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THE FERTILITY TRANSITION, THE PUBLIC AND PRIVATE COSTS OF "DEPENDENTS", AND INTERGENERATIONAL WELFARE

STUART J. WILSON

Department of Economics
University of Regina
Regina, Saskatchewan
Canada S4S 0A2

econ@uregina.ca

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Stuart J Wilson
University of Regina
Regina SK S4S 0A2 Canada
Tel: 306 337 2230
Email: stuart.wilson@uregina.ca.

Abstract

Much of the pension reform debate has been driven by the expected rise in public pay-as-you-go pension contribution rates in the face of declining fertility rates and rising life expectancies. However, little has been written regarding the potential savings in childrearing costs to those generations facing pension contribution rate increases. Are higher pension contribution rates an unacceptable expense to people who are choosing to have fewer children than previous generations? An overlapping generations modeling framework is used to examine the implications of fertility changes on the potential savings in childrearing costs, and how these adjustments may be incorporated into the pension reform debate. Model economies are calibrated to match historical and projected economic and demographic conditions in Canada.

Keywords: public pensions, childrearing, fertility transition
JEL Classification Codes: H55, E21, C68, J26

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1. Introduction

Public pay-as-you-go (PAYGO) pension systems are facing a sustainability crisis. The dependency ratio of pensioners to working-age adults has been rising because of health care improvements which have led to longer life expectancies, and because of declining fertility rates. If there is no change in the benefit formula of pensioners in the European Union, public pension contribution rates will have to rise from an average of 16% of wages to 27% by the middle of the twenty-first century (European Union Commission, 2001). In the US, projected demographic conditions would necessitate a rise in the Social Security contribution rate from 12.4% to 17.8% (Social Security Administration, 2001). The contribution rate for the Canada Pension Plan was 5.6% in 1996 and would have to rise to 14.2% by 2025 to maintain equality between contributions received and benefits paid (Oreopoulos, 1996: 4). As a result, policy-makers have been vigorously debating methods to reform public pension systems. Alternatives to gradually raising the contribution rates of these pension systems include gradually decreasing benefit rates, gradually raising the retirement age, using other government revenues to fund pension deficits, quickly raising contribution rates to pre-fund future pension deficits, or any combination of these measures. Other prominent alternatives include partial or full funding of individual government pension accounts.

The PAYGO system provides introductory generations with large benefits in relation to their contributions, and the system transfers income from future generations to their parents' and grandparents' generations. Much of the research on pension reform has focused on the losses and gains of current and future generations as a result of new pension policy in relation to the status quo. Reform measures attempt to relieve the burden on future generations by requiring current generations to accept a welfare loss through a reduction in benefits, or an increase in costs, and much of the welfare loss as a result of reform is found to fall on the baby-boom generation and a few subsequent generations (De Nardi, Imrohoroglu, and Sargent, 1999; Kotlikoff, Smetters, and Walliser, 1999; Fehr, 2000). Using a simple overlapping generations model economy without technological progress, Emery and Rongve (1999) argued that pension reform measures that shift the pension burden from the baby-bust generation to the baby-boom generation may be inter-generationally regressive, since the baby-bust generation will reap the benefits of labour scarcity, and receive higher wages than the baby-boom generation. In this case, the PAYGO pension

sustainability problem may be, “much ado about nothing,” since future generations with higher wages will be able to cover higher payroll taxes.

The pension reform literature notes the fertility decline as one cause of the public pension sustainability issue, however little attention is placed on any possible benefits of the fertility decline. If people are choosing to have fewer children, there must be some reason for doing so. It has been suggested that one of the reasons for the decline in fertility is the presence of a pension system that provides an income for the elderly. One theory of fertility posits that children are a means for income when elderly or infirm. Parents may make an investment in the human capital of their children so that their children will provide them with a return on their investment later on in life when they can no longer work and provide for themselves. Children are investment goods, and the amount of support received when elderly depends on the amount of surviving children (Caldwell, 1982; Nugent, 1985; Boldrin and Jones, 2002). However, it may become difficult to enforce the support payments from children. One of the motivating reasons for setting up a PAYGO pension system in Germany was to formalize the intergenerational transfer from children to parents (Sinn, 2004: 1336). The elderly effectively receive transfers from a government body, financed by their children and grandchildren through payroll taxes.

A common way of setting up a PAYGO pension system is to link (in many cases loosely) earnings, from which come contributions, to retirement benefits, so that a pensioner receives benefits based on his or her earnings history. An income-linked public pension reduces the need for children as investment goods, and people may decide to reduce their investment in children. The benefits of children as workers and pension contributors are effectively distributed amongst the elderly, while the costs of raising children are covered by the parents based on the investment in their children. In this type of system, some may free-ride on the children, and the investment in human capital, of others. Such a system distorts the fertility decision of adults. As a result, PAYGO systems have been seen as contributing to some extent to the decline in fertility rates and the decrease in the population growth rate (Raut, 1991; Wigger, 1999; Sinn, 2004).

The introduction of family allowances or subsidies including public education to help raise children causes an income-linked pension system to behave more like a mixed PAYGO system, whereby parents get an effective subsidy to offset their pension contributions in recognition of their investment in human capital (Kolmar, 1997: 344). Such subsidies for raising

children reduce the disincentives to have children, and help to spread some of the costs of raising children across those who receive benefits from future pension contributions.

It is a given that raising children involves significant costs. What are the private costs of raising children? Estimates of the private direct costs of raising children from household expenditure surveys vary by the age of the child, the number of children, by income, and by household preferences. Young children have low costs associated with items like food and clothing, but large costs associated with child care expenses. Older children have higher costs associated with food, clothing and transportation. Childrearing may also be associated with economies of scale as the first child is estimated to be much more expensive than subsequent children. Canadian estimates for the direct costs of raising a child to the age of 18 fall in the range of $4,000 to $9,000 per year, and the inclusion of indirect costs (e.g. lost wages of a spouse as home caregiver) can double these cost estimates (Beaujot and Kerr, 2004:79-82). US estimates for the annual expenditures on a child range from $7,000 for low income households to $16,000 for high income households (USDA, 2005:Table 1).

What are the public costs of raising children? Governments typically provide primary and secondary schooling, as well as partial or full funding for post-secondary education and health care. For the fiscal year 2000-1, Health Canada (2001) estimated that $10.2 billion was spent to provide health care for children under the age of 25. For the same fiscal year, the combined levels of government spent $37.4 billion for public elementary and secondary education, $10.6 billion on university education, and an additional $4.2 billion on community college education (Statistics Canada, n.d.: Tables 478-0014, 478-0007, and 478-0004). While many adults receive subsidized education at universities and community colleges, the majority of beneficiaries of all forms of public education are under the age of 25. By allocating all education expenditures to those under 25, public education and health care expenditures amounted to about $6,200 per child in 2000-1.

