THE RELATIONSHIP BETWEEN EXECUTIVE FUNCTION AND ARITHMETIC STRATEGY IN GRADES 4 TO 6

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Abstract

Current literature suggests that variance in conceptually-based strategy use on multi-term arithmetic problems is due to individual differences. The present study examined whether individual differences in conceptually-based strategy use was related to individual differences in executive function (EF). Through an additive and multiplicative problem-solving task, three arithmetic concepts were examined in 112 Grade 4-6 students: inversion (e.g., 2 + 5 – 5, or 7 x 4 ÷ 4), associativity (e.g., 2 + 29 – 27 or 7 x 9 ÷ 3), and equivalence (e.g., 4 + 2 + 7 = 4 + __ or 5 x 2 x 3 = 5 x __). An EF battery task was used to assess working memory, updating, inhibition, and switching components. Our primary hypothesis was that individual differences in arithmetic strategy use would be related to individual differences in EF. We also hypothesized that (1) participants would perform best and use inversion the most, followed by associativity, then equivalence, (2) there would be no grade differences (3) participants will perform better on additive than multiplicative problems. Correlational results indicated apparent links between EF capacities and arithmetic strategy use, with inversion correlating with inhibition, all three conceptual strategies correlating with updating, and equivalence correlating with all four EF components. ANOVA results indicated that participants perform best on inversion problems, but use equivalence strategies the most, individual rather than grade differences accounted for variance, and participants do better on additive problems. However, operation and concept ANOVA interactions suggest arithmetic strategy use may depend on operational and conceptual problem format.

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The Relationship between Executive Function and Arithmetic Strategy in Grades 4 to 6

A great deal of research has been conducted on the importance of mathematical understanding and cognitive abilities, as children's academic and real-world skills and successes can greatly depend on this knowledge and capacity (Bierman et al., 2008; Diamond et al., 2011; Knuth et al., 2006). In North American classrooms, children are taught a highly procedural method for solving that can conflict with conceptual understanding of mathematics. In formal education students are typically taught a repetitive, left-to-right procedure for solving two-term problems (e.g., 2 + 4 = __ or 3 x 5 = __). McNeil (2014) suggests that children may become entrenched in using this left-to-right algorithmic procedure, which can lead to misconceptions and difficulties in mathematics. A common mathematical misconception is that the equal sign is an indication that students need to perform action(s), which may not be necessary or effective for problem-solving. For instance, children will often see an equals sign in an equivalence problem and take it as a sign that they need to perform an action, such as adding the numbers on the left or adding the number on both sides of the equals sign, rather than making each side of the sign even. This misconception can be so powerful that it can be difficult to transfer instruction to new or novel problems, resulting in poor arithmetic performance (McNeil, 2014). The use of a left-to-right approach when solving different, complex, or lengthy questions (including those which involve understanding of the equal sign) can be mentally taxing, time-consuming, and prone to error (Knuth et al, 2006; Robinson, 2019). Instead of using strict, procedural approaches or knowledge to solve operations, research has shown that understanding various mathematical concepts, their underlying principles, and their relationships may produce more flexible, successful learning and solving (Robinson & LeFevre, 2012). The understanding of underlying mathematical concepts can sometimes be referred to as conceptual knowledge.

Conceptual knowledge is difficult to define, but has been previously described and measured in mathematics by an individual's ability to understand underlying principles and relationships of a particular concept, such as arithmetic (Bisanz & LeFevre, 1990). As with most developmental concepts, one would expect that conceptual knowledge and cognitive abilities improve with age and/or formal education. However, research has consistently shown that the development of conceptual knowledge does not improve based on age, but rather as a result of individual differences, such as attitudes and beliefs (Robinson & Dubé, 2012). Another potential source of individual difference may be found when examining executive function.

Executive function (EF) is defined as the skills and processes that guide, control, direct, or coordinate one's thoughts, concentration, and actions (Bull & Lee, 2014; Cragg et al., 2017; Diamond & Lee, 2011). The development and measurement of executive function remains vague and debatable. However, most of the current literature supports the idea that the executive regulatory system, which controls executive function, begins to dramatically develop when children enter formal education and can be influenced by biological, environmental, and individual factors (Bull & Lee, 2014; Diamond et al., 2011). EF is also commonly defined by the specific functions that it measures, such as working memory, updating, inhibition, and switching (Bull & Lee, 2014; Bull & Scerif, 2001; Siegler & Araya, 2005). Recently, studies have started to explore the connections between conceptual understanding and executive functions, which can increase understanding of child learning and development, and develop interventions that can improve children's skills and abilities (Diamond & Lee, 2011; Eaves et al., 2021). The current study aimed to investigate this connection and contribute data to the field, which may be highly beneficial for developing curriculum and interventions that can improve critical areas, such as mathematical success.

Conceptually-Based Arithmetic Strategies

Many research studies have devoted exploration into conceptual understanding of mathematics by measuring children's performance and demonstration of conceptually-based arithmetic strategies. Research has shown that using three-term arithmetic problems involving underlying mathematical concepts introduces participants to novel problems (Diamond & Lee, 2011; Siegler & Araya, 2005). Since children are often taught two-term problems (i.e. 2 + 4 or 3 x 5), researchers have used these novel three-term arithmetic problems to assess understanding of arithmetic concepts (McNeil, 2014; Robinson et al., 2018). Students are often unexposed to these three-term arithmetic problems which can be solved using their conceptual understanding of arithmetic. The use of arithmetic problems that are specifically designed to measure children's comprehension of underlying mathematical concepts and relationships can aid in understanding the role of conceptual knowledge in mathematical achievement. These kinds of arithmetic concepts and questions are explained in depth in the following sections.

Inversion

Piaget (1952) asserts that children's success in higher level mathematics relies on successful conception of addition and subtraction. Piaget (1952), defined inversion as the understanding that addition and subtraction are inversely related. This means that the inversion concept is based on the knowledge that adding and subtracting the same number will equal zero, and the initial number will remain unchanged. For instance, on inversion questions (8 + *26 - 26* = ___), understanding that both adding and subtracting 26 would cancel the second part of the equation and leave the first number (8) as the final answer, is a demonstration of the inversion strategy (rather than solving procedurally as 8 + 26 = 34 – 26 = 8). Thus, utilizing this strategy does not require any calculations (Baroody et al., 2009; Bisanz & LeFevre, 1990).

Bisanz and LeFevre (1990) studied three-term inversion problem solving using the format: $a + b - b =$ ___. In the thirty years since this study, the inversion strategy (sometimes referred to as the inversion shortcut) has been largely defined in the literature as a conceptually-based strategy that involves understanding the inverse relationship between addition and subtraction and has been examined using similar, three-term problems (Robinson & Dubé, 2009b; Robinson & LeFevre, 2012). Moreover, the importance of understanding the relationship between addition and subtraction has been largely emphasized. Current research shows great support in understanding the relationship between addition and subtraction, as this understanding is critical for success in later, more complex mathematics such as multiplication, division, and algebra (Knuth et al., 2006; Siegler & Araya, 2005). The use of inversion strategies also has clear advantages in improving accuracy and solution time on arithmetic problems (Bisanz & LeFevre, 1990; Robinson & Dubé, 2009b).

In recent years, researchers (see Robinson et al., 2006) have extended inversion problems beyond the addition and subtraction format by including the inverse relationship between multiplication and division ($a \times b \div b =$ ___). However, the multiplicative versions of the inversion concept have shown to be weakly understood and used, regardless of age. To illustrate, Dubé and Robinson (2010a) found that only two thirds of grade six students discovered the inversion strategy on multiplicative inversion problems. Moreover, even adults fail to demonstrate conceptual knowledge of inversion, especially with multiplication and division, which shows that individuals may not have or utilize conceptual knowledge regarding the inverse relation between multiplication and division when solving problems (Dubé & Robinson, 2010b; Eaves et al., 2021). These results can indicate that conceptual understanding and use of additive inversion is not predictive of the understanding and use of multiplicative inversion.

Additionally, the use of a multiplication and division inversion strategy may be influenced more by problem characteristics and individual differences than the addition and subtraction strategy (Robinson & LeFevre, 2012).

Associativity

In recent years, the concept of associativity has been defined as the understanding that the operations in a three-term problem ($a + b - c +$ = ___ or $a \times b \div c +$ ___) can be solved in any order to make solving more efficient (Eaves et al., 2021). Children can make addition and subtraction associativity problems ($a + b - c$) easier by subtracting the last two numbers before adding the first number (Eaves et al., 2021; Robinson, 2019). For instance, on an addition and subtraction associativity problem (3 + 9 – 6 = ___), students might utilize the associativity strategy to make computation easier by subtracting first (9 - 6 =3) and then adding the first number to this calculation (3 + 3 = 6) to find the answer. In contrast, moving from left to right (3 + 9 = 12, 12 – 6 = 6) would yield higher numbers and make problem-solving more challenging and error-prone (Robinson, 2019). Similar to inversion, associativity has also come to include multiplication and division problem formats (a x $b \div c$), but the use of associativity strategies has shown to be significantly less on these questions than on addition and subtraction questions (Robinson & LeFevre, 2012). The decreased use and understanding of conceptually-based strategies on multiplicative associativity problems may be due to the slower development and increased complexity of multiplication and division for children and adults (Dubé, & Robinson, 2018; Robinson & LeFevre, 2012).

The use of the associativity strategy is often much faster and less error-prone than standard, left-to-right computations (Downton et al., 2020; Eaves et al., 2021). However, associativity still involves calculations, which means that errors may be more likely than in concepts which do not require calculations to be performed, such as inversion. While applying conceptual knowledge to both associativity and inversion problems involves understanding the operations of addition, subtraction, multiplication, and division, associativity is considered to be more complex for learners (Robinson and Dubé, 2012; Siegler & Araya, 2005). Due to the computation involved in associativity, many studies suggest that inversion is easier, acquired earlier, preferred, and demonstrated more frequently than associativity (Robinson & Dubé, 2009b; Robinson et al., 2009, 2017). For instance, individuals may not implement the associativity concept because associativity problems closely resemble standard, control questions (i.e. $a + b$ – c which can be solved left-to-right) and they may not realize that it is easier to solve questions in this way (Eaves et al., 2021).

Additionally, research by Dubé and Robinson (2018) has shown that few children in Grades 6 through 8 have a firm comprehension of associativity, which may explain why children use standard, left-to-right procedures rather than utilizing conceptual knowledge and strategies. The associativity concept is believed to be more difficult for students to understand because it can overtax cognitive resources. Eaves et al. (2021) also suggested that misconceptions in the order of operations, such as a tendency to follow the American PEDMAS (Parentheses, Exponents, Multiplication, Division, Addition, Subtraction) acronym in a literal interpretation, may contribute to the low use of associativity strategy use while solving math problems. The difficulty in solving associativity problems may also be due to solvers having to allocate their attention to a number of criteria. Solvers have to pay attention to all three terms, the particular numbers that have been added and subtracted in the problem, simultaneously notice that the third number can be subtracted from the first, perform the subtraction, and then finally add the middle number (Robinson & Dubé, 2012).

Equivalence

The equivalence arithmetic concept can be described as the understanding that both sides of an equivalence problem ($a + b + c = a +$ ___) should be equal and interchangeable (McNeil, 2014). On equivalence problems (e.g., $5 + 28 + 3 = 5 +$ ___), employing a conceptually-based strategy would involve adding the second and third numbers on the left (28 + 3) to find the equivalent answer on the right (31). Many children also attempt to solve equivalence questions using a left-to-right procedure by adding all of the numbers on the left (5 + 28 + 3 = 36) and then subtracting the number on the right of the equal sign from that previous calculation (36 – 5) to get a final answer (31). Evidently, this process is much longer, requires more effort, and has been shown to lead to more errors and worsened solution latencies (McNeil & Alibali, 2005; Robinson, 2019). While the left-to-right approach may be more taxing and can lead to greater errors, the ability to make both sides of the equivalence problem equal and interchangeable by working from left to right also demonstrates understanding of the equivalence concept. Thus, there are two ways to correctly approach equivalence problems that both demonstrate an understanding of the equivalence concept.

Similar to associativity, research has shown that many children struggle to grasp the concept of equivalence. For instance, children who do not understand the significance of the equal sign or interpret it as a cue to “do something,” will often employ strategies that yield incorrect responses (Kieran, 1981). Knuth et al. (2006) investigated understanding of the equal sign in grade six and eight students and found that many children lack a firm understanding of the equal sign, resulting in greater incorrect responses on equations. McNeil and Alibali (2005) explain that children may attempt to add all three numbers on the left (5 + 28 + 3 = 36) and use this as the answer. Children may also attempt to add all four numbers in the equation (5 + 28 + 3 + 5 = 41) and use this as their final answer. Further, McNeil (2014) conducted a study of equivalence on children aged 7 to 11 and found that older participants actually performed worse than younger participants and that only 20% of participants successfully demonstrated conceptual understanding of equivalence. McNeil (2007) suggests that this may be due to students becoming inflexible learners as they age, since formal education has entrenched particular problem solving strategies.

