CONCEPTUALLY-BASED STRATEGY USE: INVESTIGATING UNDERLYING MECHANISMS AND DEVELOPMENT ACROSS ADOLESCENCE AND INTO EARLY ADULTHOOD

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Adam Kenneth Dube, candidate for the degree of Doctor of Philosophy in Psychology**,** has presented a thesis titled, **Conceptually-Based Strategy Use: Investigating Underlying Mechanisms and Development Across Adolescence and into Early Adulthood*,*** in an oral examination held on March 23, 2012. The following committee members have found the thesis acceptable in form and content, and that the candidate demonstrated satisfactory knowledge of the subject material.

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Abstract

Researchers have used inversion and associativity problems (e.g., 2 × 8 ÷ 8, 3 + 19 − 17, respectively) to assess whether or not individuals have the conceptual understanding that addition and subtraction and multiplication and division are inverse operations (i.e., the inversion concept, Robinson & Ninowski, 2003; Starkey & Gelman, 1982) and whether or not they understand that numbers can be decomposed and recombined in various ways and still result in the same answer (i.e., the associativity concept, Canobi, Reeve, & Pattison, 1998; Robinson, Ninowski, & Gray, 2006). It is not known when the development of these two concepts reaches adult levels. Furthermore, it is not known whether the application of these concepts during problem solving requires individuals to interrupt the execution of well-practiced procedural knowledge (e.g., Siegler & Araya, 2005).

In the present study, 40 adolescent participants per grade from Grades 7, 9, and 11 and 40 adult participants who had graduated from high school the previous academic year solved multiplication and division inversion and associativity problems. Also, participants completed a task that measured whether the execution of the inversion shortcut or associativity strategy interrupted the execution of computational strategies. The results suggest that inversion shortcut and associativity strategy use increase in Grade 9, that inversion shortcut use approaches adult levels before associativity strategy use, and that the execution of both conceptually-based strategies interrupts computational strategies. Therefore, the present study identifies adolescence as an important developmental period for inversion shortcut and associativity strategy use and provides the first evidence that applying conceptual mathematical knowledge to problem solving requires the interruption of procedural mathematical knowledge.

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Dedication

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Table of Contents

Abstract i

Acknowledgements iii

Dedication iv

Table of Contents v

List of Tables vii

List of Figures viii

List of Appendixes ix

Introduction 1

Mathematical Problem Solving 1

Development of Conceptually-Based Strategy Use 4

Theories of Conceptually-Based Strategy Use 19

Interruption of Procedures in Conceptually-Based Strategy Use 26

Goals of the Study 30

Method 32

Participants 32

Measures 34

Procedure 36

Results and Discussion 39

Mathematical Problem Solving Task 39

Interruption of Procedures Task 68

General Discussion 79

Development of Conceptually-Based Strategy Use 79

Individual Variability in Strategy Use 88

Interruption of Procedures Mechanism 92

Conclusions 99

Maturation in Adolescence 100

Individual Variability 101

Models of Mathematical Knowledge 103

References 107

List of Tables

Table 1. Strategy Use (%) on Small and Large Inversion Problems by Developmental Period. 42

Table 2. Strategy Use (%) on Small and Large Associativity Problems by Developmental Period. 45

Table 3. Percentage of Accurate Responses and Solution Latencies (ms) of Correct Responses on Inversion and Associativity Problems by Developmental Period. 48

Table 4. Correlations Between Strategy Use by Developmental Period. 60

Table 5. Strategy use (%) on Inversion and Associativity Problems by Cluster Membership. 64

Table 6. Percentage of Accurate Responses to the Interruption of Procedures Task by Strategy Use on Inversion and Associativity Problems and Number Probe Type. 72

Table 7. Response Latencies (ms) of Correct Responses on the Interruption of Procedures Task by Strategy Use on Inversion and Associativity Problems and Number Probe Type. 73

Table 8. Percentage of Accurate Responses and Response Latencies (ms) of Correct Responses by Cluster Membership on the Three Number Probe Types. 77

List of Figures

Figure 1. Associated Solution Latencies (ms) of Correct Responses for Strategies Used on Inversion (top) and Associativity Problems (bottom). 54

Figure 2. Proportion of Participants (%) in the DC, IC, NGC, and NOC Clusters by Developmental Period. 66

Figure 3. Percentage of Accurate Responses and Solution Latencies (ms) of Correct Responses by Number Probe Type. 70

List of Appendices

Appendix A: List of Three-term Problems 121

Appendix B: Trial Event Sequence 122

Appendix C: The Number of Participants in Each Cluster by Developmental Period with Adjusted Residual Scores 123

Appendix D: Research Ethics Board Approval Form 124

Conceptually-Based Strategy Use: Investigating Underlying Mechanisms and Development Across Adolescence and Into Early Adulthood

Researchers have used mathematical problem solving to systematically study how problem solving strategies are selected, how new problem-solving strategies develop, and what factors affect problem solving. The benefits of studying mathematical problem solving are that the answers are absolute, the number of strategies, although numerous, are finite, and the strategies can be clearly defined and identified by combining accuracy and reaction time data with verbal reports (Siegler, 1989). As a result, detailed models of strategy choice and development have been proposed (e.g., Siegler & Lemaire, 1997; Shrager & Siegler, 1998). These models account for how problem characteristics (e.g., problem size; Zbrodoff & Logan, 2005) and individual variability (e.g., strategy preference; Siegler, 1988a) affect problem solving. However, current models do not adequately explain how deep, abstract knowledge (i.e., conceptual knowledge) is applied to problem solving.

Mathematical Problem Solving

Researchers have identified different types of mathematical knowledge to better understand how mathematical problem solving develops. Some researchers propose that mathematical problems are solved using two qualitatively different types of mathematical knowledge, instrumental and relational (Skemp, 1976). Instrumental understanding involves knowledge of the rules of mathematics and knowing when to apply the rules during problem solving. For example, an individual with an instrumental understanding of mathematics knows the rules needed to solve the problem 6 × 8 + 2 (i.e., multiplication then addition). In contrast, relational understanding involves relating a mathematical problem to an appropriate internal abstract representation which governs the mathematical rules involved in that task (i.e., a schema, Byers & Erlwanger, 1985; Schneider & Stern, 2010; Skemp, 1976). For example, an individual with a relational understanding of mathematics understands why the problem 6 × 8 + 2 must be solved in a specific order. From this perspective, mathematical problem solving is approached from either an instrumental or a relational understanding and the transition from an instrumental to a relational understanding is a qualitative shift in the development of mathematical problem solving (Graeber, 1999).

Alternatively, there may be three types of mathematical knowledge used during problem solving, factual, procedural, and conceptual (Bisanz & LeFevre, 1990). Factual knowledge is memorized information about the arithmetic relations between numbers. Procedural knowledge is information on step-by-step goal directed activities used to solve mathematical problems (e.g., 20 – 13: 20 – 10 = 10, 10 – 3 = 7)[1]. Conceptual knowledge is knowing why a procedure works or whether it is legitimate (LeFevre et al., 2006), understanding mathematical principles (Stern, 1992), and having knowledge of mathematical principles and their interrelations (Schneider & Stern, 2010). From this

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[1] This notation is read as follows: problem: step 1, step 2, . . . = answer.

perspective, individuals can use all three types of mathematical knowledge during problem solving and the knowledge types develop iteratively within individuals (i.e., increases in one knowledge type result in increases in the other knowledge types; Rittle-Johnson, Siegler, & Alibali, 2001). Recently, researchers have even proposed that the iterative development between procedural and conceptual knowledge may be so extensive that an integration between procedural and conceptual knowledge occurs, in which knowledge of procedures and concepts is interconnected (Newton, Star, & Lynch, 2010). Alternatively, procedural knowledge may always develop before conceptual knowledge (i.e., procedures-first theories; Cowan, 2003; Fuson, 1988; Karmiloff-Smith, 1992) or conceptual knowledge may always develop before procedural knowledge (i.e., concepts-first; Canobi, 2009; Geary, 1994; Hallett, Nunes, & Bryant, 2010). Regardless of whether there are two or three types of mathematical knowledge, most researchers differentiate between mathematical problem solving that relies on an understanding of the step-by-step process of mathematics (instrumental/procedural) or a deep, abstract understanding of mathematics (relational/conceptual).

Some areas of mathematical problem solving are understudied. The majority of research has investigated problem solving that relies on an understanding of the step-by-step process of mathematics. For example, there is considerable research on the step-by-step problem solving strategies used to solve simple two-term problems (e.g., $2 + 4$, $8 - 3$, 6×7, $18 \div 2$; Campbell & Timm, 2000; Robinson, 2001; Robinson & Dubé, 2008;

Siegler, 1988b; Siegler & Shrager, 1984). In contrast, relatively less research has investigated problem solving that relies on a deep, abstract understanding of mathematics. For example, there is relatively little research on the use of conceptually-based strategies. Conceptually-based strategies require an individual to apply their deep, abstract understanding of mathematics to solve mathematical problems (Rittle-Johnson et al., 2001). Due to the lack of research investigating these strategies, it is not known whether current models of mathematical problem solving accurately explain the development of conceptually-based strategies.

Development of Conceptually-Based Strategy Use

There are many fundamental mathematical concepts that individuals can apply to solve mathematical problems, such as the commutativity, additive composition, and complement concepts. The commutativity concept (i.e., knowing that problems containing the same terms in a different order have the same answer, $a + b = b + a$; Canobi et al., 1998) can be applied to generate the min strategy (i.e., solving 2 + 15 by counting on from the larger term instead of counting on from the smaller term; Cowan & Renton, 1996). The additive composition concept (i.e., knowing that numbers are composed of other numbers, e.g., 10 = 7 + 3, 5 + 5, 2 + 3 + 1 + 4; Bryant, Christie, & Rendu, 1999) can be applied to generate the decomposition strategy (e.g., 8 × 9: 8 × 5 = 40, 8 × 4 = 32, 40 + 32 = 72). The complement concept (i.e., understanding the complementary relationship between addition and subtraction, $a + b = c$, $c - b = a$) can be applied to solve complementary problems (e.g., using the answer from 3 + 4 = 7 to solve 7 - 4 = ?; Canobi, 2005). Thus, each of these fundamental concepts can be applied to solve everyday mathematical problems. However, studying how these concepts are spontaneously applied to problem solving is difficult because many of the strategies derived from these concepts are taught in schools (Baroody, Torbeyns, & Verschaffel, 2009; Prather & Alibali, 2009).