If people decide to have fewer children and these choices are voluntary, then it is assumed these choices were made in an attempt to improve the welfare of the decision-makers, however that welfare is measured. The cost savings of having fewer children may be directed into other uses, such as increasing the consumption of parents in the present or in the future through savings, and/or increasing the consumption, and investment in human capital, of fewer children. A decline in fertility also has implications for governments. The potential cost savings

as a result of educating and providing health care for fewer children may be directed to other uses and/or improving the quality of fewer children. However, a decline in fertility raises the ratio of elderly dependents to the working age population, and increases the per worker costs of government-provided pensions and services to the elderly.

The purpose of this paper is to investigate the effects of a fertility transition on intergenerational welfare and government programs targeted to certain age cohorts, and abstracts from the effects of changes in longevity. This paper presents a simple overlapping generations model that incorporates cohort-specific private and public expenditures to help answer this question. The model is calibrated to the Canadian demographic, economic and policy environment. Model results suggest that a decline in fertility can indeed be welfare-improving because the increased costs of public pension provision and other government services to an increasing proportion of retirees can be offset by reduced costs in providing services to children. Section 2 describes the modeling environment. Section 3 presents the calibration parameters for the model economies to be investigated, with the results discussed in section 4. Section 5 provides an overview of modeling extensions, and section 6 concludes the paper.

2. Modeling Environment

A simple overlapping generations dynamic general equilibrium framework is now introduced to examine the impact of a fertility transition on private and public costs of providing for children, as well as the costs of providing public services to different age groups. Model economies are populated by agents who live for four periods, with each period corresponding to about 20 years. In the first period of life, agents are born into the economy as children who consume goods and services provided by parents and the government. In the second period of life, agents are young workers and raise children. In the third period, agents are older workers, and no longer provide for their children. In the fourth period, agents are no longer working, but are retirees. When working, agents pay mandatory contributions to a pension system as a percentage of wages, and when retired, agents receive benefits at a given rate of their prior employment earnings. Agents pay taxes to the government on income, and government uses these revenues to pay for government services and transfers (e.g. health care, education, family allowances, old age security payments) other than pensions.

2.1 Population Process

The population of the model economy is defined as follows, where N_{it} denotes the population of that cohort in the ith period of life in time t:

(1) $$N_t = N_{1,t} + N_{2,t} + N_{3,t} + N_{4,t}.$$

There is no lifetime uncertainty, so that agents live for all four periods. As such, $N_{i+1,t+1} = N_{i,t}$ for i=1,2,3, and for all t. The population will evolve over time, at the time-varying growth rate, n_t, so that $N_{i,t} = (1+n_t)N_{i,t-1}$

Agents supply $e_{i,t}$ units of labour inelastically in the ith period of life, at time t. Children and seniors do not supply labour, so that $e_{i,t} = 0$ if $i = 1$ or 4. At time t, the total labour supply in the model economy is defined as:

(2) $$L_t = N_{2,t}e_{2,t} + N_{3,t}e_{3,t}.$$

2.2 Production

Production in the economy is deterministic, and is defined by the constant returns to scale production function:

(3) $$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}.$$

As such, the returns to the factors of production, labour and capital, per unit, are:

(4) $$w_t = (1-\alpha)A_t K_t^{\alpha} L_t^{\alpha-1}, \text{ and}$$

(5) $$r_t = \alpha A_t K_t^{\alpha-1} L_t^{1-\alpha} - \delta_t.$$

Following Kotlikoff (1995, 1996), Kotlikoff, Smetters and Walliser (1999), Emery and Rongve (1999), Butler (2000) and Fehr (2000), model economies without technological progress (or human capital growth) will be examined, so that $A_t = A$. Model economies with technological progress will be discussed in section 5.

2.3 The Role of Government

The government runs a PAYGO pension system whereby benefits in any period paid to retirees are funded by contributions by workers in that same period. The government imposes a pension contribution rate, $p_{i,t}$, on labour earnings of generation i at time t, and pays out benefits to

retirees, the amount of which is denoted by b_t for retirees at time t. Pension contributions are tax-deductible, and pension benefits are taxable. These benefit levels will be determined in advance and will be set as a proportion of wage earnings when employed:

(6) $$b_t = br_t(\varphi_2 w_{t-2} e_2 + \varphi_3 w_{t-1} e_3).$$

As such, the pension budget constraint is:

(7) $$N_{2,t} p_{2,t} w_t e_2 + N_{3,t} p_{3,t} w_t e_3 = b_t N_{4,t}.$$

With the PAYGO system, $p_{2,t} = p_{3,t}$.

The government provides a range of services, including education for children and health care for all agents. Education and health care expenditures per person are denoted by s_i, are taken as given by recipients, do not enter into the utility maximization decision of agents, and are financed with general income taxes. The government provides transfers to young adults to assist child-rearing, which are tax-exempt; the government also provides transfers to seniors, which are taxable (this is to ensure consistency with Canadian programs). These transfers are included as income for recipients and enter into the utility maximization decision of agents. The government does not transfer funds to individuals in their first or third periods of life ($z_1 = z_3 = 0$). Other miscellaneous government expenditures (e.g. defense, environment, recreation) are denoted by X_t and do not enter into the consumption-savings decision of agents. For these expenditures, the government's budget constraint is:

(8) $$\sum_{i=1}^{4} N_{i,t}(s_{i,t} + z_{i,t}) + X_t = \tau_t(\sum_{i=2}^{3} N_{i,t} w_t e_i (1 - p_{i,t}) + r_t K_t + N_{4,t}(b_t + z_{4,t})).$$

2.4 Providing for Children

Young adults are assumed to provide a subsistence level of consumption to their children and do not derive any utility from this transfer. Any value of consumption for children over the subsistence level is included as consumption of the adult, from which the adult derives (altruistic) utility. The value of subsistence consumption per child, including private health care costs, is denoted by m_t, and the transfer per young adult to children is denoted by q_t, so that:

(9) $$q_t = m_t(N_{1,t} / N_{2,t}).$$

The government also provides transfers to parents to help raise children: family allowances or child tax credits. The government will pay an allowance of d_t per child, so that each young adult receives the following allowance:

(10) $z_{2,t} = d_t(N_{1,t} / N_{2,t})$.

For the purpose of the simulation exercises, there is no heterogeneity within a cohort, and no modeling of household formation. With child-rearing, homogeneity within a cohort implies that all young adults "share" the private expenses associated with the rearing of the next generation. For example, if there are ten young adults and seventeen children in the population, each young adult will receive a transfer $z_{2,t} = 1.7d_t$ from the government as a family allowance, and will privately pay $q_t = 1.7m_t$ to provide the subsistence level of consumption for children. The investigation of heterogeneity in child-rearing is left for other work.

2.5 Utility Maximization Problem

Agents do not make decisions as children, so that their consumption as children is not included in the utility function. Adult agents, born in time *t*, maximize utility over a standard composite good, which can be used for consumption, *c*, or used as a capital asset in production, *a*, with time separable preferences, subject to a discount factor β, under the following constraints:

(11) $U_{t+1} = \sum_{i=2}^{4} \beta^{i-2} \log c_{i,t+i-1}$, subject to

(12) $c_{2,t+1} = w_{t+1}e_2(1-p_{2,t+1})(1-\tau_{t+1}) + z_{2,t+1} - q_{t+1} - h_{2,t+1} - a_{2,t+1}$,

(13) $c_{3,t+2} = (w_{t+2}e_3(1-p_{3,t+2}) + r_{t+2}a_{2,t+1})(1-\tau_{t+2}) - h_{3,t+2} + a_{2,t+1} - a_{3,t+2}$,

(14) $c_{4,t+3} = (r_{t+3}a_{3,t+2} + z_{4,t+3} + b_{4,t+3})(1-\tau_{t+3}) - h_{4,t+3} + a_{3,t+2}$,

(15) $a_{i,t} \geq 0 \forall i,t$.