As with inversion and associativity, equivalence can also include a multiplicative and division format ($a \times b \times c = a \div$ ___). However, compared to inversion and associativity, relatively little research has been devoted to the multiplicative and divisive versions of the equivalence concept. Robinson et al. (2017) found that relatively few students exhibit a conceptual understanding of the equal sign and that this understanding does not appear to improve across the middle grades. Robinson et al. (2017) found that equivalence (and associativity) understanding was the weakest of the arithmetic concepts in grades 3-5 students. These findings are concerning, as this understanding forms a basis for success in algebraic mathematics that is learned in adolescence (Eaves et al., 2021; Kieran et al, 1981; Knuth et al., 2006; McNeil, 2014). While the study of multiplicative and divisive equivalence problems has been limited, even less research has explored the relation of equivalence to executive functioning.

Executive Function

The definition, measurement, analysis, and development of executive function (EF) and its various components are not concrete. However, research generally suggests that executive function can be defined by the various concepts that it includes and measures, particularly inhibition, updating, and shifting (Cragg et al., 2017; Miyake, 2000). These three components of executive function may even be referred to as a “three-domain structure” due to their importance in EF (Bull & Lee, 2014, p. 37). Bull and Scerif (2001) explain that many studies in the field are vague on their definitions of the executive and often treat it as a single system rather than a combination of various systems. There are also great debates over what the executive components consist of and how they can be measured. Some researchers suggest that updating and working memory can be defined as the same function (Cragg et al., 2017; Miyake, 2000). However, other researchers in the field argue that shifting and working memory can be distinguished as their own separate but related concepts (Bull & Lee, 2014). This approach defines updating as the processing, replacing, and recalling of information in the brain, while working memory is the amount of information that can be held in the brain for a short period of time (Bull & Lee, 2014). Nevertheless, most of the current literature considers working memory, updating, inhibition, and switching as separate, but highly interrelated components that rely on and interact with each other for successful executive functions (Bull & Lee, 2014). The current literature also generally agrees that the most appropriate method to measure EF is by examining its components using multiple measures in a cognitive battery (Bull & Scerif, 2001). Thus, using a combination of specific tasks or measures to analyze the various components of executive function may reveal information about the role and importance of executive functions in grades 4-6 children. These four executive functions and the tasks that can be used to measure them are detailed in the section below.

Working Memory & The Corsi-Forwards and Backwards Tasks

As mentioned, working memory is often considered as the amount of information that individuals can retain in their brain for a limited amount of time. While this process may involve updating, where information must be remembered and replaced continuously, working memory focuses more on the span of information that can be temporarily remembered (Bull & Lee, 2014). A common measure used to assess the capacities of working memory is the Corsi block task (Higo et al., 2014). Forwards and backwards versions of this test can be administered to determine an individual's ability to store and recall information in either direction. In these tasks, a sequence of target blocks must be encoded in the order they are presented and then participants must recall and reproduce the same order (Higo et al., 2014). For instance, a number of blocks may light up one by one in sequential order and the number of blocks that are correctly remembered in their standard (forwards) and oppositional (backwards) order may reveal information about the directional spans of working memory. The ability to encode and recall block sequences with a high number of target blocks is associated with a greater working memory span and capacity (Higo et al., 2014). Thus, the Corsi block task can be an effective tool for analyzing the capacities of working memory.

Updating & The 2-Back (N-Back) Task

Updating is an executive function where individuals hold and retrieve a small amount of information in the brain (Cragg et al., 2017; Miyake, 2000). As Meule (2017b) mentions, a common measure used to assess working memory capacities is the N-Back task and its variations. The 2-Back task, for example, is a variation of the N-Back task where participants are presented with two trials of stimuli to which they must respond. Updating can be analyzed by measuring participants' ability to successfully match stimuli on N-Back tasks through examining participants' accuracy and efficacy (reaction time) in recalling stimuli (Meule, 2017b). Participants must keep track of the previous stimuli and then respond appropriately when the stimuli that is presented matches the same stimuli that was presented n trials before (Meule, 2017b). For instance, a participant may be presented with a sequence of letters and expected to match the letter from two trials ago by holding the letter information and responding accordingly after every two stimuli presentations. Moreover, individuals must constantly update the information in their working memory as new stimuli is presented (Bull & Lee, 2014; Cragg et al., 2017). Thus, the components and capacities of working memory are utilized in such a task and can be measured using N-Back task, or its 2-Back variation.

Inhibition & The Go/No-Go Task

Inhibition is defined as the ability to monitor and manipulate information stored in the brain, suppress other distracting or irrelevant information, and avoid an improper response (Bull & Lee, 2014; Miyake et al., 2000). As Meule (2017a) mentions, one of the most popular psychological tasks that measures inhibition is the “Go/No-Go” task. In this task, individuals are typically required to perform a motor response to particular stimuli as quickly as possible and avoid the same response for alternative stimuli (Meule, 2017a). For instance, a student might be asked to tap the spacebar on their keyboard in response to a certain visual cue and avoid pressing the key when presented with a separate visual cue. Meule (2017a) explains that there is some confusion in research designs over interpretation, measurement, and definition of the markers of inhibitory control. Although most researchers measure reaction time, some researchers emphasize the calculation of omission errors (not responding on trials where participants are expected to respond), and other researchers emphasize measuring commission errors (responding on trials where participants are expected not to respond). However, most studies tend to measure the executive function of inhibition by analyzing reaction time and the number of commission errors, or false alarms, which researchers agree demonstrates whether a participant can effectively control their responses to irrelevant stimuli (Meule, 2017b). Therefore, the Go/No-Go task can be effectively used to analyze inhibitory control.

Switching & The Wisconsin Card Sorting Task (WCST)

Switching is a component of executive function that involves the ability to successfully and flexibly shift between mental sets or tasks (Bull & Lee, 2014; Miyake, 2000). Researchers have commonly implemented card sorting tasks such as the Dimensional Change Card Sorting Task (DCCS) or Wisconsin Card Sorting Task (WCST) to measure switching. These tasks require participants to sort cards according to particular criteria and then switch to other sorting criteria (Bierman et al., 2008). The Wisconsin Card Sorting Task (WCST) involves sorting cards according to colour, shape, or number. In WCST, for instance, a participant may be required to solve cards using a shape categorization before the sorting criteria changes to number, and then implement this new rule for card sorting. Successful switching is often associated with the ability to apply new sorting criteria quickly and with relatively few errors (Bull & Scerif, 2001). However, studies have shown that many children struggle to successfully switch to new rule categories when sorting, as previous sorting criteria may persevere (Bull & Scerif, 2001; Söğüt et al., 2021). As a result, a participant may attempt to apply the previous rule, such as colour, when they are required to sort the cards using a new rule, such as shape. These errors are often referred to as perseveration or preservation errors, and studies indicate that these errors are correlated with low performance on WCST and switching abilities, since the participant cannot 'switch' from the previous to the current category (Söğüt et al., 2021).

Switching is often measured with tasks that use switching rules, where retaining different pieces of information, while also inhibiting particular actions, is necessary. As a result, switching may be more difficult to examine independently, since common switching tasks also involve working memory and inhibition (Bierman et al, 2008; Bull & Scerif, 2001; Diamond et al., 2005; Söğüt et al., 2021). However, Miyake et al. (2000) suggest that performance on WCST is primarily impacted by a participants' ability to switch between particular strategies, rather than the ability to update or inhibit. Thus, the number of categories a participant can successfully switch between, with relatively little perseveration or general errors on tasks like the WCST, can be effective in measuring the switching component of executive function.

The Relationship between Executive Function Components and Arithmetic Concepts

Inversion & EF Components

Most of the research regarding the link between executive function and conceptually-based strategy use has focused on additive inversion. Research regarding the multiplicative format of inversion in relation to EF has been relatively unexamined, but the same principles involved in additive inversion are likely to apply to multiplicative inversion. Working memory and better inhibitory skills are believed to play a role in the ability to generate conceptually-based problem solving strategies and understanding of the inversion concept (Dubé & Robinson, 2010a; 2017; Robinson, 2019). Rasmussen and colleagues (2003) conducted a study on the role of working memory in preschool children using a Corsi span task and found that accuracy on inversion problems was highly correlated with better working memory. Siegler and Araya (2005) proposed a theoretical model known as the “Strategy Choice and Discovery Simulation (SCADS)” model that outlines the various strategies that individuals may have, how they decide which strategy to choose, and how strategies may be used or not used when solving certain problems. The model suggests that in order for children to discover a new strategy, they may be required to attend to important parts of the problem and then determine, from their knowledge and strategic skills, which of the potential strategies will be the most efficient. Researchers have applied this model to understanding arithmetic strategy choice in mathematics, but it may also reveal underlying executive functions that contribute to conceptual knowledge. For instance, Robinson and Dubé (2009b) apply the SCADS model to the inversion strategy, suggesting that successful solving may depend on the ability to shift attention to and encode the inverse relation between the last two terms in a problem (ex. $b - b$ or $b \div b$). This plays into executive function components such as working memory, inhibition, and switching. Updating may also play a role in inversion problem solving, as problem-solvers may be required to update the information stored in the brain while approaching the inverse portion of a problem, although this relationship is still unclear and unsupported in research. Thus, EF has important connections and applications to additive and multiplicative inversion.

Associativity & EF Components

The link between associativity and executive function components is less developed compared to inversion strategy use and EF. Eaves et al. (2021) suggest that working memory can be essential in creating mental images of the additive associativity problem ($a + b - c$), which can allow individuals to direct attention to different parts and operations in the problem, find the shortcut, evaluate the right-hand side, navigate and compare it with the left and finally, choose a favoured solution. Updating has been relatively unexamined in relation to associativity problems. However, like inversion, updating may also be essential for information and calculations, such as calculating the second portion of the question ($b - c$ or $b \div c$) and then updating memory with the number required for adding or multiplying the first number. Eaves and colleagues (2021) also suggest that inhibition may be essential while solving associativity problems because the instinct to utilize a left-to-right approach must be overpowered in order to choose a less common, yet more efficient associativity shortcut (Eaves et al., 2021). Robinson and Dubé (2009b) also suggest that encoding and shifting attention to the last two numbers on the right in an associativity problem is essential for efficient problem solving, rather than following the common left-to-right method. Shifting is often characterized by the ability to flexibly shift between mental tasks. Students' ability to flexibly apply their conceptual knowledge on both additive and multiplicative associativity problems, which are similar to left-to-right control problems, is considered critical for efficient associativity strategy applications (Robinson & LeFevre, 2012). While the research remains underdeveloped, EF components may be very useful for effective arithmetic performance on both additive and multiplicative associativity problems.

Equivalence & EF Components

The equivalence concept has been even less explored than inversion and associativity in relation to executive function components of working memory, updating, switching, and inhibition. While Rasmussen and colleagues (2003) conducted a study regarding the link between inversion strategy use and working memory, they raised an important proposition regarding the relationship between working memory and equivalence. They suggested that children who can adequately represent sets and changes to sets may be more likely to notice equivalence concepts of sets prior to and following transformations with the same number. Therefore, working memory may be an important component in equivalence problems, which requires similar notice of sets and the equivalent numbers on either side of the equal sign. Updating may be critical for computations and problem solving in equivalence problems by updating memory with the calculations of the left side of the equal sign in order to apply it to the right side and find the final answer. This is especially true if children utilize the left-to-right approach to solving equivalence problems, although the link between updating and equivalence remains unexplored in the literature. McNeil (2007) mentions that children must ignore and override their operational representations in order to solve equivalence problems correctly. Thus, the inhibition of irrelevant information in performing adequate problem strategies may be important in equivalence strategy use, but this relationship also remains unclear. The ability to effectively switch may also be useful to switch between the left and right sides of an equivalence equation, or from standard questions (i.e. $a + b - c$) to alternative problems, such as equivalence questions. However, theories regarding the importance of the executive function components in solving additive and multiplicative equivalence problems is not well supported or documented in the literature. More importantly, the distinction between additive and multiplicative equivalence problems in relation to EF has been unexplored.

