 \geq 0$

$11L2-7U3 \geq 0$

$11L4-7U3 \geq 0$

$11L1-7U4 \geq 0$

$11L2-7U4 \geq 0$

$11L3-7U4 \geq 0$

$7U2-9L1 \leq 0$

$7U3-11L1 \leq 0$

$7U4-11L1 \leq 0$

$7U1-9L2 \leq 0$

$7U3-11L2 \leq 0$

$7U4-11L2 \leq 0$

$7U1-9L3 \leq 0$

$7U2-9L3 \leq 0$

$7U4-11L3 \leq 0$

$7U1-9L4 \leq 0$

$7U2-9L4 \leq 0$

$7U3-11L4 \leq 0$

$L1+U2+U3+U4 \geq 1$

$L2+U1+U3+U4 \geq 1$

$L3+U1+U2+U4 \geq 1$

$L4+U1+U2+U3 \geq 1$

$U1+L2+L3+L4 \leq 1$

$U2+L1+L3+L4 \leq 1$

$U3+L1+L2+L4 \leq 1$

$U4+L1+L2+L3 \leq 1$

$L1-U1 \leq 0$

$L2-U2 \leq 0$

$L3-U3 \leq 0$

$L4-U4 \leq 0$

END

LP OPTIMUM FOUND AT STEP 16

OBJECTIVE FUNCTION VALUE

1) 0.3750000

VARIABLE	VALUE
U1	0.281250
U2	0.281250
U3	0.343750
U4	0.343750
L1	0.218750
L2	0.218750
L3	0.218750
L4	0.218750

<Upper Model with respect to External Approval>

Min $U1+U2+U3+U4-L1-L2-L3-L4$

S.T.

$9L2-7U1 \leq 0$

$9L3-7U1 \leq 0$

$9L4-7U1 \leq 0$

$9L1-7U2 \leq 0$

$9L3-7U2 \leq 0$

$9L4-7U2 \leq 0$

$11L1-7U3 \leq 0$

$11L2-7U3 \leq 0$

$11L4-7U3 \leq 0$

$11L1-7U4 \leq 0$

$11L2-7U4 \leq 0$

$11L3-7U4 \leq 0$

$7U2-9L1 \geq 0$

$7U3-11L1 \geq 0$

$7U4-11L1 \geq 0$

$7U1-9L2 \geq 0$

$7U3-11L2 \geq 0$

$7U4-11L2 \geq 0$

$7U1-9L3 \geq 0$

$7U2-9L3 \geq 0$

$7U4-11L3 \geq 0$

$7U1-9L4 \geq 0$

$7U2-9L4 \geq 0$

$7U3-11L4 \geq 0$

$L1+U2+U3+U4 \geq 1$

$L2+U1+U3+U4 \geq 1$

$L3+U1+U2+U4 \geq 1$

$L4+U1+U2+U3 \geq 1$

$U1+L2+L3+L4 \leq 1$

$U2+L1+L3+L4 \leq 1$

$U3+L1+L2+L4 \leq 1$

$U4+L1+L2+L3 \leq 1$

$L1-U1 \leq 0$

$L2-U2 \leq 0$

$L3-U3 \leq 0$

$L4-U4 \leq 0$

END

LP OPTIMUM FOUND AT STEP 18

OBJECTIVE FUNCTION VALUE

1) 0.3333333

VARIABLE	VALUE
U1	0.250000
U2	0.250000
U3	0.305556
U4	0.305556
L1	0.194444
L2	0.194444
L3	0.194444
L4	0.194444

<Lower Model with respect to Risk>

Max $U1+U2+U3+U4-L1-L2-L3-L4$

S.T.