Research investigating the development of the inversion and associativity concepts has provided some of the most detailed information on the development of conceptually-based strategies. Having an understanding of the concepts of inversion and associativity is important for developing an understanding of other mathematical concepts such as commutativity, additive composition, and related complement (Baroody et al., 2009), and for developing an understanding of the relationship between operations. It can also result in the use of novel conceptually-based strategies (Bryant et al., 1999; Robinson & Dubé, 2009b). Individuals who understand the inversion concept know that adding and subtracting or multiplying and dividing a number by the same number results in no change to the original number, $a + b - b = a$, $d \times e \div e = d$ (Robinson & Ninowski, 2003; Starkey & Gelman, 1982). Individuals who understand the associativity concept know that numbers can be decomposed and recombined in various ways and result in the same answer, $(a + b) - c = a + (b - c)$, $(d \times e) \div f = d \times (e \div f)$ (Canobi, Reeve, & Pattison, 1998; Robinson, Ninowski, & Gray, 2006). Individuals' understanding of inversion and associativity can be studied by determining whether an individual can apply the concepts to problem solving, producing two fast and accurate conceptually-based strategies called the inversion shortcut (e.g., 3 ~~× 24 ÷ 24~~ = 3) and the associativity strategy (e.g., 4 × 24 ÷ 8, 24 ÷ 8 = 3, 3 × 4 = 12; Rasmussen, Ho, & Bisanz, 2003; Robinson et al., 2006). The study of these conceptually-based strategies has yielded considerable information on how they develop.

The inversion shortcut. The study of individuals' understanding of the inversion concept and its application to problem solving (i.e., the inversion shortcut) originated with Piaget. Piaget (1976) proposed that the type of reasoning involved in understanding the inversion concept was similar to the type of reasoning required for formal operational thinking (i.e., hypothetico-deductive reasoning; Dubé & Robinson, 2010a). Applying the inversion concept and formal operational thinking both require an individual to possess a general theory and to deduce from it what might occur in a given situation. This work highlighted the importance of studying children's understanding of the inversion concept and was the impetus for future research on the topic (Starkey & Gelman, 1982). After Piaget's initial work, the majority of research on the inversion concept focused on identifying when children exhibit an understanding of the inversion concept (Schneider & Stern, 2009).

Inversion shortcut use has been studied across childhood and adulthood. Researchers studying preschool children's understanding of the inversion concept have found that, on average, children between three and four years of age can use the inversion shortcut to solve inversion problems (Bryant et al., 1999; Sherman & Bisanz, 2007). Preschool children's ability to use the inversion shortcut is assessed by presenting participants with a set of concrete items, to which varying numbers of items are added and removed, and then researchers ask the participants how many items remain (Bryant et al., 1999). Children are faster and more accurate when the same number of items are added and removed (i.e., inversion trials) than when different numbers of items are added and removed (Bryant et al., 1999; Rasmussen et al., 2003; Sherman & Bisanz, 2007). Preschool children's increased speed and accuracy on the inversion trials is surprising considering that the type of reasoning required to use the inversion concept is supposed to be a hallmark of formal operational thinking. This suggests that the inversion shortcut can be implemented very early in development but it does not necessarily mean that children have a complete grasp of the inversion concept.

The frequency of inversion shortcut use after the onset of formal schooling suggests that conceptually-based strategies that are used early in development can still take considerable time to mature. After the onset of formal schooling, children in Grades 2 through 4 use the inversion shortcut on just under half of three-term addition and subtraction inversion problems (Bisanz & LeFevre, 1990; Robinson & Dubé, 2009a). Further, inversion shortcut use is stable across Grades 2 through 4 and this suggests that Grade 4 children's understanding of the inversion concept is not markedly different from that of Grade 2 children. Also, the use of the inversion shortcut on fewer than half of the problems suggests that children's understanding of the inversion concept is still developing in Grade 4. Research on older children's understanding of the inverse relationship between addition and subtraction and between multiplication and division has provided evidence that children's understanding of the inversion concept continues to develop in late childhood (Robinson et al., 2006). Children in Grade 6 or 8 use the inversion shortcut less frequently on multiplication and division problems (20%) than on addition and subtraction problems (40%) and they do not exhibit any crossover (i.e., inversion shortcut use on addition and subtraction problems does not promote inversion shortcut use on multiplication and division problems; Robinson et al., 2006). Also, children's inversion shortcut use on multiplication and division problems is stable across Grades 6 through 8 (Robinson & Dubé, 2009b). This suggests that understanding of the inversion concept begins to form well before the onset of formal schooling but maturation of this understanding is prolonged. By adulthood, the inversion shortcut is used on the majority of inversion problems, regardless of the operations involved (Robinson & Ninowski, 2003). Researchers have not yet identified when individuals' understanding of the inversion concept reaches adult levels. Also, the less frequent use of the inversion shortcut on multiplication and division than on addition and subtraction inversion problems suggests that the ability to apply conceptually-based strategies during problem solving is affected by the pairs of operations involved (Robinson & Ninowski, 2003; Robinson et al., 2006). Therefore, conceptually-based strategy use is affected by a person's age (i.e., developmental period) and by problem characteristics (i.e., the operations involved).

The associativity strategy. Individuals' understanding of the associativity concept and its application to problem solving (i.e., the associativity strategy) has received relatively less attention than the inversion concept (Robinson et al., 2006). One of the first studies investigating children's understanding of associativity was conducted by Canobi et al. (1998). In their study, children from Grade 1 and Grade 2 were asked to solve a three-term single-operation problem (e.g., 6 + 2 + 3). After solving the three-term problem, children were asked to solve a two-term problem constructed of the first term and the sum of the second and third term of the recently solved three-term problem (e.g., 6 + 5). The researchers hypothesized that if children understand that numbers can be recombined and decomposed in various ways (i.e., associativity) then they should be able to use their experience with solving the three-term problem to solve the two-term problem. For 41% of the children, experience with the three-term problems facilitated problem solving on 20% of the two-term addition problems. For the remaining children, experience with the three-term problems did not facilitate problem solving on the two-term problems. As an alternative measure of conceptual understanding, the researchers provided a demonstration to participants of using the three-term problem to help solve the two-term problem and asked participants whether doing so was appropriate. The study found that 60% of the children judged the demonstration to be appropriate. Furthermore, children's judgements of the appropriateness of the demonstration were positively related to their use of the decomposition strategy (e.g., solving 5 + 6 by summing 5 + 3 + 3), which has been argued to require an understanding of associativity (Sternberg, 1985). This is the only study to investigate children's application of the associativity concept to two-term problem solving. These results suggest that children across Grades 1 and 2 can recognize the appropriateness of using a strategy based on the associativity concept but they are not likely to spontaneously use the strategy. This pattern of understanding could indicate that these children have the potential to use the associativity concept in problem solving but require further experience with the concept to master it (Heine et al., 2010; Vygotsky, 1978).

Researchers have also investigated conceptually-based strategy use based on the associativity concept using three-term mathematical problems. In some studies investigating inversion shortcut use on three-term problems (e.g., Shrager & Siegler, 1998), researchers compare performance on three-term inversion problems to performance on three-term control problems (i.e., problems on which the inversion shortcut is not typically applied, 9 + 13 - 14, 9 × 14 ÷ 21). If participants are more accurate and faster at solving inversion problems than control problems then researchers use this as one source of evidence to conclude that participants are using the inversion shortcut to solve inversion problems (Bisanz & LeFevre, 1990). In a study of preschool children's inversion shortcut use on concrete addition and subtraction problems, Klein and Bisanz (2000) found that some children spontaneously applied their understanding of associativity by rearranging the order of the control problems to make problem solving easier. In a study of adults' understanding of inversion and associativity for addition and subtraction and multiplication and division, Robinson and Ninowski (2003) constructed half of the control problems such that performing the subtraction operation before the addition operation or performing the division operation before the multiplication operation resulted in a positive, whole number (e.g., $12 + 23 - 20$, $9 \times 18 \div 6$). The researchers hypothesized that if participants understand the associativity concept then they will choose to complete the division operation before the multiplication operation (i.e., the associativity strategy), because the strategy is faster and less prone to error than completing the multiplication operation before the division operation. Studies using these adapted control problems (i.e., associativity problems) have provided detailed information on the development of the associativity concept.

Across Grades 2 through 4, children use the associativity strategy to solve approximately 20% of addition and subtraction associativity problems (Robinson & Dubé, 2009a). Similar to inversion shortcut use, the stability of associativity strategy use across Grades 2 through 4 suggests that children in Grade 4 have a similar understanding of the associativity concept as children in Grade 2. Across Grades 6 through 8, children use the associativity strategy on approximately 17% of addition and subtraction associativity problems (Robinson et al., 2006) and on 10% of multiplication and division associativity problems (Robinson & Dubé, 2012). Again, the lower frequency of associativity strategy use on multiplication and division associativity problems than on addition and subtraction problems parallels the lower frequency of inversion shortcut use on multiplication and division problems. By adulthood, the associativity strategy is used on approximately 58% of addition and subtraction associativity problems (Robinson & Ninowski, 2003) and on 44% of multiplication and division associativity problems (Dubé & Robinson, 2010a; Robinson & Ninowski, 2003). The results of these studies suggest that individuals' understanding of the associativity concept is not well developed in late childhood. Therefore, it is not known when an understanding of the associativity concept reaches adult levels. Similar to the development of the inversion shortcut, the use of the associativity strategy across development indicates that conceptually-based strategy use can be affected by the developmental period and by problem characteristics (i.e., the operation involved).