General Links between Arithmetic Concepts & EF Components

In a more general sense, the current literature suggests that executive function may be vital in performing arithmetic strategies, particularly those which are complex and involve many phases. Dubé and Robinson (2010) demonstrated that children are more likely to use conceptually-based procedures on three-term problems if they have better working memory. Bull and Lee (2014) also suggest that executive functions, such as updating, may be important in overall mathematical achievement because it can help children retain and retrieve information that is critical during solving. Inhibition and shifting have also greatly contributed to the ability to devise a plan, while problem solving and working memory is necessary in order to carry out a solution by aiding in selecting, monitoring, and implementing proper solving strategies (Viterbori et al., 2017). For instance, this could be important in selecting what strategy to use when approaching inversion, associativity, and equivalence problems. Kotsopolous and Lee (2012) conducted a study of eighth graders' problem solving using verbal behaviour, which further supports the role of EF in the various phases of problem solving. In particular, they discovered that updating can be problematic while understanding the question, inhibition can be essential in the phase where one plans which strategy to carry out, and shifting can be vital in the phase that evaluates which solution to use (Kotsopolous and Lee, 2012). Further, Bull and Lee

(2014) suggest that shifting may be essential when solving different operations, using distinct solutions, and dealing with the many steps required to solve complex, multi-term problems. Therefore, executive functions such as working memory, updating, inhibition, and switching may reveal important relationships and interactions between EF and conceptual strategy use and understanding in arithmetic.

As Diamond et al. (2011) discuss, few studies have examined the link between executive function in relation to arithmetic understanding. Nevertheless, EF skills have been well supported for their importance in arithmetic achievement (Bull & Lee, 2014; Cragg et al., 2017, Viterbori et al., 2017). Research has shown that the use of conceptually-based arithmetic strategies may be affected by or utilize particular executive function components while solving math problems. The relationship between these particular components and concepts have received little attention in the field. In addition, the relationship between additive and multiplicative operation problem formats in relation to arithmetic strategy use of inversion, associativity, equivalence and executive function skills has been largely unexplored and underdeveloped.

The Current Study

To our current knowledge, no studies have examined executive function and its updating, shifting, inhibition, and working memory components in relation to the conceptually-based strategies of inversion, associativity, and equivalence within a single study, particularly one involving both additive and multiplicative versions of the three arithmetic concepts. The present study aimed to investigate the relationship between these executive function components and arithmetic understanding, particularly in relation to additive and multiplicative inversion, negation, associativity, and equivalence. The current study aimed to explore this relationship in greater depth, although the particular nature of this relationship is unclear.

The research generally supports a link between conceptual understanding and executive function. Our primary research question aimed to determine if individual differences in conceptual understanding is related to or may be accounted for by individual differences in EF. We hypothesized that there would be a relationship between individual differences in conceptually-based strategy use and executive function. More specifically, we predicted that students who use conceptually-based arithmetic strategies would have higher performance on EF tasks. We also had a number of secondary research questions and hypotheses.

For the secondary research questions and hypotheses, we first aimed to determine whether there would be differences in the use and performance of conceptually-based strategies (inversion, associativity, and equivalence). Based on previous research studies, we hypothesized that participants would use and perform inversion the most, followed by associativity, and lastly, equivalence (Robinson et al., 2017; 2018). Second, we wanted to investigate if there were grade differences among Grade four, five, and six students in arithmetic strategy use and executive function performance. We hypothesized that there would be no observed grade differences between students in higher and lower grades, as the research supports that most variability in conceptual strategy use can be accounted for by individual differences (Robinson & Dubé, 2012). Our third and final research question was whether there was a difference in arithmetic performance based on the operation format (addition/subtraction or multiplication/division) of each problem type. We hypothesized that students would perform better on addition and subtraction problems and worse on multiplication and division problems. This is especially true as multiplication and division problems are considered more complex and even adults fail to demonstrate the conceptual knowledge of the arithmetic concepts on these problems (Dubé & Robinson, 2010b; Eaves et al., 2021; Robinson et al., 2018).

Methods

Participants

We aimed to obtain 38 participants in each grade, based on a G*Power analysis that was performed prior to participant recruitment. In total, there were 43 grade four, 40 grade five, and 29 grade six participants (N = 112). Participants were recruited via Facebook, Instagram, and Google advertisements that targeted parents throughout Saskatchewan and later expanded to parents across Canada. The study's demographics involved a majority of Caucasian, middle-class, Canadian students, primarily from Saskatchewan. Additionally, all participants had technology that allowed them to use Psytoolkit and Zoom services within their homes. Children who received parental consent and gave voluntary assent were included in the study. This study was approved by the University of Regina's Research Ethics Board and took place between November 2021 and March 2022.

Materials

The same paper sheets were used to record participants' arithmetic answers and strategies. Researchers were equipped with laptops compatible with Psytoolkit and Zoom services as well as an external video camera to record the audio of participant answers on arithmetic tasks. Additionally, all participants had technology that allowed them to use Psytoolkit and Zoom services within their homes. All students participated in a one-on-one Zoom session with a research assistant for a duration of approximately 60 to 90 minutes. In these sessions, participants completed the arithmetic task followed by the executive function cognitive battery.

Procedure

Arithmetic Task

The arithmetic problems were administered to participants through a Google Slides presentation, which was screen shared from the researcher's computer over Zoom. First, participants were read instructions verbatim. Next, participants completed two practice questions (see Appendix A) before progressing to the real problems. The real questions consisted of 12 additive problems, followed by 12 multiplicative problems (see Appendix A). There were a total of four additive questions for each of the inversion, equivalence, and associativity concepts. There were also four each of inversion, equivalence, and associativity problems included in the multiplicative section. The questions were distributed such that each problem type did not appear twice consecutively. Additionally, all problems were designed to produce a whole number. Each question was presented on an individual slide and participants were instructed to fill in the blank by providing the answer as well as an explanation of the strategy used. Participants were not given a time limit, but were asked to refrain from using a pen and paper, a calculator, or anyone's help. This was to ensure participants were solving the questions through their own knowledge. If participants were unable to answer a question, they were asked to describe the strategy they were attempting to use while solving the problem or what strategy they might use. The researchers used prompts to help the children explain how they were attempting to solve the problem if the participants were struggling or gave an unclear explanation (e.g., How were you trying to get that answer? If you were to solve this question, what would you do first?).

Participant responses and strategies were recorded by the researcher using arithmetic coding data sheets. Researchers also used an external video camera to record participants' verbal responses. The video camera did not record faces, only the participants' verbal responses, to allow for revision by researchers if potential errors in coding were made. To avoid influencing participant performance, researchers refrained from providing feedback on accuracy or strategy during the arithmetic tasks or using leading questions during participant strategy explanations. When all 24 arithmetic questions were answered and coded, the external video camera was turned off before progressing to the next section of the study. The length of time that the participant took to answer the arithmetic problems was also recorded for analysis.

In the arithmetic task, the experimenters coded participant responses based on the accuracy of their answers as well as the strategies used. For participants' accuracy, answers were coded on a three-item scale (See Appendix B). If participants gave the correct response, this was recorded as "Correct." If participants gave an incorrect answer, this was recorded as "Incorrect." Finally, if the participant's response was cut off or they did not know the answer, this was coded as "Cut Off/Did Not Know." In addition to the accuracy of responses, the experimenters also recorded the strategies that participants used while solving the problems (See Appendix B).

If the participants used one of the conceptually-based strategies (INV, N, ASSOC, EQ), they were coded as "Conceptually-Based Strategy Use." For instance, on the inversion questions ($a + b - b =$ __ or $a \times b \div b =$ __), if participants used the inversion strategy to cancel the b's and answered with the original number, this indicated the use and understanding of the inversion concept ("Inversion/INV"). If participants were able to simplify associativity problems ($a + b - c =$ __ or $a \times b \div c =$ __) by subtracting c from b before adding to the a or dividing c from b before multiplying to the a, this was an example of the associativity concept ("Associativity/ASSOC"). As previously mentioned, there are two ways to solve equivalence problems ($a + b + c = a +$ __ or $a \times b \times c = a \times$ __) using conceptually-based strategies. First, if students were able to recognize the a's on both sides of the equal sign and simply add or multiply the b and c to make both sides of the equal sign equivalent, this demonstrated the equivalence concept and was coded as “Equivalence/EQ.” Similarly, if participants approached the equivalence problem by adding or multiplying all of the numbers on the left side of the equivalence question ($a + b + c = a +$ __ or a x b x $c = a +$ __), and then figured out the number on the right side by adding or multiplying (or subtracting and dividing from the answer on the right) to determine the answer on the left, this was also coded as “Equivalence/EQ.” In both instances, participants demonstrated conceptually-based strategy use through their understanding of the equals sign and the need to make each side interchangeable.

If participants solved the question by moving from left-to-right (LR), their strategy was coded as “Left-to-Right.” For instance, the participant displayed this strategy if they moved from left to right on an inversion question ($a + b – b =$ __ or a x $b \div b =$ __) by adding or multiplying a and b and then subtracting or dividing the b at the end. On associativity problems ($a + b – c =$ __), if participants did not subtract c from b first, but rather began with a, added b, and then subtracted c, this was an example of the left-to-right strategy. In both instances, participants did not utilize the aforementioned conceptually-based strategies (INV or ASSOC), but solved by moving from one side of the equation (left) to the other (right). Additionally, children who did not demonstrate understanding of an equal sign for equivalence problems question ($a + b + c = a +$ __ or a x b x $c = a +$ __) were coded as “No-Equivalence” (No-EQ). In this case, children often attempted to solve equivalence problems by adding or multiplying all three of the numbers on the left together (*a*, *b*, and *c*) and fitting them into the blank or they would add or multiply all four of the numbers in the problem (*a*, *b, c*, and *a*) and use this as their final answer. If participants used these two strategies mentioned, it was evident that they did not understand the significance of the equals sign and thus, the equivalence concept.

On inversion questions ($a + b - b$ or $a \times b \div b$), if the participant attempted to solve the question from left to right, but realized the inversion strategy, they were coded as "Negation/NEG." For instance, if participants began by moving left to right by adding or multiplying *a* and *b* and then realized that the *b*'s would cancel, this was coded as "Negation (N) of Inversion." Negation coding could only be applied to inversion questions. While negation is conceptually-based, it is not a pure demonstration of conceptual strategy use. Therefore, for the purposes of this study, despite negation being at least partially evident of children's understanding of the inversion concept, only participants who used the inversion strategy were credited for using a conceptually-based problem strategy.

Finally, all other strategies were coded as "Weird/Other/Guessing." For instance, if students added instead of subtracted on the arithmetic questions or simply guessed instead of utilizing any particular solving strategy, this was coded as "Weird/Other/Guessing." During the session, researchers had the option to code the questions as "Other" or "Code Later" so that arithmetic answers that were not intuitive to code could be assessed again by the coder or another researcher. This ensured that the answers were coded appropriately by allowing additional analysis and insight before coding the responses into one of the five previous categories.

Executive Function Tasks

Following the arithmetic task, participants completed several different tasks to measure the various components of executive function via Psytoolkit. Researchers sent participants and their parents/guardians a link to the Psytoolkit tasks using the Zoom "chat" feature and then instructed participants on how to share their screen with the researcher. Participants or their parents/guardians were directed to enter their corresponding participant number into the box before advancing on to the tasks.

Corsi Forwards & Corsi Backwards Tasks.

The first section of the executive function tasks included the Corsi forwards and Corsi backwards tasks (see Appendix C), which were used to assess working memory span. These tasks were demonstrated by the researcher via the ‘shared screen’ process. Following the researcher’s demonstration, participants would share their screen again with the researcher so that the researcher could read the instructions and watch the participant complete the first task. The Corsi tasks involved remembering and recalling the correct order in which squares lit up. The pattern and the squares that lit up changed with each trial. As the participant progressed, more squares would light up, increasing the difficulty. For instance, the first trial involved two out of the nine squares lighting up. If the participant correctly replicated the pattern, the next trial would involve three squares to remember. This pattern would continue until all nine squares lit up. If the participant answered incorrectly, they were given another chance to complete the same target number of squares. If they correctly matched this re-creation, the task would continue to progress and the number of squares lighting up would increase. However, if they answered incorrectly twice, the task would end and the participant would move on to the backwards task.

In the Corsi backward task, the participant was instructed to use the same procedure as the forward task, but in the opposite order. In this variation, the participant had to recreate the pattern using the backwards order, starting with the last square that lit up and ending with the first square that lit up. Similar to the forward version, the order of squares to remember changed and increased in number and difficulty with each trial. In this version, participants were also given another chance to answer the order correctly but if they answered incorrectly twice, the task would end. The spans, or the number of squares that the participant successfully remembered and repeated, were recorded for both the forward and backward Corsi tasks. For instance, if a participant was successfully able to remember four squares before they had two consecutively incorrect trials, their maximum span would be recorded as four. Additionally, the average span of both of the tasks was calculated to find a mean Corsi span.