$13L2-7U1 \geq 0$

$13L3-7U1 \geq 0$

$13L4-9U1 \geq 0$

$17L1-13U2 \geq 0$

$17L3-7U2 \geq 0$

$17L4-9U2 \geq 0$

$15L1-13U3 \geq 0$

$15L2-7U3 \geq 0$

$5L4-3U3 \geq 0$

$15L1-13U4 \geq 0$

$15L2-7U4 \geq 0$

$15L3-7U4 \geq 0$

$13U2-17L1 \leq 0$

$13U3-15L1 \leq 0$

$13U4-15L1 \leq 0$

$7U1-13L2 \leq 0$

$7U3-15L2 \leq 0$

$7U4-15L2 \leq 0$

$7U1-13L3 \leq 0$

$7U2-17L3 \leq 0$

$7U4-15L3 \leq 0$

$9U1-13L4 \leq 0$

$9U2-17L4 \leq 0$

$3U3-5L4 \leq 0$

$L1+U2+U3+U4 \geq 1$

$L2+U1+U3+U4 \geq 1$

$L3+U1+U2+U4 \geq 1$

$L4+U1+U2+U3 \geq 1$

$U1+L2+L3+L4 \leq 1$

$U2+L1+L3+L4 \leq 1$

$U3+L1+L2+L4 \leq 1$

$U4+L1+L2+L3 \leq 1$

$L1-U1 \leq 0$

$L2-U2 \leq 0$

$L3-U3 \leq 0$

$L4-U4 \leq 0$

END

LP OPTIMUM FOUND AT STEP 17

OBJECTIVE FUNCTION VALUE

1) 0.5217391

VARIABLE	VALUE
U1	0.282609
U2	0.369565
U3	0.326087
U4	0.326087
L1	0.282609
L2	0.152174
L3	0.152174
L4	0.195652

<Upper Model with respect to Risk>

Min $U1+U2+U3+U4-L1-L2-L3-L4$

S.T.

$13L2-7U1 \leq 0$

$13L3-7U1 \leq 0$

$13L4-9U1 \leq 0$

$17L1-13U2 \leq 0$

$17L3-7U2 \leq 0$

$17L4-9U2 \leq 0$

$15L1-13U3 \leq 0$

$15L2-7U3 \leq 0$

$5L4-3U3 \leq 0$

$15L1-13U4 \leq 0$

$15L2-7U4 \leq 0$

$15L3-7U4 \leq 0$

$13U2-17L1 \geq 0$

$13U3-15L1 \geq 0$

$13U4-15L1 \geq 0$

$7U1-13L2 \geq 0$

$7U3-15L2 \geq 0$

$7U4-15L2 \geq 0$

$7U1-13L3 \geq 0$

$7U2-17L3 \geq 0$

$7U4-15L3 \geq 0$

$9U1-13L4 \geq 0$

$9U2-17L4 \geq 0$

$3U3-5L4 \geq 0$

$L1+U2+U3+U4 \geq 1$

$L2+U1+U3+U4 \geq 1$

$L3+U1+U2+U4 \geq 1$

$L4+U1+U2+U3 \geq 1$

$U1+L2+L3+L4 \leq 1$

$U2+L1+L3+L4 \leq 1$

$U3+L1+L2+L4 \leq 1$

$U4+L1+L2+L3 \leq 1$

$L1-U1 \leq 0$

$L2-U2 \leq 0$

$L3-U3 \leq 0$

$L4-U4 \leq 0$

END

LP OPTIMUM FOUND AT STEP 18

OBJECTIVE FUNCTION VALUE

1) 0.4800000

VARIABLE	VALUE
U1	0.260000
U2	0.340000
U3	0.300000
U4	0.300000
L1	0.260000
L2	0.140000
L3	0.140000
L4	0.180000

<Lower Model with respect to Uniqueness>

Max $U1+U2+U3+U4-L1-L2-L3-L4$

S.T.