Comparing the development of the inversion shortcut and the associativity strategy provides additional information on the development of these strategies. The frequency of inversion shortcut and associativity strategy use across development suggests that individuals have a better understanding of the inversion concept than the associativity concept. However, the developmental trajectory is similar for both concepts. In studies of children's conceptually-based strategy use, the inversion shortcut was used more frequently on inversion problems than the associativity strategy was used on associativity problems, both conceptually-based strategies were used less frequently than the left-to-right computation strategy on their respective problem types, and there were no grade differences in the frequency of conceptually-based strategy use (Dubé & Robinson, 2010b; Robinson & Dubé, 2009a, 2009b). Also, the frequency of both inversion shortcut and associativity strategy use was significantly lower on multiplication and division problems than on addition and subtraction problems (Robinson & Dubé, 2009a, 2009b; Robinson & Ninowski, 2003; Robinson et al., 2006). In studies of adults' conceptually-based strategy use, the inversion shortcut was used more frequently than the associativity strategy and the inversion shortcut was used more frequently than the left-to-right computational strategy on inversion problems, whereas the associativity strategy was used on approximately half of associativity problems (Dubé & Robinson, 2010a; Robinson & Ninowski, 2003). The similar developmental trajectories for both conceptually-based strategies could suggest that the two strategies are related. Alternatively, it could suggest that applying a deep, abstract understanding of mathematics to mathematical problem solving follows a similar developmental trajectory, regardless of the mathematical concept being applied.

Individual variability in conceptually-based strategy use. The development of inversion shortcut and associativity strategy use has been investigated across early childhood to early adolescence and in adulthood. In each of these developmental periods, researchers have identified considerable individual variability in inversion shortcut and associativity strategy use. The results of three recent studies indicate that mapping individual variability in inversion shortcut and associativity strategy use is essential for understanding the development of these conceptually-based strategies. The studies investigated conceptually-based strategy use across Grades 2, 3 and 4 on addition and subtraction problems (Robinson & Dubé, 2009a), across Grades 6, 7, and 8 on multiplication and division problems (Robinson & Dubé, 2009b), and in adulthood on multiplication and division problems (Dubé & Robinson, 2010a). Combining these studies, there is evidence that (1) there is a relationship between inversion shortcut and associativity strategy use, (2) that the strength of this relationship changes across development, and (3) that adolescence may be a particularly important time for the development of the inversion shortcut and associativity strategy.

Each of the three studies identified patterns of individual variability in conceptually-based strategy use, with patterns of strategy use changing across development. In Robinson and Dubé's (2009a) study of Grade 2, 3, and 4 children's strategy use on three-term addition and subtraction problems, children were classified as users of both the inversion shortcut and associativity strategy (Dual Concept users), users of only the inversion shortcut (Inversion Concept users), or users of neither of the conceptually-based strategies (i.e., using the left-to-right computational strategy 3 + 24 – 22: 3 + 24 = 27, 27 – 22 = 5; No Concept users). In Robinson and Dubé's (2009b) study of Grade 6, 7, and 8 children's strategy use on multiplication and division problems, children were classified as users of the inversion shortcut, of an intermediate strategy between inversion and left-to-right computational strategies (i.e., negation; Robinson et al., 2006), or only of the left-to-right computational strategies. No group of associativity strategy users was identified. However, participants were presented with only a few associativity problems and the associativity strategy was not included in the cluster analyses. In Dubé and Robinson's (2010a) study of adults' strategy use on multiplication and division problems, participants were classified similarly as in Robinson and Dubé (2009a) with adults classified as Dual, Inversion, or No Concept users.

Taken together, the results of these three studies suggest that there is a relationship between the two conceptually-based strategies, which depends on both the operation involved and the developmental period under investigation. There is a relationship between the two conceptually-based strategies early in development for addition and subtraction problems: children who used one strategy were more likely to use the other strategy (Robinson & Dubé, 2009a; 2009b). This relationship is not as strong when assessed using multiplication and division problems across late childhood and early adolescence but is strong in adulthood (Dubé & Robinson, 2010a; Robinson & Dubé, 2009b). However, the exact point in development at which the relationship between these conceptually-based strategies reaches adult levels has yet to be identified.

The identification of individual variability in conceptually-based strategy use also suggests that some individuals have a better grasp of the inversion and associativity concepts than their peers, but the reason for this is not known. Most research investigating why there is individual variability in inversion shortcut use has focused on the three types of mathematical knowledge (e.g., Baroody & Lai, 2007; Canobi & Bethune, 2008; Gilmore & Spelke, 2008). The simplest explanation would be that individuals who use conceptually-based strategies have greater mathematical knowledge than individuals who do not use the conceptually-based strategies. Greater factual knowledge should increase inversion shortcut and associativity strategy use. For example, as children learn addition facts they begin to understand that addition increases quantities, as they learn subtraction facts they begin to understand that subtraction has the opposite effect (Canobi & Bethune, 2008), and this should increase their understanding of the inverse and associative relationship between addition and subtraction. However, knowledge of mathematical facts does not predict frequency of inversion shortcut or associativity strategy use (Robinson & Dubé, 2009a, 2009b). Greater procedural knowledge should also increase inversion shortcut and associativity strategy use because procedural knowledge is believed to develop iteratively alongside conceptual knowledge (Rittle-Johnson, Siegler, & Alibali, 2001). However, children can have good conceptual knowledge of the inversion concept but poor computational skills. For example, Gilmore and Papadatou-Pastou's (2009) meta-analysis of 14 studies on children's inversion shortcut use on addition and subtraction problems investigated whether children who frequently used the inversion shortcut had better procedural knowledge of mathematics than children who used computational strategies. The study identified three groups of children. Proceptual thinkers had a good understanding of the inversion concept and good calculation skills. Procedural thinkers had a poor understanding of the inversion concept and good calculation skills. A third group had a good understanding of the inversion concept but poor computational skills. Despite the theoretical link between mathematical knowledge and conceptually-based strategy use, researchers have not been able to account for individual variability in inversion shortcut or associativity strategy use using measures of procedural and factual knowledge.

Some studies have investigated whether general abilities play a part in developing children's understanding of the inversion concept. Rasmussen et al. (2003) investigated preschool and Grade 1 children's inversion shortcut use on a concrete addition and subtraction task (i.e., adding and subtracting physical blocks) to determine whether inversion shortcut use was predicted by visual-spatial working memory capacity. They found that visual-spatial working memory capacity predicted inversion shortcut use for children in preschool but not for children in Grade 1. They concluded that preschool children with higher visual-spatial working memory capacity were better able to represent the blocks internally than preschool children with low visual-spatial working memory capacity and that by Grade 1 all children had sufficient visual-spatial working memory capacity to internally represent the blocks. Therefore, differences in visual-spatial working memory capacity predicted preschool children's inversion shortcut use because of the nature of the concrete inversion task and did not necessarily reflect children's understanding of the inversion concept. The results could not explain why there is variability in inversion shortcut use in Grade 1.

Dubé and Robinson (2010b) investigated Grade 6 and Grade 8 children's inversion shortcut use on multiplication and division problems to determine whether individual variability in inversion shortcut use was predicted by short-term memory capacity, working memory capacity, or analogical reasoning ability. Both analogical reasoning and working memory capacity independently predicted inversion shortcut use. These results suggest that the general ability to understand relationships and central executive functioning affect conceptually-based strategy use. Taken together, the investigations by Rasmussen et al. (2003) and Dubé and Robinson (2010) are examples of how general cognitive abilities affect mathematical problem solving.

In summary, despite a considerable amount of research on the development of the inversion shortcut and associativity strategy use, it is not known when the frequency of conceptually-based strategy use for multiplication and division problems reaches adult levels or when the relationship between the inversion shortcut and the associativity strategy use on multiplication and division problems reaches adult levels. The absence of grade differences in inversion shortcut and associativity strategy use on multiplication and division problems across Grades 6, 7, and 8 indicates that development of these conceptually-based strategies occurs after this developmental period. The concepts of inversion and associativity are necessary for manipulating mathematical problems in a meaningful way (e.g., algebra; Baroody et al., 2009). Thus, a study of inversion shortcut and associativity strategy use should be conducted with participants from adolescence into early adulthood to determine when these essential mathematical concepts fully develop.

Theories of Conceptually-Based Strategy Use

Two strategy choice theories have been proposed to explain the development of inversion shortcut use on three-term addition and subtraction problems, the schema-based theory (Baroody, 1994, 1999; 2003; Baroody & Ginsburg, 1986) and the strategy choice and discovery simulation model (SCADS*; Siegler & Araya, 2005). These theories must be generalized to the development of conceptually-based strategy use on multiplication and division problems and to the development of the associativity strategy, because neither theory has addressed these operations or the associativity strategy directly. Both theories were designed to make such generalizations. The schema-based theory proposes that mathematical concepts (e.g., inversion and associativity) are abstractions, which can eventually be applied to multiple problem types (Baroody, 2003). The SCADS* model is an adaptation of previous strategy selection models that was designed to better understand how conceptually-based strategies are discovered and applied to problem solving (Siegler & Araya, 2005). Thus, these models are designed to account for multiple types of conceptually-based strategy use and research is needed to determine how successful these models are at predicting the development of different conceptually-based strategies across multiple problem types. Each of these theories is successful in predicting some aspects of the development of the inversion shortcut but neither can account for the lower frequency of inversion shortcut and associativity strategy use on multiplication and division problems relative to addition and subtraction problems. Moreover, they do not explain the individual variability in inversion shortcut and associativity strategy use or account for the absence of grade differences in conceptually-based strategy use.

Schema-based theory. Baroody's schema-based theory of inversion shortcut use (Baroody, 1994; 2003; Baroody & Ginsburg, 1986) is particularly concerned with the development of the underlying conceptual understanding of inversion and how it can be applied to a variety of problem formats. The schema-based model proposes that children develop an abstract representation of the inversion concept (i.e., an inversion schema) that strengthens over time and is eventually applicable to all inversion problems, regardless of number size or context (Baroody & Lai, 2007). A strong inversion schema is one in which the inversion principle is "general in nature and, thus, logically and consistently applied" (Baroody & Lai, 2007, p. 164). The schema-based theory was developed using data from children's inversion shortcut use on three-term addition and subtraction problems (e.g., Baroody & Lai, 2007).