N-Back (2-Back) Task.

After the Corsi forward and backward tasks, participants completed the N-Back task (See Appendix D), which was administered to assess the updating component of executive function. Students completed the 2-Back task variation of the N-Back task, as this was deemed by researchers as the most appropriate for the current age group. In this task, participants completed a practice round with a modified version of the task that the researcher explained as the participant completed it. In the practice round, the two previous letters were shown before the target letter to make it easier to visualize and determine if the target letter was the same as the letter two turns ago. In the real task, the students only saw one letter at time. As a result, participants had to rely on their memory to determine if the letter on the screen was the same as the letter from two turns ago. If the participant answered correctly, they would hear a “woohoo” sound. If a participant answered incorrectly by clicking on a letter that was not a match or if they did not click on a correct match, they heard an “oops” noise. The task had a short break in the middle and the participant could click the mouse once they were ready to continue again.

On the N-Back, or 2-Back task, the researchers measured participant performance by recording the accuracy and mean response time (RT) of each participant. To calculate accuracy, researchers divided the total number of trials by the number of correct trials. In this case, there were 50 trials with a short break in the middle of the task. For the mean response time, the researchers calculated the RT based only on match trials. Since a new letter was presented for each trial and participants were expected to match the letter from two turns ago, participants would not be able to match until they were exposed to at least two letters. In other words, participants could not match letters until the third trial for trials prior to the scheduled break and until the twenty-eighth trial following the break. Thus, the first two trials (1 and 2) at the start of the task were excluded along with the first two trials following the break (26 and 27). Additionally, the trials allotted participants 3000 milliseconds (ms) to respond. Thus, trials that were 3000ms were indicated as "no response" and were excluded from the analysis. Trials where the participants were expected not to respond (correct miss) or they were expected to respond and they did not (incorrect miss) were excluded. In this regard, the RT was only recorded for trials where the participants were expected to match the letters. For instance, if a participant had 16 match trials, the response time of each trial would be added together and then divided by 16 to calculate the final RT average. Thus, the RT for each match trial was summed and then divided by the total number of match trials to calculate each participant's mean response time.

Go/No-Go (G/NG) Task.

Following the 2-Back task, participants completed the Go/No-Go task (See Appendix E) to measure inhibition. In this task, participants engaged in a practice round in which the researcher could guide them and explain the task. Participants were instructed to tap the spacebar when the green "Go" oval appeared on their screen. When the red "No-Go" oval appeared, participants were instructed to avoid pressing the space bar and simply wait for the next round. Participants were encouraged to go as fast as possible without making mistakes. After 75 trials, a break was automated for a brief period before participants could continue the following 25 trials.

The mean response time and the false alarm rate were recorded to measure participant accuracy on the Go/No-Go task. The mean response time (RT) was calculated only for go trials. Therefore, since there was a total of 75 go trials, the response time for each go trial was summed and then divided by 75 to obtain a final mean RT. False alarms are instances where participants incorrectly respond on “No-Go” trials by pressing the space key when expected not to. To calculate the total number of false alarms, the researchers simply summed the total errors on No-Go trials. For instance, if a participant accidentally pressed the space key twice during these “No-Go” trials, their false alarm rate would be two.

Wisconsin Card Sorting Task (WCST).

The final task that participants completed was the Wisconsin Card Sorting Task, or WCST (See Appendix F), which assessed the switching component of executive functioning. In this task, participants watched a demonstration through the researcher’s shared screen. Then, the participants shared their screen with the researcher again so that the researcher could instruct and guide them through a practice round. Finally, participants progressed to the actual task. Participants were instructed to match the bottom card with one of the four cards at the top of their screen using three pattern match rules. Participants could match a card if it had the same colour, shape, or number of shapes. The participants were also informed that the rule for matching the cards would change throughout the game, meaning they had to figure out the new match pattern if they started to get them wrong. In this task, each match pattern rule was used for ten consecutively correct trials before switching to a new rule. The participants either completed all six matches, or categories, or reached a total of 128 trials before the task ended.

The researchers calculated the total errors, perseveration errors, and the total number of categories that participants achieved on the WCST. When participants chose a card that did not match the target pattern/rule (either colour, shape, or number of shapes), this was recorded as an error. The sum of these errors was calculated to indicate a participant’s total number of errors. Additionally, the researchers recorded the total number of perseveration errors. Perseveration errors, also referred to as preservation errors, are errors where participants attempt to use a previous rule pattern after a new pattern has been introduced (Landry & Mitchell, 2021). For instance, if participants had previously matched the cards based on colour, they may attempt to use this colour rule again, even if the new criteria switched to shape or number of shapes. Similar to the total errors, the sum of these errors was calculated and indicated a participant's total perseveration errors.

Finally, the experimenters recorded the total number of categories that participants successfully achieved. A participant could either obtain the maximum of six categories (colour, shape, number, colour, shape, and number) or a total of 128 trials. Participants were required to achieve ten correct consecutive trials for a particular rule before they progressed to another rule. To calculate the total number of categories, the researcher analyzed the number of rule switches, or categories, that the participant went through as well as the number of consecutive trials in each rule/category to ensure that there were ten in total and they had successfully achieved the said category. For instance, if a participant was able to go through five categories (colour, shape, number, colour, and shape), but they only achieved nine consecutively correct number rule trials before the 128th trial, they would have achieved a total of five categories.

After all the executive function tasks were completed, the experimenter completed a short debriefing. The researcher thanked the child for participating in the study and told the child that their parent or guardian would be emailed an Amazon gift card and a participation certificate. The researcher then asked the participant and the parent or guardian if they had any questions that the researcher could answer. After answering any questions, thanking the participant and parent or guardian, and offering farewell, the Zoom meeting was ended by the experimenter.

Results & Discussion

Correlation Analysis: Results & Discussion

To test the primary research question, whether individual differences in conceptually-based arithmetic understanding can be accounted for or related to individual differences in executive function, we conducted a correlation analysis. We hypothesized that there would be a correlation between conceptual strategy use and executive function task performance, with students who perform better on EF tasks using more conceptually-based arithmetic strategies. We correlated conceptually-based strategy use on inversion, associativity, and equivalence problems with Corsi average span, N-Back reaction time, Go/No-Go reaction time, WCST total errors, and WCST total categories achieved. Unless otherwise noted, the alpha significance level used was 0.05 or less for all significant results.

Inversion and Executive Function

For the Corsi-forwards and Corsi-backwards span, the average of both these spans was calculated and correlated with conceptually-based strategy use. However, there were no significant correlations between Corsi average span and conceptually-based strategy use on either additive or multiplicative inversion problems. This may suggest that working memory, which the Corsi is designed to measure, is not as important in the use of inversion strategy as other executive function components. The N-Back task measured the executive function of updating through total accuracy and reaction time. N-Back reaction time was correlated with conceptually-based strategy use in the current study. There was a significant, negative correlation for additive inversion and N-Back reaction time, $r(124) = -.30$; $p = < .001$. Additionally, there was a significant negative correlation between multiplicative inversion and N-Back reaction time, $r(124) = -.22$; $p = .013$. Thus, participants who used more conceptually-based strategies on additive and multiplicative inversion problems were faster at updating information on the N-

Back task. These correlations may indicate that updating, the primary executive function measured with the N-Back task, is highly intertwined with the ability to both select and successfully carry out inversion strategies. Since updating during the N-Back task requires individuals to keep track of previous stimuli and then appropriately respond when the current stimulus matches previous stimuli, it seems logical that similar strategies would be used when analyzing the inversion question ($a + b - b$ and $a \times b \div b$). For instance, being able to quickly recognize that the latter number matches the one previous is essential in order to recognize the more efficient inversion strategy, cancel out the previous number ($b - b$ or $b \div b$), and simply select the first number as the final answer is similar to the process used during the N-Back task. Therefore, it appears children who have higher inversion strategy use are better and quicker at updating.

The Go/No-Go (GNG) task measured executive function performance through reaction time and number of false alarms. Go/No-Go reaction time was correlated with conceptual strategy use in the current study. Strategy use on additive inversion questions was negatively correlated with GNG reaction time, $r(125) = -.22$; $p = .012$. Additionally, strategy use on multiplicative inversion questions was also negatively correlated with GNG reaction time, $r(125) = -.21$; $p = .016$. Thus, participants who used more conceptually-based strategies on additive and multiplicative inversion were faster at inhibiting information on the Go/No-Go task. As suggested with the N-Back reaction time results, the negative correlation on the Go/No-Go task may provide evidence that children who use conceptually-based inversion strategies have faster and better executive function skills. Since the Go/No-Go task is intended to measure inhibition, it may be that children who perform better take less time to effectively inhibit previous numbers or strategies (i.e. the left-to-right strategy) in order to choose the inversion strategy and correctly calculate the answer. Thus, children who have higher inversion strategy use may be better and quicker at inhibiting.

The Wisconsin Card Sorting Task (WCST) measured executive function performance through the total number of errors, the total number of preservation errors, and the number of categories achieved. WCST total errors and number of categories achieved were correlated with the EF task performance measures. However, neither of the WCST task performance measures were significantly correlated with additive or multiplicative inversion strategy use. WCST is designed to measure an individual's ability to effectively sort between various switching rules. The insignificant correlation results are surprising, given that "switching" from the standard, left-to-right strategy that children typically employ when solving the inversion strategy would seem critical in conceptual strategy use. However, these results may indicate that switching between particular sorting criteria is less critical to conceptual inversion strategy use than the other forms of executive function.

Associativity and Executive Function

For additive and multiplicative associativity questions, there were only a few significant correlations with the EF task performance measures. First, there were no significant correlations between Corsi average span and conceptually-based strategy use on either additive or multiplicative associativity problems. This may also suggest that the use of working memory, which Corsi primarily measures, is not as critical in conceptually-based strategy use as the other EF components. For N-Back reaction time, there was a significant, negative correlation with the additive associativity strategy use, $r(124) = -.23$, $p = .008$. However, N-Back reaction time was not significantly correlated with multiplicative inversion strategy use. These results can help pose a similar theory to the inversion questions. Children who utilize the additive associativity strategy appear to have quicker reaction times on the N-Back task. This could indicate that children with more conceptual knowledge have faster capacities in storing and updating information about previous numbers of the associativity question, at least on additive problem formats. All of the other executive function performance measures, including GNG reaction time, WCST total errors, WCST perseveration errors, and WCST categories achieved, were insignificantly correlated with additive and multiplicative associativity use. This may indicate that associative strategy use does not rely as prominently on inhibiting or switching skills, which these two tasks measure.

Equivalence and Executive Function

While conceptually-based strategy use on inversion and associativity problems had few significant correlations with the executive task performance measures, conceptually-based strategy use on equivalence problems was significantly correlated with all of the task measures. First, Corsi average span was significantly correlated with both additive equivalence strategy use, $r(126) = .33$, $p = <.001$, and multiplicative equivalence strategy use, $r(126) = .28$, $p = .002$. Equivalence questions ($a + b + c = a +$ __ or a x b x $c = a$ x ___) require participants to look at all the numbers on opposite sides of the equals sign and utilize memory of previously calculated numbers or the totalled leftward side to accurately solve for the answer on the right or select the equivalence shortcut. These results seem to indicate the ability to effectively store information in the working memory is important for both effective strategy selection with conceptually-based additive and multiplicative equivalence problems.

N-Back reaction time was negatively correlated with both additive equivalence strategy use, $r(124) = -.37$, $p = < .001$, and multiplicative equivalence strategy use, $r(124) = -.37$, $p = < .001$. These results indicate, similar to those found with inversion and associativity problems, that an ability to update information in the brain (i.e. updating) is an essential skill required to update previous calculations and strategies in order to effectively select a conceptually-based strategy when solving equivalence questions. Equivalence problems require individuals to sum up and continually update previous numbers and calculations for effective solving. The theme of significant negative reaction time correlations, in line with our previous arguments, may suggest that children who utilize conceptually-based arithmetic strategies on equivalence questions are faster at updating.

Go/No-Go (GNG) reaction time was negatively correlated with both additive equivalence strategy use, $r(124) = -.29$, $p = < .001$, and multiplicative equivalence strategy use, $r(124) = -.230$, $p = .009$. These results could determine that an ability to effectively select and carry out the equivalence strategy is related to an ability to inhibit previous numbers or strategies and correctly solve both additive and multiplicative equivalence problems. These negative correlations may also suggest that participants who use the conceptually-based equivalence strategy are faster at inhibiting. These children may be quicker at inhibiting previous strategies and calculations for effective equivalence solving.