$11L2-5U1 \geq 0$

$11L3-5U1 \geq 0$

$11L4-5U1 \geq 0$

$17L1-5U2 \geq 0$

$17L3-5U2 \geq 0$

$17L4-5U2 \geq 0$

$9L1-5U3 \geq 0$

$9L2-5U3 \geq 0$

$9L4-5U3 \geq 0$

$15L1-5U4 \geq 0$

$15L2-5U4 \geq 0$

$15L3-5U4 \geq 0$

$5U2-17L1 \leq 0$

$5U3-9L1 \leq 0$

$5U4-15L1 \leq 0$

$5U1-11L2 \leq 0$

$5U3-9L2 \leq 0$

$5U4-15L2 \leq 0$

$5U1-11L3 \leq 0$

$5U2-17L3 \leq 0$

$5U4-15L3 \leq 0$

$5U1-11L4 \leq 0$

$5U2-17L4 \leq 0$

$5U3-9L4 \leq 0$

$L1+U2+U3+U4 \geq 1$

$L2+U1+U3+U4 \geq 1$

$L3+U1+U2+U4 \geq 1$

$L4+U1+U2+U3 \geq 1$

$U1+L2+L3+L4 \leq 1$

$U2+L1+L3+L4 \leq 1$

$U3+L1+L2+L4 \leq 1$

$U4+L1+L2+L3 \leq 1$

$L1-U1 \leq 0$

$L2-U2 \leq 0$

$L3-U3 \leq 0$

$L4-U4 \leq 0$

END

LP OPTIMUM FOUND AT STEP 18

OBJECTIVE FUNCTION VALUE

1) 1.000000

VARIABLE	VALUE
U1	0.366667
U2	0.500000
U3	0.300000
U4	0.500000
L1	0.166667
L2	0.166667
L3	0.166667
L4	0.166667

<Upper Model with respect to Uniqueness>

Min $U1+U2+U3+U4-L1-L2-L3-L4$

S.T.

$11L2-5U1 \leq 0$

$11L3-5U1 \leq 0$

$11L4-5U1 \leq 0$

$17L1-5U2 \leq 0$

$17L3-5U2 \leq 0$

$17L4-5U2 \leq 0$

$9L1-5U3 \leq 0$

$9L2-5U3 \leq 0$

$9L4-5U3 \leq 0$

$15L1-5U4 \leq 0$

$15L2-5U4 \leq 0$

$15L3-5U4 \leq 0$

$5U2-17L1 \geq 0$

$5U3-9L1 \geq 0$

$5U4-15L1 \geq 0$

$5U1-11L2 \geq 0$

$5U3-9L2 \geq 0$

$5U4-15L2 \geq 0$

$5U1-11L3 \geq 0$

$5U2-17L3 \geq 0$

$5U4-15L3 \geq 0$

$5U1-11L4 \geq 0$

$5U2-17L4 \geq 0$

$5U3-9L4 \geq 0$

$L1+U2+U3+U4 \geq 1$

$L2+U1+U3+U4 \geq 1$

$L3+U1+U2+U4 \geq 1$

$L4+U1+U2+U3 \geq 1$

$U1+L2+L3+L4 \leq 1$

$U2+L1+L3+L4 \leq 1$

$U3+L1+L2+L4 \leq 1$

$U4+L1+L2+L3 \leq 1$

$L1-U1 \leq 0$

$L2-U2 \leq 0$

$L3-U3 \leq 0$

$L4-U4 \leq 0$

END

LP OPTIMUM FOUND AT STEP 21

OBJECTIVE FUNCTION VALUE

1) 0.7954546

VARIABLE	VALUE
U1	0.275000
U2	0.386364
U3	0.225000
U4	0.375000
L1	0.113636
L2	0.125000
L3	0.113636
L4	0.113636

<Lower Model (with respect to Culture)>

Max $U1+U2+U3+U4-L1-L2-L3-L4$

S.T.