Agents must consume h_i quantities of private health care, which are not discretionary, and provide no utility. Agents in the model economies are liquidity constrained. They cannot hold negative private wealth and cannot borrow against future earnings, pension benefits or transfers. The aggregate level of capital stock held by residents is defined as:

(16) $K_{t+1}^d = N_{2,t}a_{2,t} + N_{3,t}a_{3,t}$.

Agents have perfect foresight. They perfectly predict the evolution of the population (N_t), and of technological progress (A_t), without uncertainty. That is, they can perfectly predict the evolution of wages, the rate of return, premium rates, pension benefits, and other economic variables given no change in pension policy.

2.6 Computation of Equilibria

Open and closed model economies will be investigated. The method of computing equilibria for a closed economy is slightly different than that used for an open economy. The method for computing equilibria in the open economy is simpler, and will be described first. In an open economy, the rate of return on assets is determined exogenously in world capital markets. Capital is assumed to be perfectly mobile. Funds will flow in from abroad (or domestic capital will flow out), until the return to capital is equal to the world interest rate, if the amount of domestically-supplied capital is too low (or too high). The amount of labour is defined in each period given the population process. The steps for computing equilibria are as follows:

1. Eq. (5) is solved for K_t, given the international rate of return, the level of labour, and the production parameters, A, α, and δ, for all values of t;
2. The wage rate is calculated using Eq. (4), for all values of t;
3. Pension benefits in Eq. (6), private childrearing costs in Eq. (9), and family allowances in Eq. (10) are calculated, given population cohort sizes.
4. The pension contribution rates in Eq. (7), and general income tax rates in Eq. (8), are calculated with predefined values for the benefit levels and government expenditures, for all values of t;
5. The agents' optimization problem is solved recursively as defined by Eqs. (11-15), for all values of i = 2 to 4, and all values of t;
6. The optimal level of asset holdings over time is used to calculate the level of domestic capital stock holdings over time, using Eq. (16);
7. The difference between the level of capital used in production and the level of domestic asset holdings in the open economy is equal to foreign-held capital stock in any given time period.

The method of solving equilibria in the closed economy differs slightly from that described above. In step 1, initial guess values for K_t are defined, and are then used to calculate the rates of return, r_t, in Eq. (5). Steps 2 through 6 do not change. In step 7, however, the value of foreign-held capital stock is zero for the closed economy. If the values of domestic capital stock holdings from step 6 do not equal the guess values for the capital stock from step 1, then the guess values for the capital stock are updated, and the procedural steps are repeated until the calculated values of capital stock in step 6 are equivalent to the previous guess values in step 1.

3. Model Calibration

In order to conduct the simulation experiments, the model must be calibrated for the population process, technology, preferences, and for cohort-specific expenditures. Since there are four periods of life, each period is approximately equal to 20 years. The first period of life is one in which agents are children whose consumption is provided by young adults, and receive education and health care from the government; this period is one of dependence and human capital formation. The second period is one in which young adults work and raise children. The third period of life is the empty-nest stage where adults no longer provide consumption for children, but are still in the labour force. The fourth and final stage of life corresponds to a withdrawal from the labour force. To match with Canadian statistical age categories for population cohorts and a retirement age of 65, the first stage of life is set to correspond to the period when agents are less than 25 years of age. The second stage corresponds to the period when agents are between 25 and 44 years of age. The third stage of life corresponds to the period when agents are between 45 and 64 years of age, and the fourth stage corresponds to the period when agents are 65 years of age and older. Although there is an increasing degree of overlap across these age groups (i.e. delayed child birth with child-rearing in the third stage of life, continued formal education in the second stage of life, and early or even delayed retirement), this categorization is for simplicity in the modeling exercises. The division of the stages of life into periods of approximately 20 years is convenient in the beginning of the twenty-first century, with current life expectancies of an additional 20 years for those aged 65.

3.1 Population

The calibrated population process is intended to mirror that experienced in Canada in the twentieth century, and expected to be experienced in the future: an initial steady state with constant growth, followed by a one-generation decline in the birth rate corresponding to that experienced over the 1926-1945 period, subsequently followed by a one-generation baby-boom over the 1946-1965 period, then followed by a process of no population growth.

The population growth rate in the first steady state is set to 0.5, which is approximately 2.0% on an annual basis. The first steady state is followed by a one-generation decline in the growth rate for the generation born during the interwar period: the growth rate for this generation is set to 0.35 (corresponding to 1.5% annually). This generation enters the labour force over the 1951-1970 period and is in the 25-44 age category in 1971. The baby-boom generation (born over the 1946-1965 period) is 1.75 times bigger than the preceding generation, which corresponds to an annual increase of almost three percent, and is in the 25-44 age category in 1991. The baby boom generation is all retired by 2031. Subsequent generations exhibit no growth over the previous generation (*i.e.* replacement fertility of young adults). Figure 1 shows the differences in the age-composition of the population given this population process, and the Canadian population at census points. Note that the model economies fail to replicate the smaller size of the cohort of retirees, and the larger size of the child cohort, in years before 2011; this is because the model economies do not incorporate rising life expectancies and declining mortality rates across ages. The model economies also ignore international migration. The implications of these omissions will be discussed later.

There are a range of values that are used for the annual discount factor in the macroeconomics literature for simulations without mortality risk. Many studies use 0.994 (for example Huggett, 1993). Other work has used 0.96 (Aiyagari, 1994), and 0.988 (Cooley and Soares, 1999). The one-period discount factor (β) is set to 0.820, to approximate the annual figure of 0.99, which corresponds to an annual rate of time preference of 1% (other figures were used in simulations, the results of which will be discussed later).

3.2 Earnings

The average earnings figures for both males and females in the age cohorts of the young (25-34, 35-44), and the old (45-54, 55-64), in constant 2004 dollars were converted into 2001 constant dollars using the consumer price index ratio for 2001 versus 2004 with 1992 as the base year, and with the basket content from 2001 (Statistics Canada, n.d.: Table 202-0407 for earnings, Table 326-0002 for the consumer price index). These figures were used to construct the following average earnings ratios for old workers to young workers:

(24) $(e_{45-54,t} + e_{55-64,t})/(e_{25-34,t} + e_{35-44,t})$ for t = 1980-2005, and

(25) $(e_{45-54,t-10} + e_{55-64,t})/(e_{25-34,t-10} + e_{35-44,t})$ for t = 1990-2005

Eq. (25) allows for the inclusion of rising real wages over ten years when a ten-year age cohort moves into the second ten-year period. The ratio of average earnings of old workers to young workers ranged from about 1 to 1.12 over the period from 1980 to 2005 in Canada using Eq. (24), and from 1 to 1.07 using Eq. (25). In the model economies, agents are endowed with age-dependent efficiency units. For young workers, e_2 is normalized to 1. For older workers, e_3 is set to 1.05, so that older workers earn wages that are five percent higher than those of young workers.