The schema-based theory must be generalized to both inversion shortcut use on multiplication and division problems and associativity strategy use. There are two ways in which this theory can be generalized to understand conceptually-based strategy use on multiplication and division problems. First, it could be argued that a schema strengthens over time and eventually can be applied to multiple pairs of operations. Second, it could be argued that a new schema is created for three-term multiplication and division problems (Robinson & Dubé, 2009b), but this has not been investigated. Similarly, the schema-based theory can be generalized to associativity strategy use by proposing the existence of an associativity schema. The strength of the schema-based view is that it recognizes the complexity of mathematical concepts and aims to understand how concepts change across development by measuring conceptually-based strategy use at varying developmental periods with different problem formats (e.g., word problems, abstract problems). Therefore, the schema-based theory depicts the development of conceptually-based strategies as the development of ever-strengthening conceptual understandings that translate into generalizable problem solving strategies.

The weakness of the schema-based theory is that it does not propose a step-by-step process describing the application of the inversion shortcut to problem solving, which limits the model's ability to predict variability in conceptually-based strategy use. For example, the schema-based theory could be used to understand some of the findings of Robinson and Dubé (2009a; 2009b) and Dubé and Robinson (2010a, 2010b). The various patterns of individual variability in inversion shortcut and associativity strategy use may reflect individual variability in the strength of inversion and associativity schemas. Some individuals may have stronger inversion or associativity schemas and this results in their increased frequency of inversion shortcut and associativity strategy use. However, the schema-based theory cannot account for other findings.

A fundamental assumption of the schema-based view is that the inversion schema and, by extension, the associativity schema become stronger over time until they can be applied to all problems and operations. Thus, the schema-based view does not account for the lower frequency of inversion shortcut and associativity use across Grade 6 through 8 on multiplication and division problems, as compared to inversion shortcut and associativity use on addition and subtraction problems. In addition, the theory does not explain why children in Grade 4 and Grade 8 are no more likely to apply conceptually-based strategies than children in Grade 2 and Grade 6, respectively. The schemas in the older children should be stronger than the schemas in the younger children and this should result in a grade difference in strategy use. Thus, the explanatory power of the schema-based view may be limited because it identifies the strength of a schema as the sole predictor of conceptually-based strategy use. If the schema-based view detailed the step-by-step process of conceptually-based strategy use then perhaps individual differences in how this process is executed could be used to better understand variability in strategy use.

SCADS*. The second of the two theories is Siegler and Araya's (2005) strategy choice and discovery simulation model (SCADS*). The SCADS* model is primarily concerned with identifying and mapping the process of applying the inversion shortcut to algebraic addition and subtraction inversion problems. Originally, Shrager and Siegler (1995) developed SCADS to model strategy discovery for simple two-term addition problems. The SCADS* model does not directly address the associativity strategy nor conceptually-based strategy use on multiplication and division problems but the model can be generalized to both. Three elements of SCADS are still present in SCADS* for the explicit purpose of facilitating the discovery and use of the inversion strategy on addition and subtraction problems. First, strategies can be generalized to novel problems. The inversion shortcut or associativity strategy can be discovered on one problem (2 + 3 – 3) and then generalized to solve another problem (5 + 6 – 6). Second, new strategies can be selected over well-learned strategies. Even though the left-to-right computation strategy is well practiced, the inversion shortcut or associativity strategy can be selected to solve the problem. Third, the speed and accuracy of a strategy determines whether it is selected. The reason the inversion shortcut or associativity strategy are selected over the well-learned left-to-right computation strategy is that they are faster and more accurate. However, simply understanding these three principles of SCADS* is insufficient to fully understand the development of the inversion shortcut or the associativity strategy.

In SCADS*, the development of inversion shortcut use and, by extension, associativity strategy use can by thought of as a series of competitions between these conceptually-based strategies and the well-learned left-to-right computation strategy. Every time the inversion shortcut or associativity strategy is activated, the associated strength of the strategies increases for the specific problem being solved but also for all inversion or associativity problems. In contrast, the associated strength of the left-to-right computation strategy only increases for the specific problem being solved. This affects strategy selection because strategies with a greater associated strength are selected for execution (Siegler & Shipley, 1995). For example, when a child first solves an inversion problem (e.g., 3 + 7 – 7), both the left-to-right computational strategy and the inversion shortcut are selected for possible execution. The associated strength of the left-to-right computational strategy is high because the child has frequently solved 3 + 7 and 10 – 7. In contrast, the associated strength of the inversion shortcut is low because the child has never encountered an inversion problem and, thus, has not used the inversion shortcut. As a result, the left-to-right computational strategy is executed and its associated strength increases for that problem. Also, the associated strength of the inversion shortcut increases to some degree for the specific problem and to a lesser degree for all inversion problems. The associated strength for the inversion shortcut increases because each experience with an inversion problem is an opportunity to apply the inversion concept to problem solving. When the child solves the next inversion problem, the associated strength of the left-to-right computational strategy depends on the problem characteristics (i.e., how many times the child had solved 4 + 8 and 12 – 8 for the inversion problem 4 + 8 – 8). In contrast, the associated strength of the inversion shortcut, for that particular problem, depends on how many inversion problems had been previously solved. Eventually, the associated strength of the inversion shortcut becomes greater than the associated strength of the computational strategy and the inversion shortcut is executed (Siegler & Stern, 1998). Once the inversion shortcut is successfully executed, the accuracy and speed of the strategy increases its associated strength.

This same process may also apply to the use of the associativity strategy. Like the inversion shortcut, when a child first solves an associativity problem both the left-to-right computational strategy and the associativity strategy are selected for possible execution. Then, even though the left-to-right computational strategy is executed, the associated strength of the associativity strategy increases for all associativity problems. Eventually, the associated strength of the associativity strategy is greater than the strength of the left-to-right computational strategy and the associativity strategy is executed.

Using a computer simulation based on SCADS*, Siegler and Araya (2005) successfully modeled children's inversion shortcut use on addition and subtraction problems. Data from Siegler and Stern's (1998) microgenetic study of children's addition and subtraction inversion use were entered into the simulation. Based on these data, SCADS* successfully predicted when children would discover the inversion shortcut and how frequently children would use it after discovery. Thus, it would seem that SCADS* has modeled all relevant aspects of inversion shortcut use. However, SCADS* would not predict the results of Robinson and Dubé's (2009c) microgenetic study of inversion shortcut use on multiplication and division problems. They found that the number of problems the children previously solved did not predict inversion shortcut use. Also, it would be unlikely for SCADS* to predict the individual variability and absence of grade differences in inversion shortcut and associativity strategy use found by Robinson and Dubé (2009a; 2009b). No component of the model accounts for why one group of individuals will use conceptually-based strategies (i.e., Dual Concept Users and Inversion Concept Users) whereas another group of users will use left-to-right computational strategies (No Concept Users). Further, the model cannot explain the absence of grade differences. However, Siegler and Araya had proposed a mechanism in SCADS* that may account for these limitations but this mechanism was never experimentally tested.

Interruption of Procedures in Conceptually-Based Strategy Use

According to SCADS*, conceptually-based strategies compete against left-to-right computational strategies and whichever strategy has the stronger associated strength is executed; however, an additional mechanism may be needed for conceptually-based strategies to win the competition. Left-to-right computational strategies are well practiced and their associated strength increases every time they are executed. Thus, it should require the solving of several inversion or associativity problems for the associated strength of conceptually-based strategies to surpass that of left-to-right computational strategies (Siegler & Araya, 2005). Yet some individuals use conceptually-based strategies to solve the first inversion or associativity problem they encounter and go on to solve the majority of inversion and associativity problems using conceptually-based strategies (Dubé & Robinson, 2010a; Robinson & Dubé, 2009a, 2009b; Robinson et al., 2006; Robinson & Ninowski, 2003). The use of conceptually-based strategies on the very first problem and on the majority of subsequent problems could by explained by Siegler and Araya's hypothesis that there is a mechanism which interrupts the execution of left-to-right computational strategies and allows for conceptually-based strategies to be executed.

Despite Siegler and Araya (2005) including the interruption of procedures mechanism in SCADS*, the existence of this mechanism has not been empirically tested. As a result, it is not clear whether the use of conceptually-based strategies interrupts the execution of left-to-right computational strategies or whether left-to-right computational strategies are not activated when conceptually-based strategies are executed. Determining which of these two alternatives actually occurs during conceptually-based problem solving is essential for understanding the process of strategy selection (Bajic & Rickard, 2009). If there is an interruption of procedures then two alternative strategies are activated during strategy selection. If left-to-right computational strategies are simply not activated then no interruption is necessary.

Evidence consistent with the interruption of procedures mechanism can be found in the execution of the negation strategy. The negation strategy involves an individual computing the answer for the first operation in an inversion problem (e.g., $3 \times 7 \div 7$, $3 \times 7 = 21$) but then noticing that the second operation is the inverse of the first operation and as a result the second operation negates the first operation (Bisanz & LeFevre, 1990). Siegler and Araya (2005) propose that the negation strategy is a combination of the left-to-right computational strategy and the inversion shortcut, a combination in which both strategies are activated but the interruption of the left-to-right computational strategy occurs halfway through solving the problem. This is evidence for the simultaneous activation of two strategies. If neither the inversion shortcut and the left-to-right computational strategy are activated, then how could the inversion shortcut be executed halfway through problem solving? However, the fact that both the inversion shortcut and left-to-right computation strategy are activated during negation strategy use does not necessitate that the left-to-right computational strategy is activated during inversion shortcut use. Research is needed to determine whether conceptually-based strategies and left-to-right computational strategies are both activated when conceptually-based strategies are executed. Such research will determine whether conceptually-based strategy use can be conceptualized as a race between two alternative strategies towards an answer or as the selection of a single strategy that is then used to solve the problem.