$5L2-3U1 \geq 0$

$5L3-3U1 \geq 0$

$5L4-3U1 \geq 0$

$5L1-3U2 \geq 0$

$5L3-3U2 \geq 0$

$5L4-3U2 \geq 0$

$L1-U3 \geq 0$

$L2-U3 \geq 0$

$L4-U3 \geq 0$

$L1-U4 \geq 0$

$L2-U4 \geq 0$

$L3-U4 \geq 0$

$3U2-5L1 \leq 0$

$U3-L1 \leq 0$

$U4-L1 \leq 0$

$3U1-5L2 \leq 0$

$U3-L2 \leq 0$

$U4-L2 \leq 0$

$3U1-5L3 \leq 0$

$3U2-5L3 \leq 0$

$U4-L3 \leq 0$

$3U1-5L4 \leq 0$

$3U2-5L4 \leq 0$

$U3-L4 \leq 0$

$L1+U2+U3+U4 \geq 1$

$L2+U1+U3+U4 \geq 1$

$L3+U1+U2+U4 \geq 1$

$L4+U1+U2+U3 \geq 1$

$U1+L2+L3+L4 \leq 1$

$U2+L1+L3+L4 \leq 1$

$U3+L1+L2+L4 \leq 1$

$U4+L1+L2+L3 \leq 1$

$L1-U1 \leq 0$

$L2-U2 \leq 0$

$L3-U3 \leq 0$

$L4-U4 \leq 0$

END

LP OPTIMUM FOUND AT STEP 11

OBJECTIVE FUNCTION VALUE

1) 0.2857143

VARIABLE	VALUE
U1	0.357143
U2	0.357143
U3	0.214286
U4	0.214286
L1	0.214286
L2	0.214286
L3	0.214286
L4	0.214286

<Upper Model (with respect to Culture)>

Min $U1+U2+U3+U4-L1-L2-L3-L4$

ST

$5L2-3U1 \leq 0$

$5L3-3U1 \leq 0$

$5L4-3U1 \leq 0$

$5L1-3U2 \leq 0$

$5L3-3U2 \leq 0$

$5L4-3U2 \leq 0$

$L1-U3 \leq 0$

$L2-U3 \leq 0$

$L4-U3 \leq 0$

$L1-U4 \leq 0$

$L2-U4 \leq 0$

$L3-U4 \leq 0$

$3U2-5L1 \geq 0$

$U3-L1 \geq 0$

$U4-L1 \geq 0$

$3U1-5L2 \geq 0$

$U3-L2 \geq 0$

$U4-L2 \geq 0$

$3U1-5L3 \geq 0$

$3U2-5L3 \geq 0$

$U4-L3 \geq 0$

$3U1-5L4 \geq 0$

$3U2-5L4 \geq 0$

$U3-L4 \geq 0$

$L1+U2+U3+U4 \geq 1$

$L2+U1+U3+U4 \geq 1$

$L3+U1+U2+U4 \geq 1$

$L4+U1+U2+U3 \geq 1$

$U1+L2+L3+L4 \leq 1$

$U2+L1+L3+L4 \leq 1$

$U3+L1+L2+L4 \leq 1$

$U4+L1+L2+L3 \leq 1$

$L1-U1 \leq 0$

$L2-U2 \leq 0$

$L3-U3 \leq 0$

$L4-U4 \leq 0$

END

LP OPTIMUM FOUND AT STEP 17

OBJECTIVE FUNCTION VALUE

1) 0.2857143

VARIABLE	VALUE
U1	0.357143
U2	0.357143
U3	0.214286
U4	0.214286
L1	0.214286
L2	0.214286
L3	0.214286
L4	0.214286

<Lower Model with respect to Cost>

Max $U1+U2+U3+U4-L1-L2-L3-L4$

S.T.

$27L2-23U1 \geq 0$

$L3-U1 \geq 0$

$27L4-19U1 \geq 0$

$25L1-27U2 \geq 0$

$25L3-27U2 \geq 0$

$25L4-19U2 \geq 0$

$L1-U3 \geq 0$

$27L2-23U3 \geq 0$

$27L4-19U3 \geq 0$

$23L1-27U4 \geq 0$

$L2-U4 \geq 0$

$23L3-27U4 \geq 0$

$27U2-25L1 \leq 0$

$U3-L1 \leq 0$

$27U4-23L1 \leq 0$

$23U1-27L2 \leq 0$

$23U3-27L2 \leq 0$

$U4-L2 \leq 0$

$U1-L3 \leq 0$

$27U2-25L3 \leq 0$

$27U4-23L3 \leq 0$

$19U1-27L4 \leq 0$

$19U2-25L4 \leq 0$

$19U3-27L4 \leq 0$

$L1+U2+U3+U4 \geq 1$

$L2+U1+U3+U4 \geq 1$

$L3+U1+U2+U4 \geq 1$

$L4+U1+U2+U3 \geq 1$

$U1+L2+L3+L4 \leq 1$

$U2+L1+L3+L4 \leq 1$

$U3+L1+L2+L4 \leq 1$

$U4+L1+L2+L3 \leq 1$

$L1-U1 \leq 0$

$L2-U2 \leq 0$

$L3-U3 \leq 0$

$L4-U4 \leq 0$

END

LP OPTIMUM FOUND AT STEP 8

OBJECTIVE FUNCTION VALUE

1) 0.4081633E-01

VARIABLE	VALUE
U1	0.275510
U2	0.255102
U3	0.275510
U4	0.214286
L1	0.275510
L2	0.234694
L3	0.275510
L4	0.193878

<Upper Model with respect to Cost>

Min $U1+U2+U3+U4-L1-L2-L3-L4$

S.T.