Finally, additive equivalence strategy was correlated with both WCST total errors $r(121) = -.31$, $p = <.001$ as well as total categories achieved, $r(121) = .30$, $p = <.001$. Multiplicative equivalence strategy use was also correlated with both WCST total errors, $r(121) = -.23$, $p = .010$, and total categories achieved, $r(121) = .26$, $p = .003$. An ability to accurately solve and utilize the equivalence problems may require students to switch from a standard arithmetic question ($a + b + c =$ ___ or a x b x $c =$ ___) when attempting to solve an equivalence question, where both sides of an equal sign need to be equal and interchangeable ($a + b + c = a +$ ___ or a x b x $c = a$ x __). The negative WCST total error correlations and the positive WCST categories achieved correlations may suggest that effective strategy choice is highly related to an individual's capacity to quickly and effectively switch. The positive correlation between equivalence strategy use and total categories achieved suggests that children who use the equivalence strategy were also able to achieve more categories. Further, the negative correlations between equivalence strategy use and WCST total errors suggest that children who utilize the equivalence strategy use made fewer switching errors on the WCST task. Thus, effective switching also appears to be related to an ability to effectively solve and select a conceptually-based strategy on equivalence problems.

EF and Arithmetic Strategy Correlational Summary

In support of our primary hypothesis, it appears that conceptually-based arithmetic strategies (inversion, associativity, and equivalence) are linked with executive function and its various components (working memory, updating, switching, and inhibition). Working memory, as measured through the Corsi tasks, was not correlated with the use of inversion and associativity, but was significantly correlated on both additive and multiplicative equivalence problems. Thus, equivalence questions, which are large, complex, and involve numerous calculations, may require a better working memory in order to solve effectively. Updating, as measured through the N-Back task, was correlated with all three conceptually-based strategies. Updating is an essential skill for reforming previous information, such as calculations, in the presence of new information. Thus, it appears that updating is an essential ability for selecting and performing conceptually-based arithmetic strategies including inversion, associativity, and equivalence. Inhibition, as measured through the Go/No-Go task, was related to both inversion and equivalence strategy use. Inhibition may be an essential skill for selecting appropriate strategies and which portions of an arithmetic question that an individual should or should not pay attention to (e.g., the inverse operations of an inversion problem or the parallel numbers on either side of an equivalence problem). Finally, WCST was used to measure switching capacities and was correlated with equivalence. The novel format of equivalence questions, which require participants to make both sides of the problem equal, may appear different than standard problems (e.g., $a + b = c$ or $a \times b = c$) or even inversion and associativity problems. Children may be required to 'switch' from typical problem-solving strategies (e.g., left-to-right) to adapt their solving strategies and apply their knowledge to this novel, conceptually-based problem.

Mixed Model ANOVA: Results & Discussion

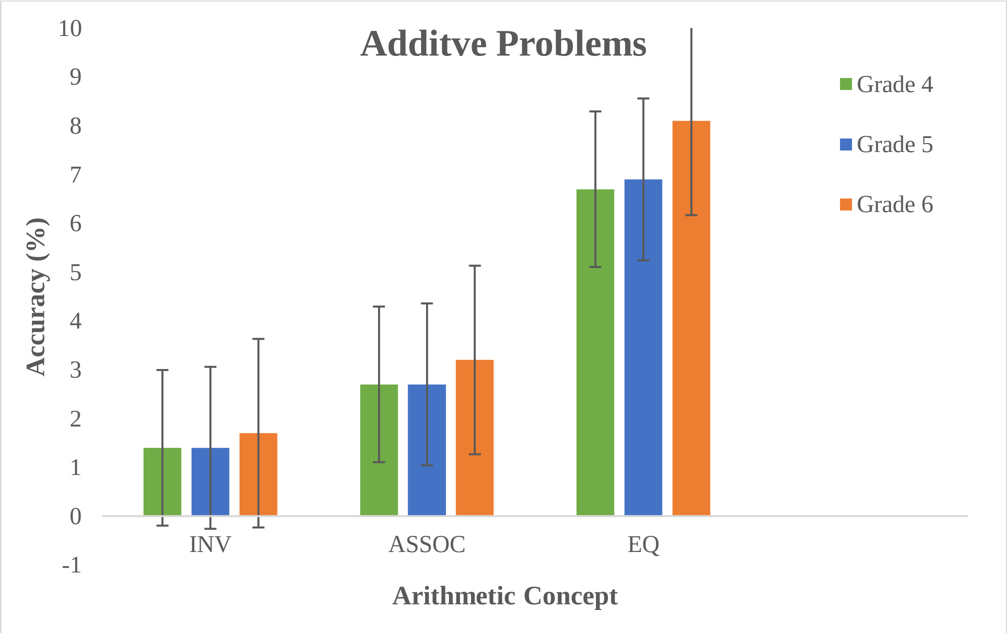
Our three secondary research questions were (1) whether there would be differences in the use and performance of conceptually-based strategies (INV, ASSOC, and EQ), (2) if there would be grade differences among grade four, five, and six students in arithmetic strategy use and executive function performance, and (3) whether there would be a difference in arithmetic performance based on the operation format (addition/subtraction and multiplication/division) of each problem type. To test the secondary research questions, two 3x3x2 mixed model analysis of variances (ANOVAs) were performed on the three conceptual concepts (INV, ASSOC, and EQ) by the three grades (four, five, and six), and the two operation problem types (addition/subtraction and multiplication/division). The first ANOVA was conducted on arithmetic accuracy and the second was conducted on arithmetic strategy. In the ANOVAs, the dependent variable was accuracy or conceptually-based strategy use and the independent variables for both ANOVAs were the three problem types (INV, ASSOC, EQ), the three grades (4, 5, and 6), and the operation problem types (addition/subtraction and multiplication/division). The within-subjects variables were the three concept problem types (INV, ASSOC, and EQ) and the two operation problem types (addition/subtraction and multiplication/division). The betweensubjects variable was the three grades (4, 5, and 6). Unless otherwise noted, the alpha significance level used 0.05 or less for all significant results.

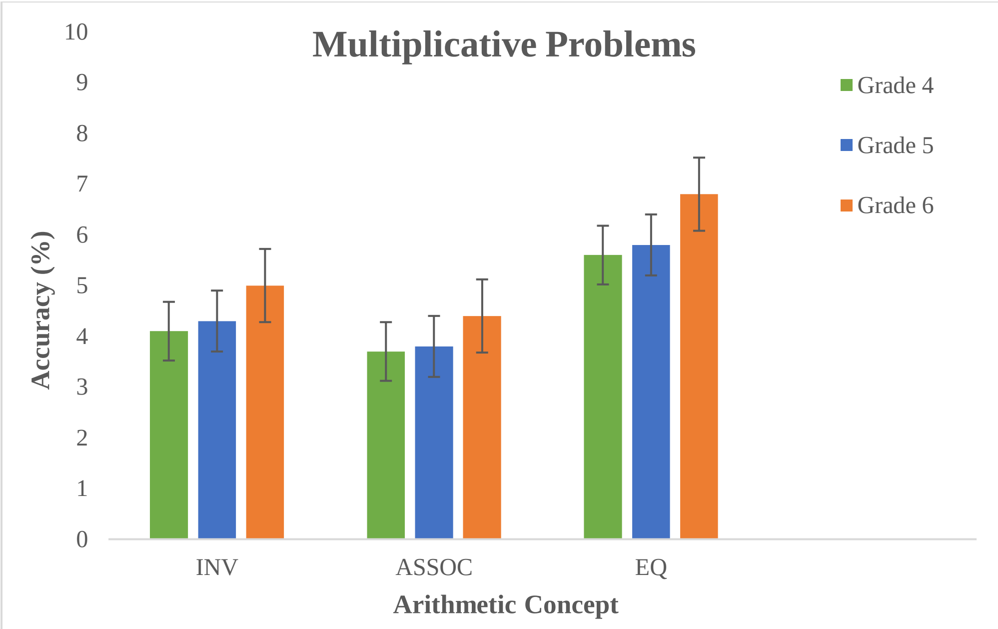
Accuracy

Consistent with our three secondary hypotheses, we expected the results would demonstrate that participants use and perform inversion the most, followed by associativity, and lastly by equivalence. The ANOVA results for arithmetic accuracy ANOVA indicated a main effect of concept, $F(2,108) = 102.25$, $p = .000$, $\eta_2^p = .609$ (see Figure 1). In line with our hypothesis regarding accuracy, students performed best on inversion questions (74.8%), followed by associativity (64.3%), and lastly on equivalence (39.1%). This theme is evident in previous research studies among elementary students that also show similar results (Robinson et al., 2017; 2018).

Figure 1

Arithmetic Accuracy (% on Inversion, Associativity, and Equivalence Problems in Grades 4,5, and 6 on Additive Problems (top panel) and Multiplicative Problems (bottom panel)





Note: 1a. Interaction between concept and operation for multiplication/division accuracy (%) with standard errors bars. INV, inversion; ASSOC, associativity; EQ, equivalence. 1b. Interaction between concept and operation for multiplication/division accuracy (%) with standard errors bars. INV, inversion; ASSOC, associativity; EQ, equivalence.

We also anticipated that there would be no grade differences in conceptual understanding. In other words, variability in conceptual strategy use would not be accounted for by grade, but due to individual differences. Regarding grade, there was no significant effect $F(2,109) = 2.61$, $p = .078$ $\eta_2^p = .046$, there was no significant interaction between grade and operation $F(2,109) = 2.16$, $p = .120$, $\eta_2^p = .038$, and there was no significant interaction between grade and concept $F(4,218) = 0.05$, $p = .993$, $\eta_2^p = .001$ for ANOVA arithmetic accuracy. Finally, there was no significant interaction between grade, concept, and operation $F(4,218) = 1.07$, $p = .572$ and $\eta_2^p = .013$ on arithmetic accuracy. Since there were no significant effects for arithmetic accuracy in relation to grade, grade by concept, grade by operation, or grade by concept by operation, these results support our hypothesis that variance in arithmetic accuracy can be accounted for by individual differences, rather than age or grade level. These results are also consistent with current literature, which demonstrates that variability in arithmetic understanding and performance is due to individual differences such as attitudes and beliefs, rather than age, grade, or gender (Robinson & Dubé, 2012).

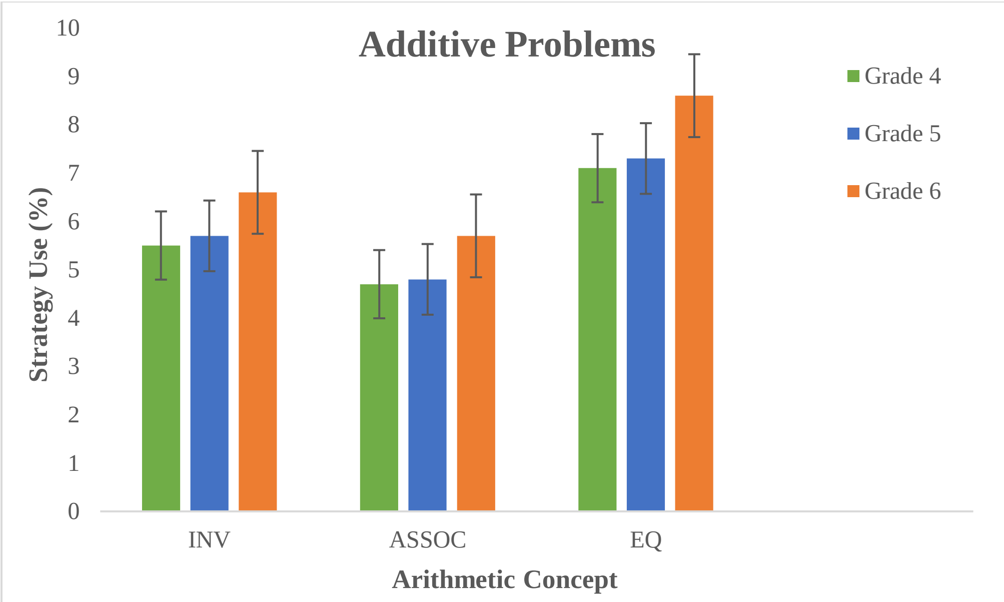
Finally, we expected that participants would perform better on addition and subtraction problems (see top panel for Figure 1) and worse on multiplication and division problems (see bottom panel for Figure 1). Participants performed significantly better on additive questions than on multiplicative questions (79.2% versus 39.7%, respectively), as evident from our accuracy ANOVA. This is consistent with previous research, which has demonstrated that multiplication and division problems are much more complex and even adults can fail to demonstrate conceptually-based arithmetic strategies on problems of this operational format (Dubé & Robinson, 2010; Eaves et al., 2012). These results indicate that children have more difficulty with multiplication and division than addition and subtraction, as we predicted in our hypothesis. The accuracy ANOVA results demonstrated a main effect of operation, $F(1,109) = 453.93$, $p = .000$, $\eta_2^p = .806$. Interestingly, the arithmetic accuracy results also demonstrated a significant interaction between operation and concept $F(2,218) = 29.852$, $p = .000$, $\eta_2^p = .295$. For accuracy, participants did best on additive inversion (97.5%), followed by associativity (89.9%) and lastly by equivalence (50.1%). For accuracy on multiplicative questions, participants did best on inversion (52.0%), followed by associativity (38.8%) and then equivalence (28.2%). Students performed noticeably better on additive questions, but these accuracy averages were almost half for the multiplicative questions. Nevertheless, the average on inversion multiplication/division questions (52.0%) was close, but still higher than accuracy averages of additive equivalence questions (50.1%). Thus, the interaction between operation and concept suggests that accuracy is largely dependent on both the operation and strategy format of the question that a participant is solving.