The average real earnings for the young cohort in 2001 was $35,119 following Eq. (24), and $33,717.4 following Eq. (25). In order to model expenditures and transfers in the simulation exercises, these items must also be normalized in comparison to earnings. In model economies with no technological progress, steady-state wages are constant. Other cost figures in the model economies are assumed to remain constant on a per person basis, and will be normalized by division by the factor of $34,000/$\hat{w}$, with $\hat{w}$ denoting the level of wages in the first steady-state.

3.3 Production

The share of income to capital is set to 0.3, following the estimate reported in Abel et al (2006: 57) and Auerbach and Kotlikoff (1998: 7). The depreciation rate is set to 0.71, which corresponds to the annual estimate of 6% by Stokey and Rebelo (1995). For the purpose of this paper, total factor productivity will be held constant following convention in Kotlikoff (1995, 1996), Kotlikoff, Smetters and Walliser (1999), Cooley and Soares (1996, 1999), Emery and Rogve (1999), Fehr (2000) and Butler (2000). De Nardi, Imrohoroglou and Sargent (1999)

incorporated annual technical progress of 1.6% in their projections for the US economy. Model economies with technological progress will be discussed in Section 5.

3.4 Expenditures

Public and private expenditures for health care by age cohort, and public expenditures for primary, secondary and tertiary education are presented in Table 1. Public education expenditures will be attributed solely to children. The calibrated values of s_i and h_i for the model economies are also show in Table 1.

Recent child-rearing cost estimates vary by the age of children, the number of children in the household, and by household income. Given estimates cited by Beaujot and Kerr (2004:81), the direct costs (excluding opportunity costs) for children range from $4,000 to $9,000 annually to the age of 18.[1] Battle (1997) suggested that the annual child-tax benefit for low income families should be set at the level of $4,000 per child, the low end of the cost range. The private costs of health care for children have been estimated at $600 per child (see Table 1). In addition, the value of university and college tuition was estimated to be $3.8 billion in 2000-0 (Statistics Canada, n.d.:Tables 478-0004,0007), or $378 per person in the child cohort. In the model economies, the figure of $5,100 in 2001 dollars will be used as non-discretionary private child-rearing costs, and additional expenditures will be considered discretionary and will provide utility for parents. The value of m in the model economies is thus set to $0.150\hat{w}$, and the value of q is determined by Eq.(9).

Welfare and employment insurance expenditures will be excluded from the analysis, as it will be assumed that all adult agents supply labour prior to retirement in life-stage 4, so that there is no unemployment, nor are there any agents on welfare. Transfers to seniors through pensions and social security, and transfers to parents for raising children, part of social expenditures, are treated separately in the analysis.

In 2000, the government transferred $24 billion in Old Age Security, Guaranteed Income Supplement and Allowance payments to 3.8 million recipients, or $6,368 per recipient (Statistics Canada 2003:29). In the model economies, z_4 is set to $0.190\hat{w}$; z_1 and z_3 are set to 0. In 2001, the federal government transferred $7,495 million in family and youth allowances, and child tax

[1] Henripin (2000) and Gauthier (1991) were cited as sources.

credits (Statistics Canada, n.d.:Table 384-0009), a per child transfer of $743 which is non-taxable. In the model economies, the value of d_t is set to $0.022\hat{w}$. The value of z_2 is determined by Eq. (10)

In Canada, the public pension system pays benefits based on a formula that compares a contributor's best earnings years to the average industrial wage. For average wage earners, annual benefits are equal to twenty-five percent of the average wage of the preceding five years.[2] Since workers in the model economies work two periods and receive benefits in one period, the model economy government operates the pension system so that benefits received by retirees are equal to 25% of their earnings in the pre-retirement period: $br = 0.25$, with $\varphi_2 = 0$, and $\varphi_3 = 1$ in Eq. (6).

Total government expenditures for 2001 totaled $459,680 million in 2000-1. The amount excluding health, social welfare, education, and debt charges was $150,505 million (Treff and Perry, 2005: Table 1.2), an amount equal to 14.3% of GNP. Miscellaneous government expenditures will be set equal to fifteen percent of gross national product in the first steady-state, and held constant at that rate thereafter.

4. Results

In this section, a set of simulations involving model economies with no technological progress, no change in employment rates, and involving fixed expenditures per person in given cohorts are examined. The simulations examine the impact of a fertility transition on the economy, all else equal. Additional adjustments will be discussed in Section 5. The open economy and closed economy results are presented in Tables 2 and 3 respectively. The results are presented in a format to link cohorts most directly affected by the time-relevant variables. For example, the table links in a row generation 31, the baby-boom generation, born in period 31, with $c_{2,\,t+1}$ (their consumption in the second period of life, in time period 32), $c_{3,\,t+2}$ (their third period consumption in time period 33), $c_{4,\,t+3}$, $a_{2,\,t+1}$ (their level of assets chosen in the second period of life, period 32, to be used as productive capital in period 33), $a_{3,\,t+2}$, p_{t+1} (the pension payroll tax rate on wages in the second period of life), $\eta_{\,t+1}$ (description below), τ_{t+1} (the tax rate in the second period of life), w_{t+1} (wages in the second period of life), and r_{t+2} (the income received on

[2] See Rosen et al (2003: 245).

assets in their third period of life). Note also that this generation is affected by p_{t+2}, w_{t+2}, r_{t+3}, and τ_{t+2} and τ_{t+3} (tax rates in the third and fourth periods of life), and that these data are presented in subsequent rows in the table. The data that affects the baby-boom generation, Generation 31, are all highlighted for convenience. The variable η measures a "direct dependency burden" of child-rearing costs and pension premia to support retirees, less government transfers for raising children, when individuals are in the second period of life (young workers raising children).

4.1 Open Economy Results

In the open economy with no technological progress, wages and rates of return on assets are constant. Intergenerational welfare disparity is caused only by differences in cohort-specific expenditures and transfers, and pension contribution and general income tax rates, as a result of fertility changes over time. The variables of primary interest for the generation born in time t are p_{t+1}, p_{t+2}, η_{t+1}, τ_{t+1}, τ_{t+2} and τ_{t+3}; these are the variables that affect the intertemporal consumption and savings decisions of this generation. Generations born in period 26 and earlier are in the steady state, and all have equal levels of utility. Although generations 27, 28, and 29 are part of the steady-state population growth process, they are affected by fertility changes starting with generation 30 (the generation born during the interwar period that includes the Great Depression). Generation 27 receives a welfare boost because of a lower tax rate in the fourth period of life, due to a decreased burden in public expenditures on children born in period 30. Generation 28 benefits from a lower third period tax rate that more than offsets the higher fourth period tax rate resulting from higher public expenditures on the baby boom generation. Generation 29 experiences lifetime utility better than that of the steady state; this is because of a lower second-period "direct dependency burden" (η), and lower second and fourth period taxes, which more than offset the higher third period pension premium and tax rates. Generation 30 receives lifetime utility lower than the steady-state level primarily because of the increased costs of raising the baby-boom generation (generation 31); their pension payroll tax rates are also higher, and the tax rate in their second period of life is higher than the respective steady-state value. Lower income taxes in the third and fourth period of life provide only partial relief.