If the interruption of procedures is a means by which conceptually-based strategies overcome left-to-right computational strategies, this would help explain why the frequency of conceptually-based strategy use is lower for multiplication and division inversion and associativity problems than for addition and subtraction problems, why there is individual variability in conceptually-based strategy use, and why the frequency of conceptually-based strategy use has not approached adult levels by early adolescence. First, left-to-right computational strategies used to solve multiplication and division inversion and associativity problems are faster than left-to-right computational strategies used to solve addition and subtraction inversion and associativity problems (Robinson et al., 2006; Robinson & Ninowski, 2003). Because a strategy's speed relates to its associated strength (Siegler & Shipley, 1995), left-to-right computational strategies have even greater associated strengths for multiplication and division problems than for addition and subtraction problems. This increased speed leaves less time for the strategies to be interrupted. Second, the interruption of procedures mechanism involves the monitoring and control of automatically-activated numerical information (i.e., interrupting the product of the multiplication operation), which requires resources from central executive processing (Conway & Engle, 1994; van der Sluis, de Jong, & van der Leij, 2007). There is evidence that children's central executive functioning predicts frequency of inversion shortcut use on multiplication and division problems (Dubé & Robinson, 2010b). Third, there is evidence that maturation of the prefrontal cortex across late adolescence leads to significant increases in executive functioning, bringing adolescents' executive functioning to an adult level (Toga, Thompson, & Sowell, 2006). Therefore, the minimal grade differences in conceptually-based strategy use found prior to late adolescence could be explained by executive functioning not being fully developed. However, there is no research to determine whether the interruption of procedures mechanism is actually involved in conceptually-based strategy use. Thus, the next step in investigating the development of inversion shortcut and associativity strategy use is to determine the role of the interruption of procedures mechanism.

Goals of the Study

The study had two goals. The first goal was to map the developmental trajectory of inversion shortcut and associativity strategy use on multiplication and division problems across early to late adolescence (i.e., across Grades 7, 9, and 11) and into early adulthood (i.e., adults who recently graduated from high school). Previous research has identified that adults use both conceptually-based strategies more frequently than children, that both children and adults use the inversion shortcut more frequently than the associativity strategy (Robinson & Ninowski, 2003; Robinson et al., 2006), and that use of one strategy relates to the use of the other strategy for both children and adults (Dubé & Robinson, 2010a; Robinson & Dubé, 2009a). However, no research has mapped the development of conceptually-based strategy use on multiplication and division problems across early to late adolescence and into early adulthood. Thus, there is no information on when the increase in strategy use occurs, whether the inversion shortcut is always used more frequently than the associativity strategy, or whether there is a change in the relationship between the use of the two strategies.

The second goal was to determine whether the interruption of procedures mechanism is involved in inversion shortcut and associativity strategy use. Siegler and Araya (2005) have proposed that the use of left-to-right computational strategies must be interrupted in order for the inversion shortcut to be executed. However, there has not yet been an empirical test of whether this mechanism is involved in either the use of the inversion shortcut or the use of the associativity strategy on multiplication and division problems. Evidence for the existence of this mechanism would help determine whether the selection of conceptually-based strategies involves the simultaneous interruption of alternative strategies.

In the present study, adolescents from Grades 7, 9, and 11 as well as adults who graduated from high school the previous academic year and were enrolled in their first year of university solved three-term multiplication and division inversion and associativity problems. While they solved the problems, the extent to which participants interrupted alternative strategies was measured. A two year age gap between each of the developmental periods was chosen because it covered development across adolescence and into early adulthood, and previous research has not found significant differences in conceptually-based strategy use between sequential grades (Robinson & Dubé, 2009a, 2009b). Multiplication and division inversion and associativity problems were presented for two reasons. First, there is information on individual variability in conceptually-based strategy use on multiplication and division problems for later childhood and adulthood, but there is no information on strategy use beyond early adolescence. Thus, it is not known when the pattern of Dual, Inversion, and No Concept users first arises for multiplication and division problems or whether this pattern holds for the developmental period beyond early adolescence. Second, it is not known whether the interruption of procedures occurs during conceptually-based strategy use on multiplication and division problems. This investigation will provide information on the development of inversion shortcut and associativity strategy use across a previously unstudied developmental period and provide a test of whether the mechanism proposed in SCADS* affects conceptually-based strategy use.

Method

Participants

Participants were 40 adolescent students per grade from Grades 7, 9, and 11 and 40 adult university students who had graduated from high school the previous academic year. Grade 7 participants (20 girls) had a median age of 12 years and 11 months (range: 12 years, 4 months to 13 years, 10 months), Grade 9 participants (22 girls) had a median age of 14 years and 6 months (range: 14 years, 0 months to 15 years, 11 months), Grade 11 participants (18 girls) had a median age of 16 years and 9 months (range: 16 years, 2 months to 17 years, 11 months), and Adult participants (33 women) had a median age of 18 years and 6 months (range: 17 years, 6 months to 19 years, 8 months). Most of the adolescent participants (*n* = 107) were recruited in the second half of the school year from 9 schools within the same mid-sized Canadian city.

It was possible that some of participants may not be representative of the general population but this was not supported. To be able to test 40 participants in each grade, some Grade 7 students (*n* = 13) were recruited after the completion of the academic year from a summer science camp. Due to the academic focus of the science camp, participants recruited from the science camp could have been more likely to use conceptually-based strategies than participants recruited from the schools. However, preliminary analyses comparing frequency of conceptually-based strategy use between Grade 7 students recruited from the school and Grade 7 students recruited from the science camp indicated that this was not the case, $p_s > .05$. Also, adult participants attending university could be more likely to use conceptually-based strategies than adult participants not attending university. However, 79% of Canadian adults aged 18 to 20 enroll in university (Statistics Canada, 2012). Thus, they are representative of the population under investigation (i.e., adults who have graduated from high school the previous academic year).

As a token of appreciation, adolescent participants were entered in a draw to win a gift certificate. Participants who withdrew from the study, for any reason, were also entered into the draw. Parental consent for participation in the study and entrance into the draw was obtained for participants under the age of 16. Adult participants were recruited in the first half of the academic year from the University of Regina's Psychology participant pool and received class credit for participation. Every participant who volunteered for the study was given the opportunity to participate.

Measures

Mathematical problem solving task. During the session, participants were presented with 40 three-term multiplication and division problems (20 inversion problems, $d \times e \div e$, and 20 associativity problems, $d \times e \div f$; half of each type were small problems, $d \times e < 25$ and half were large problems, $d \times e > 25$). Problems were individually presented on a 15" LCD screen using the psychological software E-prime (see Appendix A). For the associativity problems, the answer for the division operation was a whole number (e.g., $3 \times 8 \div 4$, $8 \div 4 = 2$), so as not to discourage the use of the associativity strategy by having the division operation yield a fraction (e.g., $3 \times 4 \div 8$, $4 \div 8 = 1/2$). No more than two problems of each type or size were presented consecutively.

The purpose of the mathematical problem solving was to determine the frequency of inversion shortcut and associativity strategy use, the relationship between inversion shortcut and associativity strategy use, and to investigate individual variability in inversion shortcut and associativity strategy use.

Interruption of procedures task. During the session, a task similar to one used by LeFevre, Bisanz, and Mrkonjic (1988) was administered. A single item from the interruption of procedures task was presented with every mathematical problem. After stating the answer to each mathematical problem, a number probe was presented in the center of the screen and participants stated whether the number was present in the just solved mathematical problem (e.g., 3 × 7 ÷ 7, was 16 present?). There were three number probe types. Half of the number probes were present in the recently solved problem (i.e., operands, $n = 20$; e.g., 3 × 7 ÷ 7, was 7 present?) and half were not present. For trials in which the number probe was not present, half of the number probes were the answer to the multiplication operation (i.e., product, $n = 10$; e.g., 3 × 7 ÷ 7, was 21 present?) and half were not the answer of the multiplication operation (i.e., unrelated, $n = 10$; e.g., 3 × 7 ÷ 7, was 15 present?). Further, the probe was never the sum or the difference of any of the operands in the problem (e.g., 3 × 7 ÷ 7, was 10, 4, or 14 present?), because presenting numbers simultaneously activates associated responses (LeFevre et al., 1988).

The purpose of the task was to determine whether conceptually-based strategy use requires the interruption of left-to-right computational strategies. In a similar task, LeFevre et al. (1988) found that participants were less accurate and slower to reject the sum or product of an operation (e.g., 5 + 1, was 6 present?) than another unrelated number (e.g., 5 + 1, was 9 present?). The sum of the operation is activated in memory and interferes with stating that it was not present in the previously solved problem. In contrast, an unrelated number is not activated in memory and, thus, does not interfere (LeFevre et al., 1988; Orrantia, Rodriguez, & Vicente, 2010). Similarly, participants should be less accurate and slower to reject the product of the multiplication operation from inversion and associativity problems (e.g., 3 x 7 ÷ 7, was 21 present?) than an unrelated number (e.g., 3 × 7 ÷ 7, was 11 present?). However, if conceptually-based strategy use requires the interruption of left-to-right computational strategies then the product of the multiplication operation may not be activated or may be interrupted. Therefore, evidence for the interruption of the left-to-right computational strategy would be found if the accuracy and speed of rejecting the product of the multiplication operation (3 × 7 ÷ 7, was 21 present?) are greater for trials solved with conceptually-based strategies than for trials solved using the left-to-right computational strategy.

Procedure

Participants were tested individually in a videotaped session lasting approximately 20 minutes. At the beginning of each session, to familiarize them with the response procedure, participants solved four two-term practice problems (i.e., $d \times e$) that were each immediately followed by a number probe (2 operands, 2 unrelated). Following the practice problems, experimenters instructed participants to, “Try your best to be as accurate and fast as possible but this is not a test.” This was the only instruction given regarding accuracy and speed.