$27L2-23U1 \leq 0$

$L3-U1 \leq 0$

$27L4-19U1 \leq 0$

$25L1-27U2 \leq 0$

$25L3-27U2 \leq 0$

$25L4-19U2 \leq 0$

$L1-U3 \leq 0$

$27L2-23U3 \leq 0$

$27L4-19U3 \leq 0$

$23L1-27U4 \leq 0$

$L2-U4 \leq 0$

$23L3-27U4 \leq 0$

$27U2-25L1 \geq 0$

$U3-L1 \geq 0$

$27U4-23L1 \geq 0$

$23U1-27L2 \geq 0$

$23U3-27L2 \geq 0$

$U4-L2 \geq 0$

$U1-L3 \geq 0$

$27U2-25L3 \geq 0$

$27U4-23L3 \geq 0$

$19U1-27L4 \geq 0$

$19U2-25L4 \geq 0$

$19U3-27L4 \geq 0$

$L1+U2+U3+U4 \geq 1$

$L2+U1+U3+U4 \geq 1$

$L3+U1+U2+U4 \geq 1$

$L4+U1+U2+U3 \geq 1$

$U1+L2+L3+L4 \leq 1$

$U2+L1+L3+L4 \leq 1$

$U3+L1+L2+L4 \leq 1$

$U4+L1+L2+L3 \leq 1$

$L1-U1 \leq 0$

$L2-U2 \leq 0$

$L3-U3 \leq 0$

$L4-U4 \leq 0$

END

LP OPTIMUM FOUND AT STEP 9

OBJECTIVE FUNCTION VALUE

1) 0.7224490E-01

VARIABLE	VALUE
U1	0.275510
U2	0.255102
U3	0.275510
U4	0.230000
L1	0.270000
L2	0.230000
L3	0.270000
L4	0.193878

<Lower Model (with respect to Financial Guarantee)>

Max $U1+U2+U3+U4-L1-L2-L3-L4$

S.T.

$11L2-13U1 \geq 0$

$11L3-9U1 \geq 0$

$11L4-7U1 \geq 0$

$13L1-7U2 \geq 0$

$13L3-9U2 \geq 0$

$13L4-7U2 \geq 0$

$17L1-7U3 \geq 0$

$17L2-13U3 \geq 0$

$17L4-7U3 \geq 0$

$11L1-7U4 \geq 0$

$11L2-13U4 \geq 0$

$11L3-9U4 \geq 0$

$7U2-13L1 \leq 0$

$7U3-17L1 \leq 0$

$7U4-11L1 \leq 0$

$13U1-11L2 \leq 0$

$13U3-17L2 \leq 0$

$13U4-11L2 \leq 0$

$9U1-11L3 \leq 0$

$9U2-13L3 \leq 0$

$9U4-11L3 \leq 0$

$7U1-11L4 \leq 0$

$7U2-13L4 \leq 0$

$7U3-17L4 \leq 0$

$L1+U2+U3+U4 \geq 1$

$L2+U1+U3+U4 \geq 1$

$L3+U1+U2+U4 \geq 1$

$L4+U1+U2+U3 \geq 1$

$U1+L2+L3+L4 \leq 1$

$U2+L1+L3+L4 \leq 1$

$U3+L1+L2+L4 \leq 1$

$U4+L1+L2+L3 \leq 1$

$L1-U1 \leq 0$

$L2-U2 \leq 0$

$L3-U3 \leq 0$

$L4-U4 \leq 0$

END

LP OPTIMUM FOUND AT STEP 16

OBJECTIVE FUNCTION VALUE

1) 0.3636364

VARIABLE	VALUE
U1	0.250000
U2	0.295455
U3	0.386364
U4	0.250000
L1	0.159091
L2	0.295455
L3	0.204545
L4	0.159091

<Upper Model (with respect to Financial Guarantee)>

Min $U1+U2+U3+U4-L1-L2-L3-L4$

S.T.