The baby-boom generation and their offspring (generations 31 and 32) are the biggest benefactors of the decline in fertility. These two generations reap the benefits of the fertility transition: a lower "direct dependency burden", with relatively fewer children to support than

preceding generations, and relatively fewer retirees to support than subsequent generations. Subsequent generations in the second steady state (starting with generation 33) are better off than those in the first steady state with lower child-rearing expenditures and income tax rates that more than offset the effect of the higher pension payroll tax rates. In the model simulations, the decline in fertility lowers the combined private and public dependency burden of children and retirees on the working population. The benefactors are the baby-boom generation that precipitated the fertility transition, and their children, in an open economy setting.

4.2 Closed Economy Results

In the closed economy setting, wages and rates of return will adjust to changes in the size of the labour force and the stock of capital. The set of variables that are of primary interest expands to include wages and interest rates. Generation 27 receives a slight welfare boost because of lower fourth period taxes, similar to that experienced in an open economy setting. Generation 28 suffers a welfare loss with the future changes in fertility since they receive lower wages in the third period of life, and lower interest rates combined with higher taxes in the fourth period, which more than offset lower taxes in the third period of life. Generation 29 enjoys higher than steady state utility levels with lower child-rearing costs, lower second and fourth period tax rates, higher wages in the third period of life, and higher interest rates in the final period. These benefits more than offset the detrimental effects of higher third period taxes and lower second period interest rates. Generation 30 pays higher child-rearing costs and taxes in the second period of life in order to raise the baby-boom generation. This generation pays higher pension premium rates than previous generations, receives lower third period wages and fourth period interest rates, but receives some benefits from higher second period wage rates and lower income tax rates in the third and fourth periods of life.

The baby-boom generation achieves higher levels of lifetime utility than any previous generation due to a reduction in child-rearing costs, much lower income tax rates, and higher wages in the third stage of life, even with the large drop in interest rates (the interest rate drops from 7.0% to 4.4% on an annual basis from t=32 to t=34). However, unlike in the open economy case, subsequent generations in a closed economy see further improvements in lifetime utility. This is because the beneficial effects of higher wage rates, lower direct child-rearing costs, and lower income tax rates to service cohort-specific expenditures and transfers far outweigh the

detrimental effects of higher pension payroll tax rates and lower interest rates. Notice that while the pension payroll tax rate (p) has almost doubled for future generations, the total burden on young workers in terms of child-rearing costs and pension contributions, less government family allowances (η, the second period "direct dependency burden"), rises only eight percent from the first steady state to the second.

4.3 Discussion

These results show how generations are affected by fertility changes over time, all else equal. While pension payroll tax rates increase with a larger dependency rate burden of seniors to the working age population, the decreased dependency rate burden of children lowers both public and private child-bearing costs, and decreases income tax rates. In closed economies, real wages also increase with the decline in fertility. The results suggest that future generations obtain higher levels of consumption and utility as a result of the decline in fertility. Without technological progress, and without any changes to existing government policies, future generations, including the baby-boom generation, are all better off in both open and closed economies, and the rising pension payroll tax rates are not a cause for concern.

Emery and Rongve (1999) investigated the effect of a one-period baby-boom in an overlapping generations context with individuals who lived for two periods, as young workers, and as retirees. Their model economies did not incorporate child-rearing costs or cohort-specific private and public expenditures and government transfers. As such, the model results lead to somewhat different inferences than the ones presented herein. In the open Emery and Rongve model economy, the baby boom generation achieves higher utility than any other generation because their pension premium rate is lower than any other to support a relatively smaller elderly population; the generation that follows the baby-boom suffers with lower utility because their pension premium rate rises in order to support a relatively large elderly population; the pre-baby boom generation experiences no adverse welfare effects as a result of population changes. With the additions to the modeling environment described in this paper, the baby-boom generation and their children are the biggest benefactors of population dynamics due to the combination of low private child-rearing costs and lower tax rates to support public programs, including those for children and the elderly. It is the pre-boom generation that experiences lower welfare than any

other because they raise the baby-boom generation and bear the private costs of raising a large child cohort.

In the closed economy, the Emery and Rongve modeling results suggest that the baby-boom generation is worse off as a result of population changes because they experience lower wage rates when working, and lower interest rates on their savings, than any other generation; the post-baby-boom generation is better off because they benefit from higher wages. With the additions to the modeling environment described in this paper, the baby-boom generation is better off than previous generations because they experience lower private child-rearing costs and lower tax rates to support public expenditures and transfers, while subsequent generations experience even higher levels of utility than the baby-boomers due to higher wages and lower income tax rates to support government expenditures and transfers.

In both open and closed model economies investigated in this section, subsequent generations enjoy higher levels of private consumption as a result of the decline to replacement fertility, even with the prospect of higher PAYGO pension premium rates. Even though these model economies are very simple in construction, they provide clues as to how a decline in fertility can be welfare-enhancing in terms of consumption, when fertility is the only item changing in the simulation and cohort-specific expenditures are kept constant with total expenditures varying with demographic change.

5. Additional Extensions and Discussion

The modeling results presented thus far do not incorporate other features that will now be addressed, specifically (i) the progress consistent with twentieth century labour, technology and public policy evolution, (ii) different pension benefit replacement rates, (iii) different preference parameters, and (iv) below-replacement fertility rates.[3] In addition, the treatment of social welfare to adults of working age, changing life expectancy, migration, and the effects of within-cohort heterogeneity will be briefly discussed in this section.

Model economies that were calibrated to include the evolution of labour productivity, labour force participation changes (especially that of females), the increasing role of government in providing education, the introduction of universal health care, public pensions and other social

[3] To preserve brevity, modeling results that include these adjustments are available on request from the author.

security programs, consistent with the twentieth century experience of Canada, were also examined. The modeling results suggested that all generations are substantially better off than their predecessors. The decline in fertility causes pension payroll taxes to increase, but this increase is offset by the decline in private child-rearing costs and rising real wages.