For each trial, the mathematical problem and number probe for the interruption of procedures task were presented to participants in the following sequence (see Appendix B). First, a trial began with participants focusing their gaze on a 1 × 1 mm fixation dot, which was exposed for 500 ms. Second, the mathematical problem was presented and remained on the screen until it was solved by participants. Third, once participants solved the mathematical problem and a 500 ms response stimulus interval had elapsed, the number probe for the interruption of procedures task was presented in the middle of the screen and remained on the screen until participants responded. Fourth, the recently solved mathematical problem was presented again and participants made a retrospective verbal report of strategy use.

For each trial, participants produced three responses. They stated the answer for the mathematical problem, stated whether the number probe from the interruption of procedures task was present in the previously solved problem, and verbally reported the strategy used to solve the mathematical problem. Verbal responses and key presses were used to collect solution latencies and accuracy data because voice-activated instruments were found to not reliably trigger due to the noise in the school environment. The data collection procedure for each response was as follows:

Response to mathematical problem. Participants responded to the mathematical problem by verbally stating the answer and simultaneously pressing the space bar. Accuracy and solution latency data (as measured from the time the mathematical problem appeared on the screen to the time when participants pressed the space bar) for solving the mathematical problem were collected. In instances when participants were unable to answer the mathematical problem, the researcher instructed participants to advance the computer onto the next step by pressing the ‘space bar’ while coding the response as a cut-off. If a cut-off occurred, the remainder of the trial sequence progressed unaltered.

Response to interruption of procedures task. Participants stated whether the number probe was present in the recently solved problem by verbally responding either ‘yes’ or ‘no’ and simultaneously pressing either ‘y’ or ‘n’ on the keyboard. Accuracy and response latency data (as measured from the time the number appeared on the screen to the time when participants pressed a response key) for the interruption of procedures task were collected.

Verbal report of strategy use. Participants verbally reported the strategy they used to solve the mathematical problem. For the verbal report protocol, the researcher asked, “How did you solve that problem?” In instances when participants were unable to answer the mathematical problem the researcher asked, “How did you try to solve that problem?” Verbal reports were written down by the researcher and coded into the computer (e.g., inversion shortcut coded as 1) before progressing to the next trial.

Results and Discussion

All results were collapsed across sex, classroom, and school as these factors yielded no significant effects. Significant interactions were interpreted using Tukey's honestly significant difference (HSD) test, unless otherwise stated.

Mathematical Problem Solving Task

Frequency of strategy use on inversion and associativity problems across development. During data collection, strategies were placed into five main categories and the interrater reliability was 96.5%. Two raters coded 20 participants' verbal reports (12.5% of all of participants) and compared agreements and disagreements of strategy coding to arrive at the interrater reliability score. After the interrater reliability was established, the data were screened for incorrectly coded strategies and incorrectly timed solution latencies using the original written verbal report transcriptions while viewing the videotaped sessions. Based on previous research (see Robinson et al., 2006; Robinson & Dubé, 2009c), the five strategy categories were the inversion shortcut, left-to-right, associativity, negation, and other. A strategy was coded as the inversion shortcut when participants identified the inverse relationship in an inversion problem and stated that they did not perform any calculations. A typical inversion shortcut strategy verbal report followed the pattern, "I saw the × 6 ÷ 6 and I just knew the answer was the first number." A strategy was coded as left-to-right when participants stated that they performed the computation for the multiplication operation and then performed the computation for the division operation. A strategy was coded as associativity when participants stated that they performed the computation for the division operation and then performed the computation for the multiplication operation. The associativity strategy cannot be used on inversion type problems because doing so results in the inversion shortcut. In previous research, some researchers have coded the associativity strategy as the right-to-left strategy (e.g., Robinson & Ninowski, 2003). In the present study and more recent research (e.g., Dubé & Robinson, 2010a), the term associativity strategy was chosen because it identifies the mathematical concept used to solve the problem. A strategy was coded as negation when participants stated that they performed the computation for the multiplication operation but then noticed that the division operation was the inverse and as a result they knew the answer was the first number. By definition, the negation strategy can only be used on inversion type problems. A typical negation strategy verbal report followed the pattern, "I did the multiplication part but then I saw that the dividing part was the opposite so I knew the answer was the first number." A strategy was coded in the "other" category if participants solved a problem by guessing or using an atypical strategy (e.g., "The answer is five because five is my favorite number"). The types of verbal reports provided by participants in the present study did not differ from the types of verbal reports provided by participants in previous studies.

The frequency of inversion shortcut and negation strategy use on small and large inversion problems was analyzed using a 2 (size: small, large) × 2 (strategy: inversion shortcut, negation) × 4 (developmental period: grade 7, 9, 11, and adult) ANOVA to determine the effects of problem size and developmental period on the frequency of participants' strategy use (see Table 1). Inversion shortcut and negation strategy use were analyzed because both are indicators that individuals have some understanding of the inversion concept (Siegler & Stern, 1998). The left-to-right and "other" strategy were not included in the analysis because the interdependence of all the strategies (i.e., an increase in one strategy necessitates a decrease in another strategy) increases the chance of making a type 1 error if all the strategies are analyzed together. The inversion shortcut was used more frequently (M = 49.2%) than negation (M = 18.2%), $F(1, 156) = 53.47$, $MSE = 2875.43$, $p < .001$, $\eta_p^2 = .26$. The inversion shortcut and negation strategy were used just as frequently on small problems as on large problems (respectively: small = 46.8%, large = 51.6%, small = 16.3%, large = 20.1%), $F(1, 156) = .51$, $MSE = 78.57$, $p = .48$. A significant Strategy × Developmental Period interaction enabled a finer-grained interpretation of the data, $F(3, 156) = 11.93$, $MSE = 2875.43$, $p < .001$, $\eta_p^2 = .19$, HSD = 18.4. Grade 7 participants' frequency of inversion shortcut use did not significantly differ from their frequency of negation strategy use. Furthermore, Grade 7 participants used the inversion shortcut less frequently and the negation strategy more frequently than participants from all other developmental periods, which did not differ from each other.

Table 1. Strategy Use (%) on Small and Large Inversion Problems by Developmental Period

	Developmental Period			
	Grade 7	Grade 9	Grade 11	Adult
Strategy	*M* (*SE*)	*M* (*SE*)	*M* (*SE*)	*M* (*SE*)
Inversion Shortcut	25.7 (6.0)	50.5 (6.0)	53.6 (6.0)	67.1 (6.0)
Small	24.0 (6.0)	47.8 (6.0)	50.8 (6.0)	64.5 (6.0)
Large	26.8 (6.1)	53.3 (6.1)	56.5 (6.1)	69.8 (6.1)
Negation	35.5 (3.6)	15.1 (3.6)	13.0 (3.6)	8.8 (3.6)
Small	32.5 (3.6)	12.8 (3.6)	13.0 (3.6)	6.8 (3.6)
Large	39.0 (4.1)	17.5 (4.1)	13.0 (4.1)	10.8 (4.1)
Left-to-Right	38.4 (5.2)	34.3 (5.2)	33.1 (5.2)	24.0 (5.2)
Small	43.0 (5.4)	39.3 (5.4)	36.3 (5.4)	28.8 (5.4)
Large	33.8 (5.3)	29.3 (5.3)	30.0 (5.3)	19.3 (5.3)

The pattern of strategy development on inversion problems indicates that the inversion shortcut and negation strategy were applied equally often to both small and large problems and that inversion shortcut use increased from grade 7 to Grade 9. The equal application of the inversion shortcut and negation strategy to both problem sizes suggests participants understand that the inversion principle underlying both of these strategies applies to all problems, regardless of problem size. Inversion shortcut use increased markedly between Grade 7 and Grade 9, after which increases in inversion shortcut use were not significant. This indicates that there is either an increase in adolescents' understanding of the inverse relation between multiplication and division between Grade 7 and 9 or that another factor (e.g., an increase in working memory capacity) enabled participants in Grade 9 to apply their understanding of the inversion principle to generate the inversion shortcut. The marked increase in inversion shortcut use between Grade 7 and Grade 9 could occur in Grade 8. However, previous research has not found grade differences in inversion shortcut use between Grade 7 and Grade 8 participants (Robinson & Dubé, 2009b). Increases in inversion shortcut use primarily came at the cost of decreases in negation strategy use. In Grade 9, inversion shortcut use increased by 24.8% over the previous grade whereas negation strategy use decreased by 20.4%. This supports Siegler and Stern's (1998) hypothesis that negation strategy use precedes the development of inversion shortcut use. The switch from negation strategy use to inversion shortcut use between Grade 7 and Grade 9 could imply that participants in both grades understand the underlying inversion principle, but participants in Grade 9 were better able to apply the inversion concept at the onset of problem solving. In contrast, participants in Grade 7 were more likely to apply the inversion concept after completing the multiplication operation (i.e., to use negation). Alternatively, the marked increase in inversion shortcut use and decrease in negation strategy use between Grade 7 and Grade 9 could indicate that conceptual understanding of the inverse relationship between multiplication and division largely occurs between Grades 7 and 9. Finally, adult participants did not use the inversion shortcut on all inversion problems. This could indicate that individuals' understanding of the inverse relationship between multiplication and division is not fully developed in early adulthood. Alternatively, adults' continued use of computational strategies may serve to practice their procedural and factual mathematical problem solving skills. This is in line with the overlapping waves perspective of strategy choice (Siegler, 1996), which proposes that individuals practice backup strategies in case direct retrieval of an answer fails.

The frequency of associativity strategy use on small and large associativity problems was analyzed using a 2 (size: small, large) × 4 (developmental period: grade 7, 9, 11, and adult) ANOVA to determine the effects of problem size and developmental period on the frequency of participants' strategy use (see Table 2). The associativity strategy was used more frequently on large problems (M = 36.5%) than on small problems (M = 32.1%,), $F(1, 156) = 15.03$, MSE = 101.89, $p < .001$, $\eta_p^2 = .09$. Associativity strategy use gradually increased across development with significant differences between Grade 7 and Grade 11 participants as well as between Grade 9 and Adult participants, $F(3, 156) = 8.06$, MSE = 2765.42, $p < .001$, $\eta_p^2 = .13$, HSD = 21.7. There were no significant interactions between problem size and developmental period.