$11L2-13U1 \leq 0$

$11L3-9U1 \leq 0$

$11L4-7U1 \leq 0$

$13L1-7U2 \leq 0$

$13L3-9U2 \leq 0$

$13L4-7U2 \leq 0$

$17L1-7U3 \leq 0$

$17L2-13U3 \leq 0$

$17L4-7U3 \leq 0$

$11L1-7U4 \leq 0$

$11L2-13U4 \leq 0$

$11L3-9U4 \leq 0$

$7U2-13L1 \geq 0$

$7U3-17L1 \geq 0$

$7U4-11L1 \geq 0$

$13U1-11L2 \geq 0$

$13U3-17L2 \geq 0$

$13U4-11L2 \geq 0$

$9U1-11L3 \geq 0$

$9U2-13L3 \geq 0$

$9U4-11L3 \geq 0$

$7U1-11L4 \geq 0$

$7U2-13L4 \geq 0$

$7U3-17L4 \geq 0$

$L1+U2+U3+U4 \geq 1$

$L2+U1+U3+U4 \geq 1$

$L3+U1+U2+U4 \geq 1$

$L4+U1+U2+U3 \geq 1$

$U1+L2+L3+L4 \leq 1$

$U2+L1+L3+L4 \leq 1$

$U3+L1+L2+L4 \leq 1$

$U4+L1+L2+L3 \leq 1$

$L1-U1 \leq 0$

$L2-U2 \leq 0$

$L3-U3 \leq 0$

$L4-U4 \leq 0$

END

LP OPTIMUM FOUND AT STEP 17

OBJECTIVE FUNCTION VALUE

1) 0.3636364

VARIABLE	VALUE
U1	0.250000
U2	0.295455
U3	0.386364
U4	0.250000
L1	0.159091
L2	0.295455
L3	0.204545
L4	0.159091

<Lower Model with respect to Experience>

Max $U1+U2+U3+U4-L1-L2-L3-L4$

S.T.

$L2-U1 \geq 0$

$15L3-13U1 \geq 0$

$15L4-13U1 \geq 0$

$5L1-3U2 \geq 0$

$15L3-13U2 \geq 0$

$15L4-13U2 \geq 0$

$17L1-9U3 \geq 0$

$17L2-15U3 \geq 0$

$17L4-13U3 \geq 0$

$25L1-9U4 \geq 0$

$25L2-15U4 \geq 0$

$25L3-13U4 \geq 0$

$3U2-5L1 \leq 0$

$9U3-17L1 \leq 0$

$9U4-25L1 \leq 0$

$U1-L2 \leq 0$

$15U3-17L2 \leq 0$

$15U4-25L2 \leq 0$

$13U1-15L3 \leq 0$

$13U2-15L3 \leq 0$

$13U4-25L3 \leq 0$

$13U1-15L4 \leq 0$

$13U2-15L4 \leq 0$

$13U3-17L4 \leq 0$

$L1+U2+U3+U4 \geq 1$

$L2+U1+U3+U4 \geq 1$

$L3+U1+U2+U4 \geq 1$

$L4+U1+U2+U3 \geq 1$

$U1+L2+L3+L4 \leq 1$

$U2+L1+L3+L4 \leq 1$

$U3+L1+L2+L4 \leq 1$

$U4+L1+L2+L3 \leq 1$

$L1-U1 \leq 0$

$L2-U2 \leq 0$

$L3-U3 \leq 0$

$L4-U4 \leq 0$

END

LP OPTIMUM FOUND AT STEP 20

OBJECTIVE FUNCTION VALUE

1) 0.3333333

VARIABLE	VALUE
U1	0.250000
U2	0.250000
U3	0.283333
U4	0.383333
L1	0.150000
L2	0.250000
L3	0.216667
L4	0.216667

<Upper Model with respect to Experience>

Min $U1+U2+U3+U4-L1-L2-L3-L4$

S.T.