Model economies with higher pension benefits rates were investigated. When the benefit rate was increased to fifty percent of period 3 wage earnings, the intergenerational welfare effects were similar to those reported in section 4 for closed economies without technological progress. The baby-boom generation and their children experienced welfare improvements compared to their predecessors. Future generations were better off than those born before the baby-boom, but not as well off as the baby-boom generation and their children since these two generations benefit from pension contribution rates that are lower than those for future generations. The benefits from the public and private reduction in the child dependency burden are still greater than the costs of the increased public dependency burden of the elderly. When the benefit rate was increased to seventy-five percent in closed economies and to fifty percent in open economies without technological progress, future generations were worse off than generations born before the baby boom. This is because the much higher pension benefits cause a much larger increase in the elderly dependency burden, which more than offsets the benefits from a lower public and private child dependency burden. However, the baby-boom generation and their children still experienced welfare improvements compared to their predecessors because their pension premium rates remain low.[4] In model economies consistent with twentieth century labour productivity growth, increased labour force participation, and the evolution of social policy, benefit rates at 50%, 75%, and 100% of period three wages were investigated and generations remained substantially better off than their predecessors. However, pension premium rates of 36% of wages combined with income tax rates of 40% when the pension benefit replacement rate is 100%, might be considered excessive and politically unsustainable even with an increasing standard of living over time. These results suggest that economies with high pension benefit replacement rates and without technological progress should consider pension

[4] One might argue that, although the current pension replacement rate for an average wage-earner is 25% of pre-retirement wages, the total transfer to the elderly is higher. The Old Age Security, Guaranteed Income Supplement and Allowance payments to the elderly were almost equal to those of the Canadian public pension plans in 2000, and were financed in a pay-as-you-go fashion through general tax revenues: $24 billion in OAS, GIS and Allowance payments to 3.8 million recipients, and $25 billion in CPP and QPP payments to 3.6 million recipients (Statistics Canada, 2003: 29, 42-45).

reform that shifts some of the burden of pensions from future generations to the baby-boom generation and their children since these generations are the prime benefactors of the fertility transition to replacement fertility. A shift of the burden of future pension obligations onto current generations is regressive in economies with sufficiently high technological progress (in the model economy cases, 1% per year TFP growth is sufficiently large to keep welfare rising over time).

Model economies with different preference parameters were investigated. When agents have higher rates of time preference (lower discount parameter β), agents prefer to consume more, earlier in life. Carroll and Samwick (1997) suggested a range of point estimates for the rate of time preference for the US in the 1980s from 5% to 14% per year. Model economies populated by agents with annual rates of time preference equal to ten percent produced the same relative intergenerational welfare results as those presented herein, although rates of interest are substantially higher (15% annual interest rate with a 10% annual rate of time preference, versus 6% annual interest rate with a 1% annual rate of time preference).

Additional simulations where the fertility rate was allowed to drop below replacement were also examined. In model economies similar to those examined in Section 4, the completed fertility rate could drop to 1.5 (or 0.75 per young adult) before future generations were worse off relative to generations in the first steady state in open model economies without technological progress, and the rate could drop to 1.3 in closed economies without technological progress.

The model economies examined herein do not incorporate employment uncertainty or agents who do not supply labour, effectively eliminating the need for employment insurance and social welfare programs targeted at the working age population. The model economies also do not allow for unbalanced government budgets. In the 2000-2001 fiscal year, the combined spending on social welfare programs and debt charges was $172 billion (Treff and Perry, 2005: Table 1.2). Program spending specifically targeted to seniors (OAS, GIS, the Allowance) amounted to $24 billion (Statistics Canada, 2003: 29), which reduces the omitted program spending to $148 billion, or 32% of total government spending. For the model economies, incorporating these features would not only cause taxes to increase to finance this additional spending, but the model economies would also require a reduction in employment and production, further increasing the tax burden of those receiving taxable income. However, if program spending on non-workers in the working age population remains constant as a

proportion of GDP, with the number of non-workers fixed as a proportion of workers in the working age population, this omission in the model economies would appear to have a minimal effect on modeling results since relative program spending on these items would not be significantly altered during the demographic transition to replacement fertility. Furthermore, employment rates of each age group (genders combined) have increased over the twentieth century, decreasing the dependency rate of non-workers to workers in the working age population, and improving the ability of the economy to provide these services. It is only if program spending is more generous than the economy's ability to provide (or willingness to tax), that problems arise - specifically the resulting debt charges that may be passed on to future generations. On this point, Canadians can relate to the effects of increased program spending and debt in the 1980s and early 1990s as Canada had expanded social programs and dealt with two major recessions. The model economies enforce balanced budgets over each period (about 20 years) and ignore business and political cycles.

The model economies do not incorporate international migration. Canada has admitted an average of over two hundred thousand immigrants since 1990 (Citizenship and Immigration Canada, 2003: 3). Immigration at current levels for Canada is expected to provide insurance against a decline in population as the native fertility rate has dropped below the replacement rate (Beaujot and Matthews, 2000). The model economies compensate for no immigration by simulating replacement fertility. Modeling immigration would cause the costs of raising immigrants as children to be borne by the immigrants themselves, their parents, and foreign governments, instead of the domestic government.

The model economies do not include heterogeneity within a cohort group. They do not include a measurement of utility derived from children, other than that proxied by the utility value of consumption on children above the subsistence level, by young parents. Heterogeneity with respect to income and child-rearing decisions both would be important to model, to better understand if and how individuals from different income groups make different decisions on the quantity of children, and the amount of resources they dedicate to their children (the decision of "quality"). There has been an intense debate in Canada regarding transfers to parents and subsidized daycare, with many parents complaining to the government to help them raise children. It would be interesting to examine to what extent the Canadian pension and tax systems provide disincentives and/or incentives to raise children, and to what extent adults with children

are behind in their savings for retirement relative to childless adults. In addition, the model economies ignore other adjustments that affect intergenerational and intragenerational welfare. Members of families transfer income in various ways: older workers help out their ailing parents with transfers for private health care; retirees transfer income and wealth to their children and grandchildren; workers in the empty nest stage transfer income to their children to help setup new households or improve their human capital through education. Families provide insurance against bad outcomes across generations. While government social programs and other insurance products reduce the need for individuals to rely on family, decreased family size reduces the ability of families to spread risk across family members. At the extreme, adults who choose to have no children have limited access to family-provided insurance.

The model economies do not incorporate changes in life expectancy over time; the model economies focus on demographic changes resulting solely from fertility rate changes. As a result, the model economies have predominantly higher elderly population shares than the actual Canadian case in the periods before period 34 (representing the 2011-2031 time period in Figure 1). In 1941, the life expectancy of males at birth was 63 years, and at 60 years of age males could expect to live an additional 16 years on average (Urquhart and Buckley, 1983: Series B65, B71). In 2000, the life expectancy of males at birth was 77 years, and males aged 60 years could expect to live an additional 21 years, on average. Females experienced similar increases in life expectancies, although their life expectancies at every age have been higher than those of males. The implications of these changes are that the fiscal burden of the elderly is overstated in the earlier periods of the model economies. If these trends continue, the fiscal burden of the elderly may also rise above that in the model economies in later periods. Given the modeling results presented herein, the decline in fertility need not result in pension reform. However, continual changes in life expectancy may necessitate reform. The most frequent policy recommendation by researchers in the field is that the pension benefit-contribution link must be strengthened (De Nardi, Imrohoroglu and Sargent, 1999; Kotlikoff, Smetters and Walliser, 1999; Fehr, 2000; Sinn, 2000). In many cases, this recommendation is made to eliminate labour-leisure distortions during normal working years, with pension policy that does not penalize or compensate with a reduction in benefits for those who retire early or strategically reduce their labour supply during their working years. This recommendation can also be made to stabilize the ratio of working years to retirement years. If people are living longer, then the benefit-contribution link may be stabilized

by increasing the age of retirement. The sustainability of other public programs targeting retirees, including health care and transfers, is threatened the same way – either the retirement age must be increased or tax contribution rates raised to raise funding levels, or benefit rates must be reduced to lower spending levels. De Nardi, Imrohoroglu and Sargent (1999) showed the dramatic effect of gradually raising the retirement age on the elderly dependency rate, and on the fiscal ability to provide social services to the elderly.