Table 2. Strategy Use (%) on Small and Large Associativity Problems by Developmental Period

	Developmental Period			
	Grade 7	Grade 9	Grade 11	Adult
Strategy	*M* (*SE*)	*M* (*SE*)	*M* (*SE*)	*M* (*SE*)
Associativity	14.1 (5.8)	29.9 (5.8)	39.4 (5.8)	53.9 (5.8)
Small	12.8 (6.7)	26.5 (6.0)	36.3 (6.0)	53.0 (6.0)
Large	15.5 (6.6)	33.3 (6.0)	42.5 (6.0)	54.8 (6.0)
Left-to-Right	85.8 (5.9)	70.0 (5.9)	60.0 (5.9)	45.8 (5.9)
Small	87.3 (6.1)	73.3 (6.1)	63.5 (6.1)	46.8 (6.1)
Large	84.3 (5.9)	66.8 (5.9)	56.5 (5.9)	44.8 (5.9)

The pattern of strategy development on associativity problems indicates that participants were more likely to use the associativity strategy on large problems than on small problems and that associativity strategy use gradually increased across adolescence and into adulthood. The more frequent application of the associativity strategy to large problems could have occurred for multiple reasons. One, participants' understanding of the associativity concept was not fully developed and as a result not equally applied to small multiplication and division problems. Associativity strategy use may develop first on large problems because these problems are more difficult to solve using left-to-right computational strategies (Robinson et al., 2006). Thus, using the associativity strategy on large problems would provide a greater benefit over left-to-right computational strategies than using it on small problems. Two, participants' understanding of associativity is not as readily activated on small problems. Three, participants' proficiency with solving small associativity problems using the left-to-right strategy is sufficient for them not to select the associativity strategy. The pattern of associativity strategy use across development indicates that participants' understanding of the associativity concept on multiplication and division problems gradually increased and did not markedly increase between any two sequential developmental periods (cf. inversion shortcut use). This gradual increase could suggest that the strength of the associativity concept increases as a function of experience with mathematics. Adolescents' experience with multiplication and division increases every year and this experience gradually increases their understanding of the associative relationship between the two operations. Alternatively, gradual maturation of other cognitive skills and resources (e.g., executive functioning) across adolescence may gradually increase their ability to apply their knowledge of the associativity concept during problem solving or gradually increase their understanding of the associativity concept.

Accuracy and solution latencies on inversion and associativity problems across development. Accuracy and solution latency data can serve as a check on the validity of the verbal reports of strategy use (Bisanz & LeFevre, 1991; Robinson, 2001). Four 2 (size: small, large) × 4 (developmental period: grade 7, 9, 11, and adult) ANOVAs were performed on the percentage of correct responses and the median solution latencies of correct responses for inversion and associativity problems (see Table 3).

Table 3. Percentage of Accurate Responses and Solution Latencies (ms) of Correct Responses on Inversion and Associativity Problems by Developmental Period.

	Developmental Period			
	Grade 7	Grade 9	Grade 11	Adult
	M (*SE*)	*M* (*SE*)	*M* (*SE*)	*M* (*SE*)
Accuracy				
Inversion Problems	89.8 (2.3)	94.9 (1.2)	94.9 (1.6)	97.5 (3.4)
Small	93.5 (1.9)	95.3 (1.2)	96.0 (1.4)	97.0 (0.8)
Large	86.3 (3.0)	94.5 (1.5)	93.8 (2.1)	98.0 (0.6)
Associativity Problems	76.3 (2.9)	84.0 (2.6)	88.8 (2.1)	91.3 (1.5)
Small	92.3 (2.3)	95.3 (1.9)	96.8 (0.8)	97.5 (1.0)
Large	60.3 (4.2)	72.8 (4.0)	80.8 (4.0)	85.0 (2.4)
Solution Latencies				
Inversion Problems	6028 (613)	3251 (333)	3142 (290)	2414 (273)
Small	5647 (601)	3068 (279)	2905 (236)	2238 (236)
Large	6929 (696)	3622 (403)	3534 (423)	2663 (326)
Associativity problems	8324 (583)	5030 (362)	4653 (445)	3769 (344)
Small	7182 (593)	4221 (300)	3774 (283)	3257 (314)
Large	13647 (1665)	7209 (699)	7370 (853)	5934 (842)

For accuracy, there were main effects of problem size and developmental period on inversion and associativity problems, $F(1, 156) = 8.07$, $MSE = 52.99$, $p = .005$, $\eta_p^2 = .05$, $F(1, 156) = 156.43$, $MSE = 220.19$, $p < .001$, $\eta_p^2 = .50$, respectively. However, significant Size × Developmental Period interactions enabled a finer-grained interpretation of the data. Grade 7 participants were more accurate on small inversion problems than on large inversion problems but Grade 9, Grade 11, and Adult participants were just as accurate on small inversion problems as on large inversion problems, $F(3, 156) = 4.76$, $MSE = 52.99$, $p = .003$, $\eta_p^2 = .08$, HSD = 5.0. Also, Adult, Grade 11, and Grade 9 participants were more accurate on large inversion problems than Grade 7 participants. There were no significant differences in accuracy on small inversion problems among the developmental periods. Adult participants were more accurate on large associativity problems than Grade 9 participants, who were more accurate on large associativity problems than Grade 7 participants, $F(3, 156) = 6.67$, $MSE = 220.19$, $p < .000$, $\eta_p^2 = .11$, HSD = 10.2. There were no significant differences in accuracy on small associativity problems among the developmental periods.

For solution latencies, there were main effects of problem size and developmental period on inversion and associativity problems. Participants were faster on small inversion problems ($M = 3465$) than on large inversion problems ($M = 4187$ ms), $F(1, 156) = 33.04$, $MSE = 1263762.4$, $p < .000$, $\eta_p^2 = .17$. Similarly, participants were faster on small associativity problems ($M = 4608$ ms) than on large associativity problems ($M =$

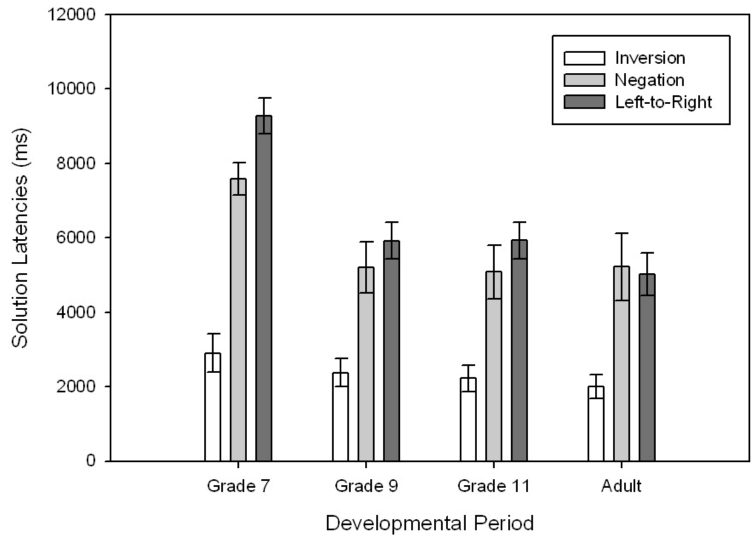
8540 ms), $F(1, 156) = 75.71$, $MSE = 16333455$, $p < .001$, $\eta_p^2 = .33$. However, a significant Size × Developmental Period interaction indicated that Adult participants were just as fast on large associativity problems as on small associativity problems, $F(3, 156) = 3.67$, $MSE = 16333455$, $p = .014$, $\eta_p^2 = .07$, HSD = 2786. Adult, Grade 11, and Grade 9 participants were faster on both inversion and associativity problems than Grade 7 participants, $F(3, 156) = 17.01$, $MSE = 13411712$, $p < .001$, $\eta_p^2 = .25$, HSD = 1483, $F(3, 156) = 14.76$, $MSE = 36875298$, $p < .001$, $\eta_p^2 = .22$, HSD = 1637, respectively.

Support for the verbal report data would be found if differences in the accuracy and solution latency data among the developmental periods correspond to differences in conceptually-based strategy use. There were improvements in accuracy on large inversion and associativity problems and improvements in solution latencies on inversion and associativity problems between participants in Grades 7 and 9 (see Table 3). Correspondingly, there were increases in inversion shortcut and associativity strategy use between participants in Grade 7 and Grade 9 (see Table 2). Evidence for the validity of the verbal reports can also be found by interpreting the presence or absence of problem size effects in the accuracy and solution latency data (i.e., small problems are more easily solved than large problems; Ashcraft, 1992; Brauwer, Verguts, & Fias, 2006; Campbell & Graham, 1985). The use of conceptually-based strategies should result in a flattened or reduced problem size effect (Bisanz & LeFevre, 1990) because conceptually-based strategies are fast and accurate, regardless of problem size. For inversion problems, participants who had lower frequencies of inversion shortcut use (i.e., Grade 7 participants) were more accurate on small inversion problems than on large inversion problems but participants who had higher frequencies of inversion shortcut use (i.e., Grade 9, 11, and Adult participants) were just as accurate on small as on large inversion problems. For associativity problems, participants who had lower frequencies of associativity strategy use (i.e., Grade 7, 9, and 11 participants) were faster on small associativity problems than on large associativity problems but participants who had higher frequencies of associativity strategy use (i.e., Adult participants) were just as fast on small as on large associativity problems. Therefore, the accuracy and solution latency data support the validity of the verbal reports.

Strategy execution on inversion and associativity problems across development. Information on strategy execution can provide a more detailed interpretation of mathematical development and serves as another check of the validity of the verbal reports of strategy use. A strategy's associated solution latency on correctly-solved problems yields information on procedural, factual, and conceptual knowledge because procedural errors, a weak factual network, and/or weak conceptual knowledge result in longer solution latencies (Bajic & Rickard, 2009). For example, a participant who takes longer to execute a conceptually-based strategy may have weaker conceptual knowledge than a participant who quickly executes the strategy. Moreover, confidence criteria may also affect how quickly participants select and execute strategies. Confidence criteria refers to a threshold that must be exceeded for a strategy to be executed (Siegler, 1988a). Confidence criteria can be strategy-specific or global. For example, individuals with a low confidence criterion for a specific strategy are more likely to execute the strategy than individuals with a high confidence criterion for that strategy (Siegler, 1988a). Also, some individuals have lower confidence criteria for all strategies than other individuals. Low confidence criteria result in executing strategies faster than high confidence criteria, because executing a strategy with high confidence criteria often involves performing checks to ensure the strategy produced the correct answer (Siegler, 1988b). Therefore, differences in solution latencies between strategies or among developmental periods could indicate differences in confidence criteria.