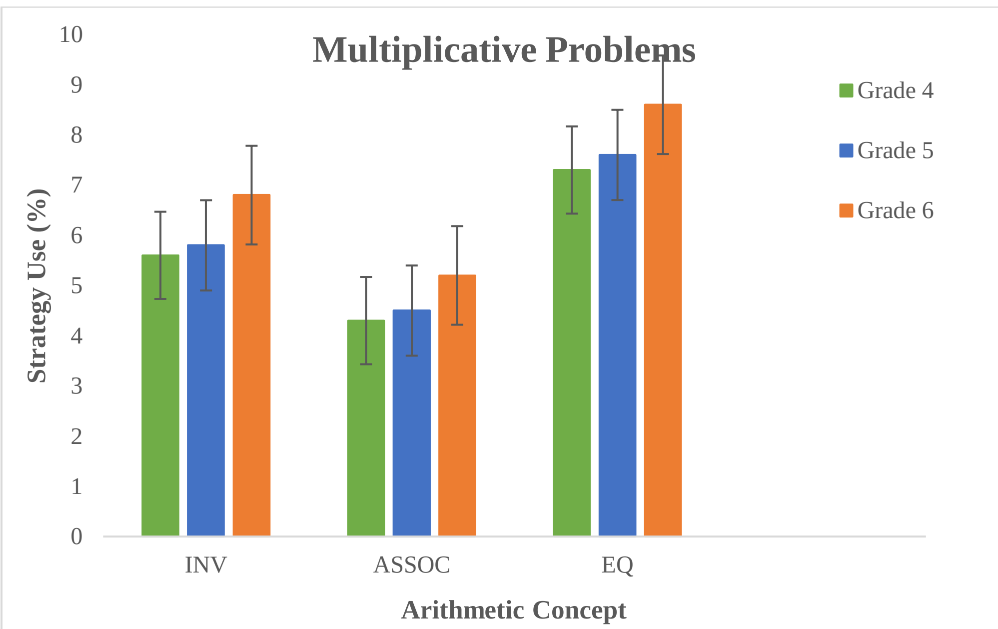
Conceptually-Based Strategy Use

Consistent with our three secondary hypotheses, we expected the results would demonstrate that participants use conceptually-based problem solving strategies on inversion problems the most, followed by associativity problems, and lastly by equivalence problems (see Figure 2). The ANOVA results for arithmetic strategy indicated a main effect of concept $F(2,108) = 36.35$, $p = .000$, $\eta_2^p = .250$. However, the ANOVA results on strategy use (Figure 2) demonstrated that participants were most likely to use equivalence (56.3%), followed by inversion (39.4%), and then associativity (20.3%). Thus, while the equivalence concept was deemed to be more complex and difficult to grasp than the inversion strategy, participants demonstrated use of this strategy over inversion and associativity. Contrary to our hypothesis, these results seem to suggest that participants were more likely to utilize the equivalence strategy than the inversion and associativity strategies, but were unsuccessful in their calculations, perhaps due to the length of the equation and the complexity of the calculations.

Figure 2

Conceptually-Based Strategy Use (% on Inversion, Associativity, and Equivalence Problems in Grades 4,5, and 6 on Additive Problems (top panel) and Multiplicative Problems (bottom panel)





Note: 1a. Interaction between concept and operation for additive strategy use (%) with standard errors bars. INV, inversion; ASSOC, associativity; EQ, equivalence.

1b. Interaction between concept and operation for multiplicative strategy use (%) with standard errors bars. INV, inversion; ASSOC, associativity; EQ, equivalence.

We also anticipated that there would be no grade differences in conceptual understanding. Instead, variability in conceptual strategy use would be accounted for by individual differences, rather than grade. The ANOVA results for arithmetic strategy use followed a very similar trend to that of arithmetic accuracy. There was no significant effect $F(4,109) = 1.74$, $p = .181$ $\eta_2^p = .031$ for grade. There was also no significant interaction between grade and operation $F(2,218) = .776$, $p = .463$, $\eta_2^p = .014$, nor grade and concept $F(4,218) = 0.22$, $p = .920$, $\eta_2^p = .004$. Finally, there was no significant interaction between grade, concept, and operation $F(4,218) = 0.87$, $p = .484$, $\eta_2^p = .016$ on arithmetic strategy. Since there were no significant effects found on arithmetic strategy use in relation to grade, grade by concept, grade by operation, or grade by concept by operation, these results also support our hypothesis that variance in conceptually-based strategy use can be accounted for by individual differences, rather than age or grade level.

As mentioned previously, we expected that participants would have higher conceptually-based strategy use on addition and subtraction problems (see top panel for Figure 2) than on multiplication and division problems (see bottom panel for Figure 2). The strategy ANOVA results demonstrated a main effect of operation $F(1,109) = 43.21$, $p = .000$, $\eta_2^p = .284$. There was also a two-way main effect between operation and concept on the arithmetic strategy ANOVA, $F(2,108) = 42.13$, $p = .000$, $\eta_2^p = .279$. For strategy on additive questions, participants utilized the equivalence strategy the most (54.9%), followed by inversion (55.9%) and then associativity (28.1%). For strategy on multiplicative problems, participants used equivalence the most (57.6%), then inversion (22.9%), and finally associativity (12.5%). This suggests that conceptually-based strategy use is situational, depending on both the operation and strategy format. For instance, it appears that participants were more likely to use the equivalence strategy on multiplicative questions, but were almost equally likely to use inversion and equivalence strategies on additive problems. Therefore, individual variance in conceptually-based strategy use may rely closely on the conceptual and operational format of the problem.

Conclusion

Our primary hypothesis that students who perform better on the executive function tasks would have better performance on arithmetic strategy use was partially supported by the correlational results. As measured by the N-Back and Go/No-Go tasks, effective inversion strategy use was associated with faster updating and inhibition capacities. Effective associativity strategy use was also associated with quicker updating, as measured on the N-Back task. Last, it appears that successful equivalence strategy use, in particular, requires greater executive function component skills than inversion and associativity, since all of the EF performance measures (Corsi tasks, N-Back, Go/No-Go, and WCST) were correlated with equivalence strategy use. The complexity of the equivalence problem format may require children to utilize various components of executive function in order to apply a conceptually-based arithmetic strategy on such problems, including working memory, updating, inhibition, and switching. Higher performances on the conceptually-based problems were associated with better performance on the executive function battery. Thus, in line with our hypothesis, individual differences in conceptually-based arithmetic performance appear to be related to or accounted for by individual differences in executive function.

As we hypothesized, students performed best on inversion, followed by associativity, and then equivalence problems. However, contrary to our hypothesis, participants utilized the equivalence strategy the most, followed by inversion, and then associativity. These results are in line with previous studies that found similar themes of arithmetic accuracy and strategy use (see

Robinson et al., 2018). There were no effects for grade found, which supports our hypothesis and current literature that variance in arithmetic accuracy and strategy use is based on individual differences (Robinson & Dubé, 2009b; 2012). Finally, participants performed significantly better on additive questions than on multiplicative questions. These results also support our hypothesis and current literature that children struggle more with multiplication and division and tend to do better on additive problems (Robinson et al., 2018; Robinson & LeFevre, 2012). Participants were more likely to use the equivalence strategy on multiplicative questions, but were almost equally likely to use inversion and equivalence strategies on additive problems. Therefore, individual variance in conceptually-based strategy use may rely closely on the conceptual and operational format of the problem.

Limitations

The current study has a number of limitations. First, **s**ince the current study is correlational, the links found are not causative, but based on correlation and interpretation. Second, while the remote, technological format of the current study was beneficial in many ways, it also had a number of limitations. Remote data collection improved cost effectiveness, eased in participant recruitment and involvement, helped provide a comfortable environment for participants by conducting the research from their homes, and increased the ecological validity and generalizability to environments outside of a laboratory. However, there were a number of technological problems associated with using Zoom, including device delays, and video, microphone, and Wi-Fi issues, which could affect participant understanding and performance. Further, conducting the study outside of the lab reduces control over confounding variables, such as distractions in the participants' home and parental or sibling influence.

The current study's participant sample had access to technology and were primarily Caucasian, middle-class, public school students. This population is the majority in most Canadian schools, indicating the current study can be generalized to most Canadian public school students. However, the participant sample may be restricted in its generalizability to minority students, such as ethnic minorities or those of lower socioeconomic class. Additionally, assessing students based on their grade may prevent consideration of the length or depth of learning that children have been exposed to. For instance, a child at the beginning of the fourth grade will not have as much arithmetic knowledge or practice as a child at the end of the fourth grade. There was also no assessment of children's intelligence levels, meaning "gifted" children or children with intelligence and learning disabilities were not accounted for. Finally, the current study used verbal responses, which require participants to both utilize and explain the conceptual strategy as a reflection of their conceptual understanding. Verbal responses are a common and reliable method used in current studies on conceptually-based arithmetic knowledge, but they may not clearly reflect or estimate children's implicit conceptual knowledge and understanding (Crooks & Alibali, 2014). For instance, a child may be able to effectively and accurately use a conceptually-based arithmetic strategy, but lack the ability to communicate this procedure to the researcher.

Strengths, Implications, & Future Directions

To our knowledge, the current study is the first to examine the link between conceptually-based arithmetic strategies (inversion, associativity, and equivalence) in relation to executive function and its components (working memory, updating, inhibition, and switching). The connection between executive function components and conceptually based arithmetic strategies can have great theoretical implications by increasing understanding of arithmetic strategies and concepts, as well as the relationship between these concepts and EF. While previous studies have examined arithmetic strategies, conceptual understanding, and EF in similar age and grade categories, most studies have examined EF or arithmetic strategies independently. The contributions of the current study are particularly important because research regarding executive function and arithmetic task use, understanding, and development remains relatively unexplored. Studying the relationship between these skills and concepts may help fill gaps in current literature. This study and its results can also help fill gaps in literature regarding the influence of operation (addition/subtraction and multiplication/division) on arithmetic strategy use, as research regarding operation and strategy has been insufficiently analyzed. Further, arithmetic and EF studies have examined similar age categories, but few have examined both the EF components and arithmetic concepts in grade four, five, and six students. Future studies need to be conducted to determine the relationship between arithmetic accuracy and strategy in relation to executive function and its components. Further studies should also be conducted to examine the relationship between executive function, arithmetic accuracy and strategy, and conceptual and operational problem format.

In a practical sense, the current study may be useful for researchers, parents, and educators to increase their understanding of children’s development in order for appropriate interventions, strategies, and curriculum to be developed and implemented in the home and classroom to improve arithmetic and EF skills. Some researchers suggest parents and teachers can help improve children’s conceptually-based arithmetic understanding by teaching children about the arithmetic concepts (inversion, associativity, and equivalence) and mixing problem formats (e.g., three-term problems with multiple operations rather than two-term, single operation problems), which are typically not included in American and Canadian curricula

(McNeil, 2014; Robinson, 2019). Additionally, parents and teachers should encourage flexible problem-solving that uses multiple approaches to solving rather than encouraging the same, “proper” solution of a left-to-right format. It would also be useful for children to learn to look at the entirety of the problem before they begin solving, which may help them notice that there are easier strategies for problem-solving (Robinson & LeFevre, 2012).