7. Conclusion

Many nations are debating how to ensure the sustainability of public pay-as-you-go pension systems. Without any changes to the existing benefit formula, pension contribution rates would have to rise due to a decline in fertility and rising life expectancies. In this paper, the effects of the fertility decline have been examined using a dynamic overlapping generations general equilibrium model. The model is calibrated to the Canadian demographic, economic, and policy environment. The innovation in this paper is a formal treatment of both private and public child-rearing costs.

Model results suggest that policymakers and economists need not be concerned with the rise in future contribution rates as a byproduct of the decline in fertility in economies like Canada with modest public pension programs, because the private and public cost savings of raising fewer children are too large to be ignored: the decline in fertility lowers the combined public and private dependency burden of children and retirees on the working population. In economies with more generous public pensions (with benefits replacing over half of pre-retirement wages), and without technological progress, reforms that shift some of the future public pension burden onto current generations (the baby-boom generation and their children) would increase intergenerational equity. Technological progress, however, acts like a cure to the pension sustainability problem by allowing future generations to enjoy greater welfare than previous generations even with more generous pension systems, when facing a decline in fertility.

It appears that a stronger reason for public concern may be rising longevity and the extending length of retirement, and not the fertility transition. People are living longer and can be productive longer. Gradually raising the age of retirement can stabilize the ratio of working to

retirement years for public pensions, stabilize the ratio of retirees to workers, and ensure the sustainability of other government programs that provide benefits to the elderly.

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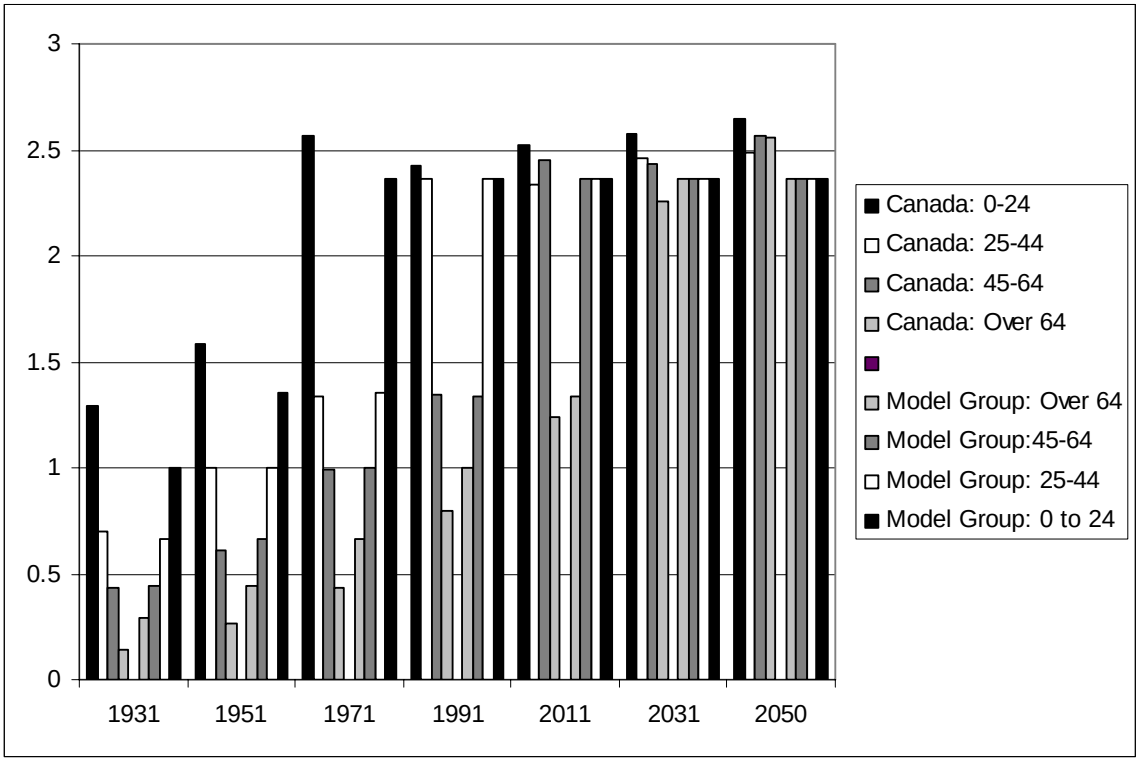

Figure 1: Population Shares for Canada and Model Economies

Note: Population shares are normalized with 1=Population of the 25-44 years of age group in 1951.

Table 1: 2000-2001 Public Expenditures Attributed to Age Cohorts

Age Cohort	2001 Population (in millions)	2000-2001 Public Education Expenditures (primary, secondary, tertiary, in millions)	Cohort-Specific Public Health Expenditures, 2000-2001 (in millions)	Cohort-Specific Public Health Expenditures Per Cohort-Member, 2000-2001 in dollars (as a ratio of the per capita figure)	Total of Cohort-specific Public Expenditures per cohort-member - Calibrated value of s_i	Cohort-Specific Private Health Expenditures, 2000-2001 (in millions)	Cohort-Specific Private Health Expenditures Per Cohort-Member, 2000-2001 in dollars (as a ratio of per capita figure)	Cohort-Specific Private Health Expenditures Per Cohort-Member - Calibrated value of h_i
0-24	10.082	$52, 256 ($5,183/child)	$10,218	$1,013 (0.45)	0.182w	$5,988	$594 (0.64)	Included in q
25-44	9.605	---	$12,621	$1,314 (0.59)	0.039w	$6,051	$630 (0.68)	0.019w
45-64	7.411	---	$13,431	$1,812 (0.81)	0.053w	$7,689	$1,038 (1.12)	0.031w
65+	3.923	---	$32,819	$8,366 (3.76)	0.246w	$8,891	$2,266 (2.46)	0.067w
Total	31.021	$52,256	$69,089	$2,227 (1.00)	---	$28,619	$923 (1.00)	---

Sources: Statistics Canada (n.d.) Table 051-0001 for population, and Tables 478-0004, 478-0007, 478-0014 for public education expenditures; Health Canada (2001), Table 2A for public health expenditures, Table 3A for private health expenditures.