$L2-U1 \leq 0$

$15L3-13U1 \leq 0$

$15L4-13U1 \leq 0$

$5L1-3U2 \leq 0$

$15L3-13U2 \leq 0$

$15L4-13U2 \leq 0$

$17L1-9U3 \leq 0$

$17L2-15U3 \leq 0$

$17L4-13U3 \leq 0$

$25L1-9U4 \leq 0$

$25L2-15U4 \leq 0$

$25L3-13U4 \leq 0$

$3U2-5L1 \geq 0$

$9U3-17L1 \geq 0$

$9U4-25L1 \geq 0$

$U1-L2 \geq 0$

$15U3-17L2 \geq 0$

$15U4-25L2 \geq 0$

$13U1-15L3 \geq 0$

$13U2-15L3 \geq 0$

$13U4-25L3 \geq 0$

$13U1-15L4 \geq 0$

$13U2-15L4 \geq 0$

$13U3-17L4 \geq 0$

$L1+U2+U3+U4 \geq 1$

$L2+U1+U3+U4 \geq 1$

$L3+U1+U2+U4 \geq 1$

$L4+U1+U2+U3 \geq 1$

$U1+L2+L3+L4 \leq 1$

$U2+L1+L3+L4 \leq 1$

$U3+L1+L2+L4 \leq 1$

$U4+L1+L2+L3 \leq 1$

$L1-U1 \leq 0$

$L2-U2 \leq 0$

$L3-U3 \leq 0$

$L4-U4 \leq 0$

END

LP OPTIMUM FOUND AT STEP 17

OBJECTIVE FUNCTION VALUE

1) 0.3731183

VARIABLE	VALUE
U1	0.250000
U2	0.250000
U3	0.283333
U4	0.403226
L1	0.145161
L2	0.241935
L3	0.209677
L4	0.216667

<Lower Model with respect to Scope Change>

Max $U1+U2+U3+U4-L1-L2-L3-L4$

S.T.

$19L2-11U1 \geq 0$

$L3-U1 \geq 0$

$L4-U1 \geq 0$

$7L1-5U2 \geq 0$

$21L3-19U2 \geq 0$

$21L4-19U2 \geq 0$

$19L1-15U3 \geq 0$

$19L2-11U3 \geq 0$

$L4-U3 \geq 0$

$23L1-15U4 \geq 0$

$23L2-11U4 \geq 0$

$23L3-19U4 \geq 0$

$5U2-7L1 \leq 0$

$15U3-19L1 \leq 0$

$15U4-23L1 \leq 0$

$11U1-19L2 \leq 0$

$11U3-19L2 \leq 0$

$11U4-23L2 \leq 0$

$U1-L3 \leq 0$

$19U2-21L3 \leq 0$

$19U4-23L3 \leq 0$

$U1-L4 \leq 0$

$19U2-21L4 \leq 0$

$U3-L4 \leq 0$

$L1+U2+U3+U4 \geq 1$

$L2+U1+U3+U4 \geq 1$

$L3+U1+U2+U4 \geq 1$

$L4+U1+U2+U3 \geq 1$

$U1+L2+L3+L4 \leq 1$

$U2+L1+L3+L4 \leq 1$

$U3+L1+L2+L4 \leq 1$

$U4+L1+L2+L3 \leq 1$

$L1-U1 \leq 0$

$L2-U2 \leq 0$

$L3-U3 \leq 0$

$L4-U4 \leq 0$

END

LP OPTIMUM FOUND AT STEP 4

OBJECTIVE FUNCTION VALUE

1) 0.2222222

VARIABLE	VALUE
U1	0.263889
U2	0.263889
U3	0.263889
U4	0.319444
L1	0.208333
L2	0.152778
L3	0.263889
L4	0.263889

<Upper Model with respect to Scope Change>

Min $U1+U2+U3+U4-L1-L2-L3-L4$

S.T.

$19L2-11U1 \leq 0$

$L3-U1 \leq 0$

$L4-U1 \leq 0$

$7L1-5U2 \leq 0$

$21L3-19U2 \leq 0$

$21L4-19U2 \leq 0$

$19L1-15U3 \leq 0$

$19L2-11U3 \leq 0$

$L4-U3 \leq 0$

$23L1-15U4 \leq 0$

$23L2-11U4 \leq 0$

$23L3-19U4 \leq 0$

$5U2-7L1 \geq 0$

$15U3-19L1 \geq 0$

$15U4-23L1 \geq 0$

$11U1-19L2 \geq 0$

$11U3-19L2 \geq 0$

$11U4-23L2 \geq 0$

$U1-L3 \geq 0$

$19U2-21L3 \geq 0$

$19U4-23L3 \geq 0$

$U1-L4 \geq 0$

$19U2-21L4 \geq 0$

$U3-L4 \geq 0$

$L1+U2+U3+U4 \geq 1$

$L2+U1+U3+U4 \geq 1$

$L3+U1+U2+U4 \geq 1$

$L4+U1+U2+U3 \geq 1$

$U1+L2+L3+L4 \leq 1$

$U2+L1+L3+L4 \leq 1$

$U3+L1+L2+L4 \leq 1$

$U4+L1+L2+L3 \leq 1$

$L1-U1 \leq 0$

$L2-U2 \leq 0$

$L3-U3 \leq 0$

$L4-U4 \leq 0$

END

LP OPTIMUM FOUND AT STEP 6

OBJECTIVE FUNCTION VALUE

1) 0.2620120

VARIABLE	VALUE
U1	0.263889
U2	0.283784
U3	0.263889
U4	0.319444
L1	0.202703
L2	0.152778
L3	0.256757
L4	0.256757

<Lower Model with respect to Size>

Max $U1+U2+U3+U4-L1-L2-L3-L4$

S.T.