Diamond and Lee (2011) suggest that teachers and parents can help improve executive function development through “diverse activities” which can include computer training, games, physical activities, mindfulness, and education curriculum (pp. 959). These diverse activities are beneficial to executive function by requiring children to play with ideas, provide thoughtful responses, and maintain focus (Diamond & Lee, 2011). However, Diamond and Lee (2011) suggest that focusing only on executive function is not sufficient, as it is imperative to address emotional, social, and physical development as well. Thus, parents and educational professionals should seriously consider teaching students and children arithmetic and executive function skills through these or similar practices. EF and arithmetic skills have critical applications towards improving student success in later, more complex mathematics such as algebra, as well as other important life skills, such as financial literacy and budget balancing. Thus, our study and results have important theoretical and practical implications, particularly in its application to the research field and Canadian and American school systems.

References

Baroody, A.J., Lai, M., Li, X., & Baroody, A.E. (2009). Preschoolers' understanding of subtraction-related principles. *Mathematical Thinking and Learning, 11*, 41-60. https://doi.org/10.1080/10986060802583956

Bierman, K. L., Nix, R. L., Greenberg, M. T., Blair, C., & Domitrovich, C. E. (2008). Executive functions and school readiness intervention: Impact, moderation, and mediation in the Head Start REDI program. *Development and Psychopathology, 20*(3), 821-843. https://10.1017/S0954579408000394

Bisanz, J., & LeFevre, J. (1990). Strategic and nonstrategic processing in the development of mathematical cognition. In D.F. Bjorklund (Ed.), *Children's strategies: Contemporary views of cognitive development* (pp. 213-244). Hillsdale, N.J.: Lawrence Erlbaum.

Bull, R. & Lee, K. (2014). Executive functioning and mathematics achievement. *Child Development Perspectives, 8*(1), 36-41. http://doi.org/10.1111/cdep.12059

Bull, R., & Scerif, G. (2001). Executive functioning as a predictor of children's mathematics ability: Inhibition, switching, and working memory. *Developmental Neuropsychology, 19*(3), 273-293. https://doi.org/10.1207/S15326942DN1903_3

Bryant, P., Christie, C., & Rendu, A. (1999). Children's understanding of the relation between addition and subtraction: Inversion, identity, and decomposition. *Journal of Experimental Child Psychology, 74*(3), 194-212. https://doi.org/10.1006/jecp.1999.2517

Cragg, L., Keeble, S., Richardson, S., Roome, H., & Gilmore, C. (2017). Direct and indirect influences of executive functions on mathematics achievement. *Cognition, 162*(1), 12-26. http://doi.org/10.1016/j.cognition.2017.01.014

Crooks, N.M., & Alibali, M.W. (2014). Defining and measuring conceptual knowledge in mathematics. *Developmental Review, 34*(4), 344-377. https://doi.org/10.1016/j.dr.2014.10.001

Diamond, A., Carlson, S. M., & Beck, D. M. (2005). Preschool children's performance in task switching on the dimensional change card sort task: Separating the dimensions aids the ability to switch. *Developmental Neuropsychology, 28*(2), 689-729. https://doi.org/10.1207/s15326942dn2802_7

Diamond, A., & Lee, K. (2011). Interventions shown to aid executive function development in children 4 to 12 years old. *Science (American Association for the Advancement of Science), 333*(6045), 959-964. https://doi.org/10.1126/science.1204529

Downton, A., Russo, J., & Hopkins, S. (2020). Students' understanding of the associative property and its applications: Noticing, doubling and halving, and place value. *Mathematics Education Research Journal*, 1-20. https://doi.org/10.1007/s13394-020-00351-w

Dubé, A.K., & Robinson, K.M. (2010a). Accounting for individual variability in inversion shortcut use. *Learning and Individual Differences, 20*(6), 687-693. http://doi.org/10.1016/j.lindif.2010.09.009

Dubé, A.K., & Robinson, K.M. (2010b). The relationship between adults' conceptual understanding of inversion and associativity. *Canadian Journal of Experimental Psychology, 64*(1), 60-66. http://doi.org/10.1037/a0017756

Dubé, A. K., & Robinson, K. M. (2018). Children's understanding of multiplication and division: Insights from a pooled analysis of seven studies conducted across 7 years. *British Journal of Developmental Psychology, 36*(2), 206-219. http://doi.org/10.1037/a0017756

Eaves, J., Gilmore, C., & Attridge, N. (2021). Conceptual knowledge of the associativity principle: A review of the literature and an agenda for future research. *Trends in Neuroscience and Education, 23*(1), 100152. https://doi.org/10.1016/j.tine.2021.100152

Higo, K., Minamoto, T., Ikeda, T., Osaka, M. (2014). Robust order representation is required for backward recall in the Corsi blocks task. *Frontiers in Psychology, 5(*1), 1-10. https://doi.org/10.3389/fpsyg.201401285

Kieran, C. (1981). Concepts associated with the equality symbol. *Educational Studies in Mathematics, 12*(3), 317-326. https://doi.org/10.1007/BF00311062

Kotsopoulos, D., & Lee, J. (2012). An analysis of math congress in an eighth grade classroom. *Mathematical Thinking and Learning, 14*(3), 181-198. https://doi.org/10.1080/10986065.2012.682958

Knuth, E. J., Stephens, A. C., McNeil, N. M., & Alibali, M. W. (2006). Does understanding the equal sign matter? Evidence from solving equations. *Journal for Research in Mathematics Education, 37*(4), 297-312. https://doi.org/10.2307/30034852

Landry, O., & Mitchell, P. (2021). An examination of preservative errors and cognitive flexibility in autism. *PLoS ONE, 16*(1), 1-19. https://doi.org/10.1371/journal.pone.0223160

McNeil, N.M. (2007). U-shaped development in math: 7-year-olds outperform 9-year-olds on equivalence problems. *Developmental Psychology, 43*(1), 687-695. https://doi.org/10.1037/0012-1649.43.3.687

McNeil, N.M. (2014). A change-resistance account of children's difficulties understanding mathematical equivalence. *Child Development Perspectives, 8*(1), 42-47. https://doi.org/10.1111/cdep.12062

McNeil, N.M., & Alibali, M. (2005). Why won't you change your mind? Knowledge of operational patterns hinders learning and performance on equations. *Child Development, 76*(4), 883-899. https://doi.org/10.1111/j.1467-8624.2005.00884.x

Meule, A. (2017a). Reporting and interpreting task performance in the Go/No-Go affective shifting tasks. *Frontiers in Psychology*, *8*(1). 701. https://doi.org/10.3389/fpsyg.2017.00701

Meule, A. (2017b). Reporting and interpreting working memory performance in n-back tasks. *Frontiers in Psychology*, *8*(1), 352. https://doi.org/10.3389/fpsyg.2017.00352

Miyake, A., Friedman, N., Emerson, M., Witzki, A., Howerter, A., & Wager, T. (2000). The unity and diversity of executive functions and their contributions to complex "frontal love" tasks: A latent variable analysis. *Cognitive Psychology*, *41*(1), 49-100. https://doi.org/10.1006/cogp.1999.0734

Piaget, J. (1952). *The origins of intelligence in children.* International Universities Press.

Rasmussen, C. Ho, E., & Bisanz, J. (2003). Use of the mathematical principle of inversion in young children. *Journal of Experimental Child Psychology, 89*(1), 89-102. https://doi.org/10.1016/S022-0965(03)00031-6

Robinson, K.M. (2019). Arithmetic concepts in the early school years. In: K.M. Robinson, H.K., Osana, & D. Kotsopoulo (Eds.) *Mathematical learning and cognition in early childhood* (pp.165-185). Springer International Publishing. http://doi.org/10.1007/978-3-030-12895-1_10

Robinson, K.M., & Dubé, A.K. (2012). Children's use of arithmetic shortcuts: The role of attitudes in strategy choice. *Child Developmental Research, 2012*(1), 1-10. http://doi.org/10.1155/2012/459385

Robinson, K., & Dubé, A. (2009). Children’s understanding of the inverse relation between multiplication and division. *Cognitive Development*, *24*, 310-321. https://doi.org/10.1016/j.cogdev.2008.11.001

Robinson, K.M., & Dubé, A.K., & Beatch, J. (2017). Children’s understanding of additive concepts. *Journal of Experimental Child Psychology, 156,* 16-28. https://doi.org/10.1016/j.jecp.2016.11.009

Robinson, K.M., & LeFevre, J. (2012) The inverse relation between multiplication and division: Concepts, procedures, and a cognitive framework, *Educational Studies in Mathematics, 79*(3), 409-428. http://doi.org/10.1007/s10649-011-9330-5.

Robinson, K.M., Ninowski, J.E., & Gray, M.L. (2006). Children’s understanding of the arithmetic concepts of inversion and associativity. *Journal of Experimental Child Psychology, 94*(4), 349-362. http://doi.org/10.1016/j.cep.2006.03.004

Robinson, K.M., Price, J.A.B., & Demyen, B. (2018). Understanding arithmetic concepts: Does operation matter? *Journal of Experimental Child Psychology, 166*(1), 421-436. http://doi.org/10.1016/j.jecp.2017.09.003

Siegler, R.S., & Araya, R. (2005). A computational model of conscious and unconscious strategy discovery. In Kail, R.V. (Ed.), *Advances in child development and behaviour (Vol. 33)* (pp. 1-42). Elsevier.

Söğüt, M., Göksun, T., & Altan-Atalay, A. (2021). The role of numeracy skills on the Wisconsin card sorting test (WCST) performances of 5-to-8-year old Turkish children. *British Journal of Developmental Psychology, 39* (1), 231-246. https://doi.org/10.1111/bjdp.12353

Viterbori, P., Traverso, L, & Usai, M. (2017). The role of executive function in arithmetic problem-solving processes: a study of third graders. *Journal of Cognition and Development*, *18*(5), 595-616. https://doi.org/10.1080/15248372.2017.1392307

Appendix A
Grades 4-6 Arithmetic Questions

Practice Problems:

1. 9 - 6 = _____
2. 2 x 5 = _____

Real Problems (Addition and Subtraction):

1. 2 + 5 – 5 = _____ (Inversion)
2. 5 + 28 + 3 = 5 + _____ (Equivalence)
3. 3 + 9 – 6 = _____ (Associativity)
4. 4 + 2 + 7 = 4 + _____ (Equivalence)
5. 8 + 26 – 26 = ______ (Inversion)
6. 2 + 29 - 27 = ______ (Associativity)
7. 6 + 3 + 5 = 6 + _____ (Equivalence)
8. 3 + 24 – 24 = _____ (Inversion)
9. 7 + 25 – 22 = _____ (Associativity)
10. 6 + 3 – 3 = _______ (Inversion)
11. 8 + 27 + 6 = 8 + _____ (Equivalence)
12. 5 + 4 – 2 = _____ (Associativity)

Real Problems (Multiplication and Division):

13. 5 x 2 x 3 = 5 x _____ (Equivalence)
14. 7 x 4 ÷ 4 = _____ (Inversion)
15. 5 x 6 ÷ 2 = _____ (Associativity)
16. 3 x 5 ÷ 5 = _____ (Inversion)
17. 4 x 6 ÷ 3 = _____ (Associativity)

18. 6 x 5 x 3 = 6 x _____ (Equivalence)

19. 7 x 9 ÷ 3 = _____ (Associativity)

20. 6 x 2 ÷ 2 = _____ (Inversion)

21. 4 x 8 x 2 = 4 x ____ (Equivalence)

22. 8 x 9 ÷ 9 = _____ (Inversion)

23. 3 x 4 x 7 = 3 x ____ (Equivalence)

24. 2 x 8 ÷ 4 = _____ (Associativity)

Appendix A. This appendix features all of the arithmetic problems, including the additive and the multiplicative sections as well as the arithmetic concept that each problem is designed to study.