Table 2: Open Economy Simulation Results with Demographic Change and No Technological Progress

Generation And Time	**Relative Cohort Size**	$\overline{U}_{t+1}$	$c_{2,t+1}$	$c_{3,t+2}$	$c_{4,t+3}$	$a_{2,t+1}$	$a_{3,t+2}$	p_{t+1}	η_{t+1}	τ_{t+1}	w_{t+1}	r_{t+2}
26	0.296	0.000	0.1144	0.2022	0.3573	0.0368	0.1241	0.0686	0.1205	0.4223	0.4624	2.0000
27	0.444	0.012	0.1145	0.2024	0.3625	0.0367	0.1235	0.0686	0.1205	0.4223	0.4624	2.0000
28	0.667	0.017	0.1147	0.2054	0.3579	0.0365	0.1276	0.0686	0.1205	0.4224	0.4624	2.0000
29	1.000	0.123	0.1201	0.2092	0.3828	0.0461	0.1277	0.0686	0.1116	0.4081	0.4624	2.0000
30	1.350	-0.069	0.1073	0.1963	0.3682	0.0215	0.1156	0.0729	0.1373	0.4375	0.4624	2.0000
31	**2.363**	**0.527**	**0.1350**	**0.2534**	**0.4644**	**0.0619**	**0.1638**	**0.0694**	**0.0913**	**0.3843**	**0.4624**	**2.0000**
32	2.363	0.501	0.1357	0.2487	0.4539	0.0724	0.1602	**0.0732**	0.0930	**0.3559**	**0.4624**	**2.0000**
33	2.363	0.281	0.1247	0.2275	0.4135	0.0564	0.1431	0.1280	0.1184	**0.3824**	0.4624	2.0000
34	2.363	0.256	0.1237	0.2249	0.4084	0.0554	0.1410	0.1280	0.1184	0.3871	0.4624	2.0000
35	2.363	0.238	0.1230	0.2233	0.4054	0.0543	0.1397	0.1280	0.1184	0.3918	0.4624	2.0000
36	2.363	0.235	0.1228	0.2229	0.4047	0.0541	0.1394	0.1280	0.1184	0.3926	0.4624	2.0000
37	2.363	0.233	0.1227	0.2228	0.4044	0.0540	0.1393	0.1280	0.1184	0.3930	0.4624	2.0000
38	2.363	0.233	0.1227	0.2228	0.4044	0.0540	0.1393	0.1280	0.1184	0.3931	0.4624	2.0000
39	2.363	0.233	0.1227	0.2227	0.4043	0.0540	0.1393	0.1280	0.1184	0.3932	0.4624	2.0000
40	2.363	0.233	0.1227	0.2227	0.4043	0.0540	0.1393	0.1280	0.1184	0.3932	0.4624	2.0000
41	2.363	0.233	0.1227	0.2227	0.4043	0.0540	0.1393	0.1280	0.1184	0.3932	0.4624	2.0000
42	2.363	0.233	0.1227	0.2227	0.4043	0.0540	0.1393	0.1280	0.1184	0.3932	0.4624	2.0000
43	2.363	0.233	0.1227	0.2227	0.4043	0.0540	0.1393	0.1280	0.1184	0.3932	0.4624	2.0000
44	2.363	0.233	0.1227	0.2227	0.4043	0.0540	0.1393	0.1280	0.1184	0.3932	0.4624	2.0000
45	2.363	0.233	0.1227	0.2227	0.4043	0.0540	0.1393	0.1280	0.1184	0.3932	0.4624	2.0000
46	2.363	0.233	0.1227	0.2227	0.4043	0.0540	0.1393	0.1280	0.1184	0.3932	0.4624	2.0000

Note: $\overline{U}_i$ denotes the lifetime value of utility relative to the first steady-state level of lifetime utility.

Table 3: Closed Economy Simulation Results with Demographic Change and No Technological Progress

Generation And Time	**Relative Cohort Size**	$\overline{U}_{t+1}$	$c_{2,t+1}$	$c_{3,t+2}$	$c_{4,t+3}$	$a_{2,t+1}$	$a_{3,t+2}$	p_{t+1}	η_{t+1}	τ_{t+1}	w_{t+1}	r_{t+2}
26	0.296	0.000	0.1021	0.2142	0.4493	0.0464	0.1415	0.0686	0.1108	0.3983	0.4250	2.5896
27	0.444	0.012	0.1021	0.2143	0.4564	0.0464	0.1412	0.0686	0.1108	0.3983	0.4250	2.5904
28	0.667	-0.016	0.1024	0.2181	0.4274	0.0461	0.1435	0.0686	0.1108	0.3983	0.4249	2.5987
29	1.000	0.144	0.1106	0.2168	0.4871	0.0508	0.1427	0.0687	0.1026	0.3855	0.4245	2.3655
30	1.350	-0.072	0.0963	0.2163	0.4355	0.0397	0.1378	0.0707	0.1261	0.4123	0.4380	2.7343
31	**2.363**	**0.280**	**0.1273**	**0.2562**	**0.3947**	**0.0565**	**0.1591**	**0.0729**	**0.0848**	**0.3635**	**0.4172**	**2.2028**
32	2.363	0.443	0.1589	0.2448	0.3819	0.0546	0.1448	**0.0681**	0.0849	**0.3393**	**0.4483**	**1.4040**
33	2.363	0.481	0.1601	0.2497	0.3898	0.0634	0.1493	0.1116	0.1118	**0.3742**	0.5143	1.4433
34	2.363	0.433	0.1567	0.2446	0.3843	0.0586	0.1455	0.1291	0.1203	0.3750	0.5103	1.4459
35	2.363	0.430	0.1559	0.2450	0.3852	0.0595	0.1460	0.1281	0.1197	0.3751	0.5100	1.4669
36	2.363	0.422	0.1553	0.2441	0.3842	0.0587	0.1454	0.1286	0.1197	0.3755	0.5079	1.4691
37	2.363	0.421	0.1551	0.2442	0.3844	0.0588	0.1455	0.1281	0.1194	0.3755	0.5077	1.4727
38	2.363	0.420	0.1550	0.2440	0.3842	0.0587	0.1454	0.1281	0.1194	0.3756	0.5073	1.4731
39	2.363	0.420	0.1550	0.2441	0.3843	0.0587	0.1454	0.1281	0.1194	0.3756	0.5073	1.4736
40	2.363	0.419	0.1550	0.2440	0.3842	0.0587	0.1454	0.1281	0.1194	0.3756	0.5072	1.4737
41	2.363	0.419	0.1550	0.2440	0.3842	0.0587	0.1454	0.1281	0.1193	0.3756	0.5072	1.4738
42	2.363	0.419	0.1550	0.2440	0.3842	0.0587	0.1454	0.1281	0.1193	0.3756	0.5072	1.4738
43	2.363	0.419	0.1550	0.2440	0.3842	0.0587	0.1454	0.1280	0.1193	0.3756	0.5072	1.4738
44	2.363	0.419	0.1550	0.2440	0.3842	0.0587	0.1454	0.1280	0.1193	0.3756	0.5072	1.4738
45	2.363	0.419	0.1550	0.2440	0.3842	0.0587	0.1454	0.1280	0.1193	0.3756	0.5072	1.4738
46	2.363	0.419	0.1550	0.2440	0.3842	0.0587	0.1454	0.1280	0.1193	0.3756	0.5072	1.4738

Note: $\overline{U}_i$ denotes the lifetime value of utility relative to the first steady-state level of lifetime utility.