$7L2-U1 \geq 0$

$21L3-5U1 \geq 0$

$21L4-5U1 \geq 0$

$11L1-3U2 \geq 0$

$11L3-5U2 \geq 0$

$11L4-5U2 \geq 0$

$11L1-3U3 \geq 0$

$11L2-3U3 \geq 0$

$11L4-5U3 \geq 0$

$3L1-U4 \geq 0$

$3L2-U4 \geq 0$

$9L3-5U4 \geq 0$

$3U2-11L1 \leq 0$

$3U3-11L1 \leq 0$

$U4-3L1 \leq 0$

$U1-7L2 \leq 0$

$3U3-11L2 \leq 0$

$1U4-3L2 \leq 0$

$5U1-21L3 \leq 0$

$5U2-11L3 \leq 0$

$5U4-9L3 \leq 0$

$5U1-21L4 \leq 0$

$5U2-11L4 \leq 0$

$5U3-11L4 \leq 0$

$L1+U2+U3+U4 \geq 1$

$L2+U1+U3+U4 \geq 1$

$L3+U1+U2+U4 \geq 1$

$L4+U1+U2+U3 \geq 1$

$U1+L2+L3+L4 \leq 1$

$U2+L1+L3+L4 \leq 1$

$U3+L1+L2+L4 \leq 1$

$U4+L1+L2+L3 \leq 1$

$L1-U1 \leq 0$

$L2-U2 \leq 0$

$L3-U3 \leq 0$

$L4-U4 \leq 0$

END

LP OPTIMUM FOUND AT STEP 20

OBJECTIVE FUNCTION VALUE

1) 1.083333

VARIABLE	VALUE
U1	0.458333
U2	0.458333
U3	0.458333
U4	0.375000
L1	0.125000
L2	0.125000
L3	0.208333
L4	0.208333

<Upper Model with respect to Size>

Min $U1+U2+U3+U4-L1-L2-L3-L4$

S.T.

$7L2-U1 \leq 0$

$21L3-5U1 \leq 0$

$21L4-5U1 \leq 0$

$11L1-3U2 \leq 0$

$11L3-5U2 \leq 0$

$11L4-5U2 \leq 0$

$11L1-3U3 \leq 0$

$11L2-3U3 \leq 0$

$11L4-5U3 \leq 0$

$3L1-U4 \leq 0$

$3L2-U4 \leq 0$

$9L3-5U4 \leq 0$

$3U2-11L1 \geq 0$

$3U3-11L1 \geq 0$

$U4-3L1 \geq 0$

$U1-7L2 \geq 0$

$3U3-11L2 \geq 0$

$1U4-3L2 \geq 0$

$5U1-21L3 \geq 0$

$5U2-11L3 \geq 0$

$5U4-9L3 \geq 0$

$5U1-21L4 \geq 0$

$5U2-11L4 \geq 0$

$5U3-11L4 \geq 0$

$L1+U2+U3+U4 \geq 1$

$L2+U1+U3+U4 \geq 1$

$L3+U1+U2+U4 \geq 1$

$L4+U1+U2+U3 \geq 1$

$U1+L2+L3+L4 \leq 1$

$U2+L1+L3+L4 \leq 1$

$U3+L1+L2+L4 \leq 1$

$U4+L1+L2+L3 \leq 1$

$L1-U1 \leq 0$

$L2-U2 \leq 0$

$L3-U3 \leq 0$

$L4-U4 \leq 0$

END

LP OPTIMUM FOUND AT STEP 25

OBJECTIVE FUNCTION VALUE

1) 0.9607843

VARIABLE	VALUE
U1	0.360294
U2	0.323529
U3	0.323529
U4	0.264706
L1	0.088235
L2	0.051471
L3	0.085784
L4	0.085784