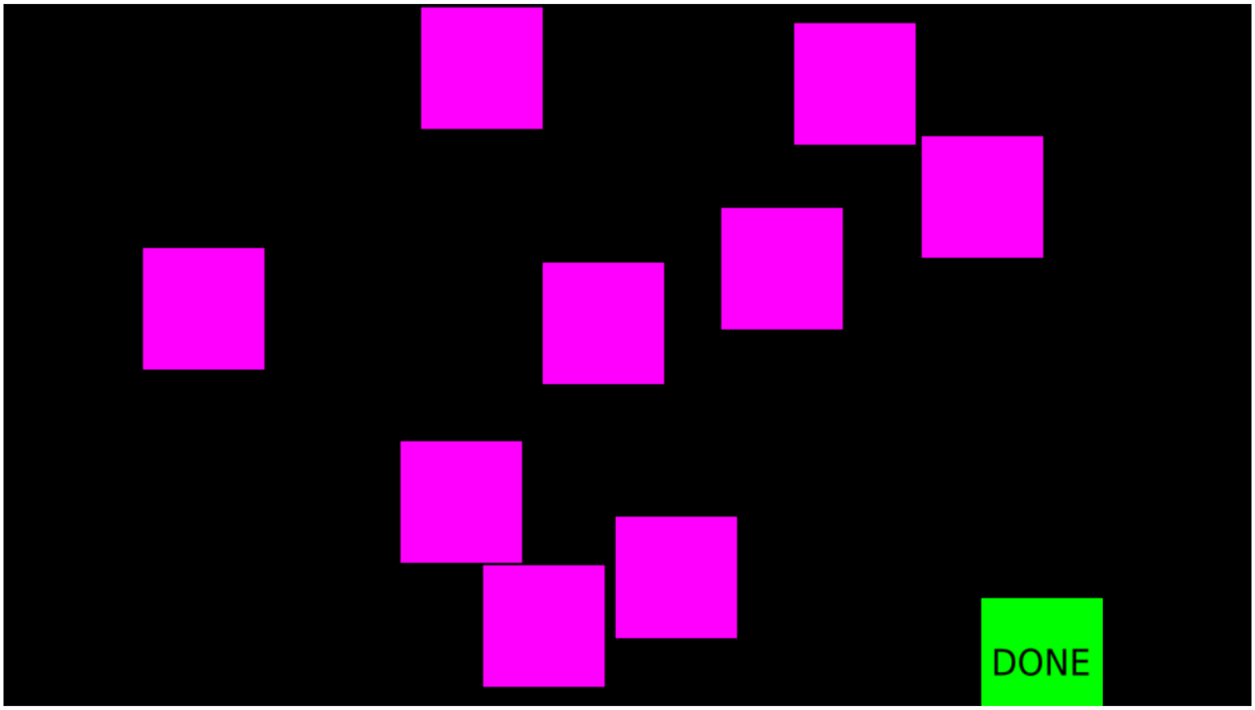
Appendix B
Arithmetic Task Coding Guide

Accuracy	
1	Correct
2	Incorrect
3	Cut off or does not know. No answer.
Strategy	
1	Conceptually-based strategy e.g., Inversion (knowing the answer is first number because multiplying and dividing by the same number leaves 1); Associativity (performing division/subtraction before multiplication/addition to simplify); Equivalence (adding or multiplying only the second and third numbers because other numbers cancel out on each side)
2	Left-to-Right (calculating the answer to the left operation followed by the right operation for associativity and inversion problems; for equivalence problems, solving left side of equation completely before finding number for the right side that satisfies equation)
3	Negation (only for inversion questions; calculates answer for first operation but then realizes they do not need to do the second operation because answer will just be original number)
4	No Equivalence (i.e. does not understand the meaning of the equal sign for equivalence problems; often just adds/multiplies all numbers on left side then says I do not know what to do next or then takes that number and just adds or multiplies it with the number on the right side; sometimes will refuse to answer and says the number on the other side confuses them)

5	Does not know, is guessing, or weird (e.g., adding instead of subtracting)
6	Other (should not be used often, make note of all types of strategies that you code as "other" so we can keep track)
7	Code later (needs further thought/confirmation from supervisor/other RA)

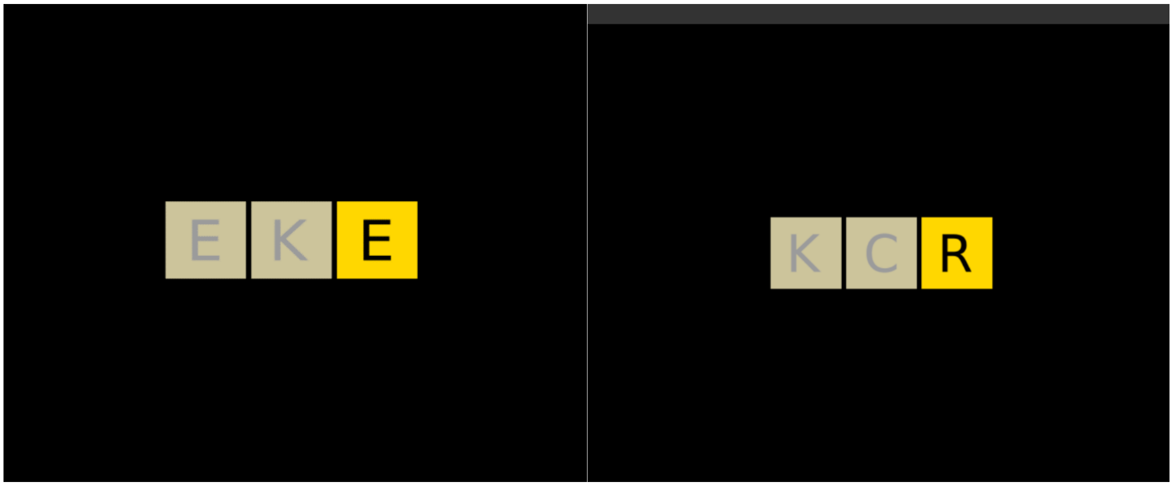
Appendix B. This appendix contains the arithmetic coding guide that researchers used to code participants' accuracy and strategy use on the arithmetic problem task.

Appendix C
Corsi- Forwards and Backwards Tasks



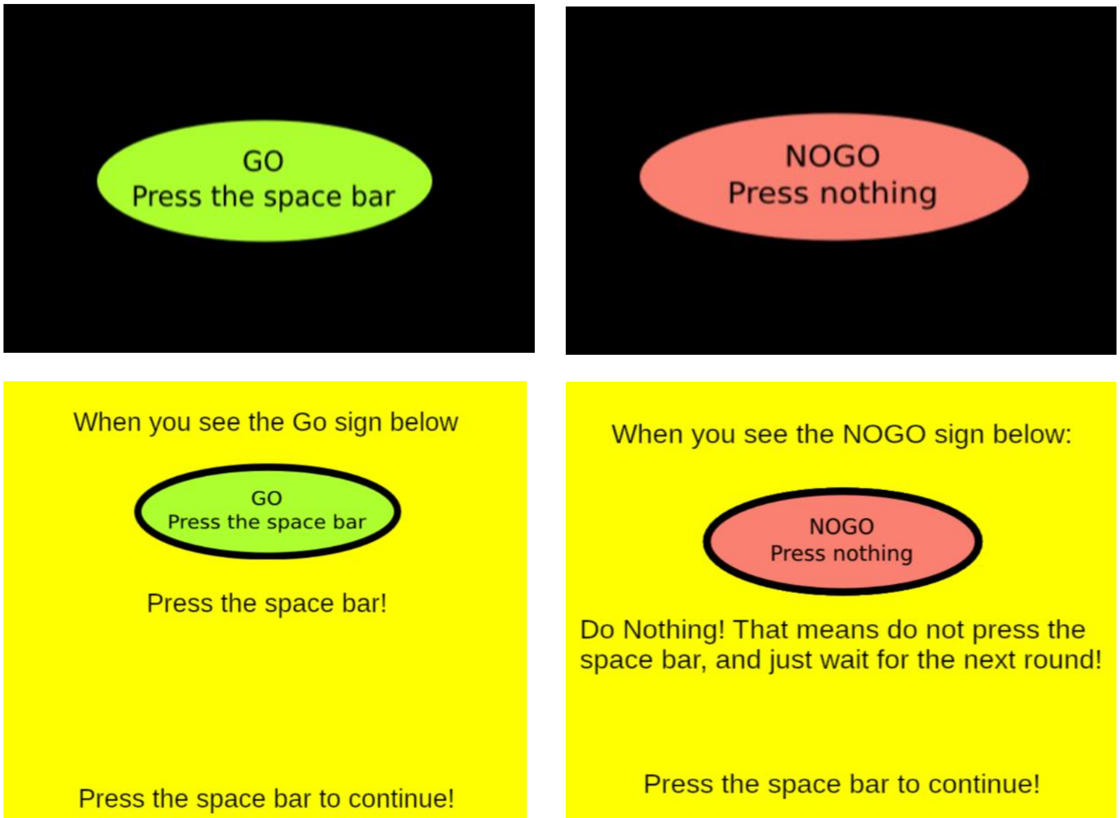
Appendix C. This appendix features what participants would view on their screen. At the beginning of the Corsi task, only two squares will light up and turn yellow to click in the forwards (Corsi Forwards) or backwards (Corsi Backwards) direction. As the participant progresses, the number of squares lighting up will increase. Additionally, the order of squares changes thus, the pattern is different for each trial.

Appendix D
N-Back (2-Back) Task



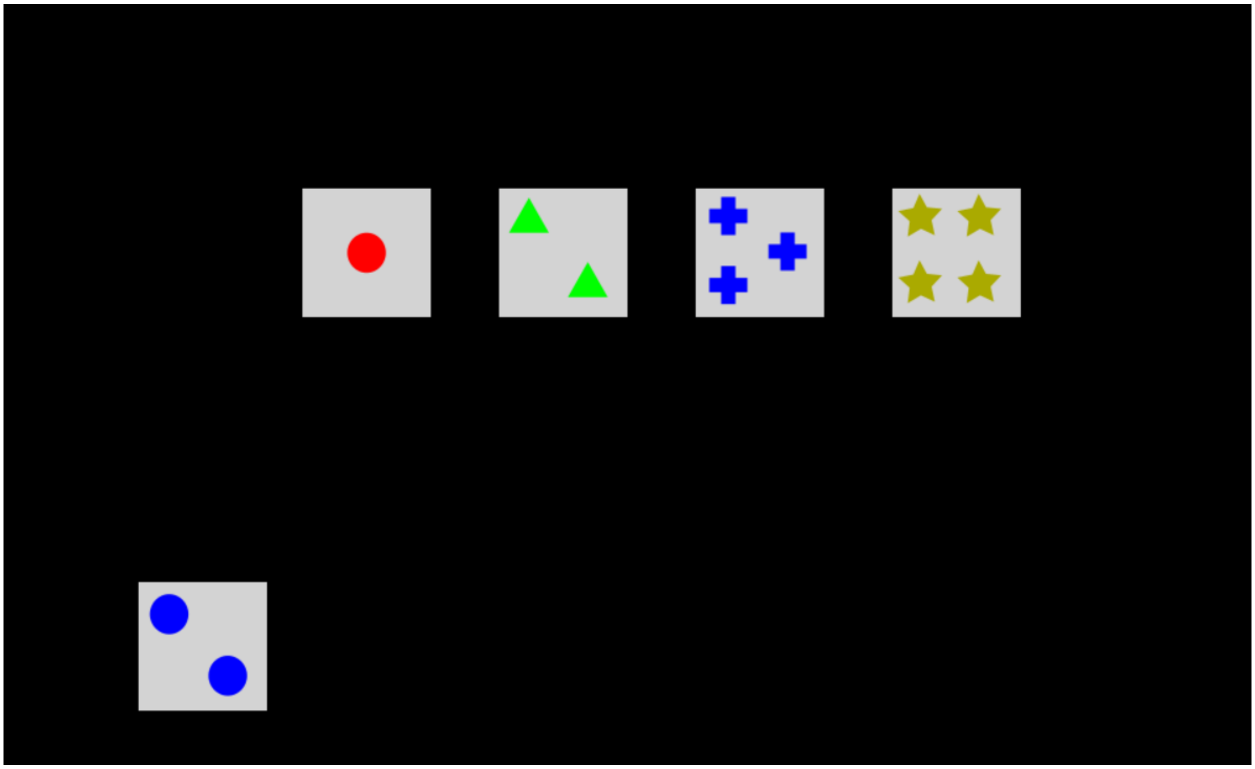
Appendix D. This appendix features what the participants would view and play on their screen during the practice round of the 2-Back task (N-Back). In the practice round, the two previous letters are shown to make it easier to determine if the letter of interest in yellow is a match. However, in the real task only one yellow letter was shown at a time. As a result, participants had to pay close attention and rely on their memory to specify whether the letter was the same as two turns ago or not. For instance, the visual on the left in the appendix features a match trial and the second visual on the right features an unmatched trial during the practice round.

Appendix E
Go/No-Go Task



Appendix E. This appendix features what participants would view and play on their screen during the practice round of the Go/No-Go Task. In the practice round, participants were instructed to push the space bar on the keyboard when they saw the green “Go” button on their screen. When participants saw the red “No-Go” button on their screen, they were instructed to wait for it to go away and avoid pressing any keys. Participants were also instructed to go as fast as possible without making mistakes.

Appendix F
Wisconsin Card Sorting Task



Appendix F. This appendix is an example of the Wisconsin Card Sorting Task (WCST) that participants completed. In this task, participants were instructed to match the card at the bottom to one of the four cards on top based on three match options. Participants could match the cards based on colour, shape, or the number of shapes. In this example, a shape match would be the first card because they are both circles, a number match would be the second card because both cards have two shapes on them, and a colour match would be the third card because it is also blue.