As in Robinson and Dubé (2008), all instances of each of the strategies used on inversion (Figure 1, top) and associativity (Figure 1, bottom) problems were collapsed across problem type and problem size to provide the means and 95% confidence intervals of solution latencies on correctly solved problems for each grade (see Figure 1). The data were collapsed across problem size because the overall developmental pattern was unaffected by this variable. The information in the figures can be interpreted using the 95% confidence intervals. The analysis of graphical data using confidence intervals is well suited for interpreting patterns in complex data sets (Jarmasz & Hollands, 2009), because the size of the confidence interval yields information on differences between strategies and the variability within each strategy's associated solution latency (Masson &

Loftus, 2003). For differences between strategies, strategies with confidence intervals that overlap by no more than half the length of one side of an interval can be interpreted as significantly different (Masson & Loftus, 2003). For variability within a strategy, the smaller the confidence interval the smaller the variability in the time taken to execute the strategy (Masson & Loftus, 2003).



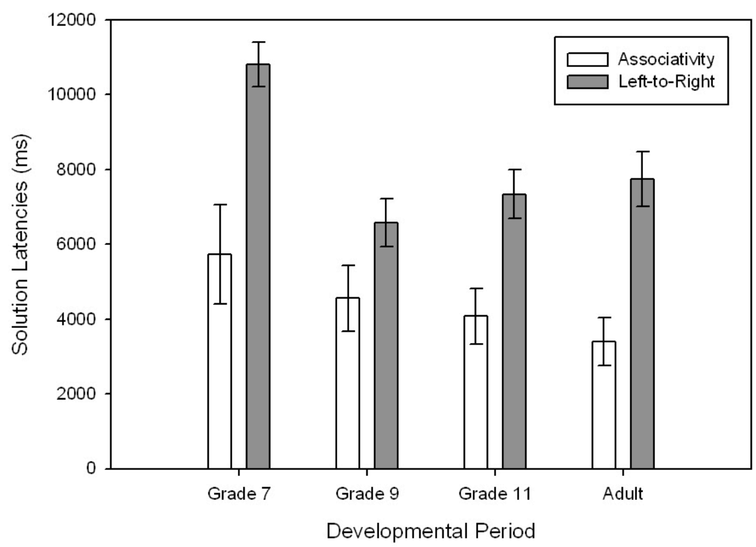


Figure 1. Associated solution latencies (ms) of correct responses for strategies used on inversion (top) and associativity problems (bottom). Error bars are 95% confidence intervals.

The strategy execution data support the validity of the verbal reports. On inversion problems, the execution of the left-to-right strategy should be the slowest because it requires two computations (i.e., multiplication and division), the negation strategy should be faster because it requires one computation (i.e., multiplication), and the inversion shortcut should be the fastest because it only requires the recognition that the second and third term of the problem are identical. This expected pattern is reflected in Figure 1. However, Adult participants executed the negation strategy with equal speed as the left-to-right computational strategy. This could indicate that the speed of completing the division operation is as fast in the left-to-right strategy as applying the inversion principle in the negation strategy. Given that the negation strategy requires an individual to stop performing the left-to-right strategy and switch to the inversion shortcut, this strategy switch might incur a cost and increase the solution latency of the negation strategy. Indeed, switch costs are robust and occur on a variety tasks that require individuals to change from a prepared response procedure to a new response procedure (Campbell & Arbuthnott, 2010; Lee, Ng, & Ng, 2009; Spencer & Wiley, 2008). Alternatively, adult participants may have unconsciously used the negation strategy but reported it as the left-to-right strategy (see Siegler & Stern, 1998). On associativity problems, the execution of the left-to-right strategy should be the slowest because completing the multiplication operation first results in a double digit number which must then be reduced. In contrast, the associativity strategy should be faster because completing the division operation first results in a single digit number which is multiplied by another single digit number. This expected pattern is again reflected in Figure 1.

The strategy execution data suggest that there are marked improvements in factual and procedural knowledge of mathematics between participants in Grade 7 and 9. There was marked improvement in the execution of the left-to-right strategy on inversion and associativity problems between participants in Grade 7 and 9 (see Figure 1). This improvement suggests that Grade 9 participants were either faster at retrieving answers than Grade 7 participants or that Grade 9 participants chose more efficient procedures than Grade 7 participants. For example, the problem $3 \times 7 \div 7$ can be solved with the left-to-right strategy by using retrieval (e.g., retrieving $3 \times 7 = 21$, retrieving $21 \div 7 = 7$) or by using several different procedures, such as derived facts (e.g., breaking 3×7 into smaller problems: $2 \times 7 = 14$, $14 + 7 = 21$, $21 \div 7 = 7$), special tricks (e.g., 3×7: three touchdowns equal 21 points), or counting. Quickly solving this problem can be achieved either by using retrieval or by selecting the most efficient procedure. Therefore, the improvement in left-to-right strategy execution between Grade 7 and 9 suggests that Grade 9 participants were using retrieval more frequently than Grade 7 participants or that Grade 9 participants were selecting more efficient procedures than Grade 7 participants. Thus, Grade 9 participants were demonstrating greater factual and procedural knowledge of mathematics than Grade 7 participants.

The strategy execution data also suggest that conceptual knowledge of inversion and associativity increased between Grade 7 and Grade 9. Having a strong understanding of a mathematical concept results in the ability to apply that concept to problems regardless of problem characteristics (Baroody & Lai, 2008). Thus, the decreased variability in inversion shortcut and associativity strategy execution between Grade 7 and Grade 9 participants suggests that Grade 9 participants more evenly applied the conceptually-based strategies to problems regardless of problem characteristics than Grade 7 participants. Also, the improved execution of the negation strategy between Grade 7 and Grade 9 participants could be an indicator of increased understanding of the inversion concept. Grade 9 participants may have had a better conceptual understanding of the inversion concept than Grade 7 participants and as a result were faster in applying the inversion concept once the multiplication operation was complete. Alternatively, the improved execution of the negation strategy could be a result of Grade 9 participants' improved ability to quickly complete the multiplication operation.

The gradual improvement in inversion shortcut and associativity strategy execution could indicate a decrease in the confidence criteria for these strategies across development. Individuals' self-evaluation of their ability to execute the inversion shortcut and associativity strategy effectively may be increasing across development, which results in faster strategy execution. Increased confidence in the effectiveness of these conceptually-based strategies across adolescence and early adulthood could suggest that the concepts of inversion and associativity are being integrated into participants' overall understanding of mathematics. When the inversion and associativity concepts are applied to problem solving they are transformed into procedures. Conceptual knowledge that is strongly integrated is more easily and quickly proceduralized (Anderson, 1983; Schneider & Stern, 2010). An individual who possesses an understanding of a particular mathematical concept must continually integrate it into an overall understanding of mathematics to generate procedures (Ma, 1999). For example, an individual who understands the inverse relationship between multiplication and division can integrate this concept with the understanding of fractions to generate the procedure for division by fractions. Thus, the improved execution of the inversion shortcut and associativity strategy could reflect how the inversion and associativity concepts are integrated across adolescence and early adulthood and, as a result, are easier to proceduralize.

Relationships among conceptually-based strategies. Previous studies of inversion and associativity have found a relationship between the frequency of inversion shortcut and associativity strategy use (e.g., Dubé & Robinson, 2010a; Robinson & Dubé, 2009a). To determine whether participants who made use of one arithmetic concept were more likely to also use the other arithmetic concept, correlational analyses between inversion shortcut and associativity strategy use as well as between negation strategy and associativity strategy use were performed. Correlations were not performed between inversion shortcut and negation strategy use because both strategies are based on the inversion concept and the frequencies of the strategies are interdependent. Overall, the correlation between inversion shortcut and associativity strategy use was $r = .77$, $p < .001$ and between negation strategy and associativity strategy use was $r = -.49$, $p < .001$; the correlations for the respective developmental periods are presented in Table 4. There were significant moderate to strong positive correlations between inversion shortcut and associativity strategy use and significant small to moderate negative correlations between negation strategy and associativity strategy use across development. Dunn and Clark's (1969) z procedure was used to determine whether differences in the correlations among developmental periods were significant. Dunn and Clark's z procedure was used because it controls for inflated Type 1 error rates while maintaing sufficient statistical power (Hittner, May, & Silver, 2003). For the procedure, each correlation value was transformed into a z-score, the z values for the differences among the developmental periods were calculated, and the z values for the differences were compared to the critical value of z = 1.96 at $p = .05$. If the z value for the difference exceeds the critical value then the difference between two correlations is significant (Dunn & Clark, 1969). For the correlation between inversion shortcut and associativity use, the differences among the developmental periods were not significant, all $z_s \leq 1.78$. For the correlation between negation strategy and associativity strategy use, the differences among the developmental periods were not significant, all $z_s \leq .69$.

Table 4. Correlations Between Strategy Use by Developmental Period

	Developmental Period			
	Grade 7	Grade 9	Grade 11	Adult
Inversion Shortcut and Associativity Strategy Use	.61	.66	.81	.81
Negation Strategy and Associativity Strategy Use	-.44	-.37	-.42	-.50

All correlations are significant at $p < .05$

The relationships among the conceptually-based strategies did not change across adolescence and into early adulthood. This suggests that the factors underlying these relationships remain constant across this developmental period. To further investigate the relationship between the conceptually-based strategies, descriptive statistics were used to determine whether individuals who used the associativity strategy used the inversion shortcut at least once. In previous research, children and adults who used the associativity strategy also used the inversion shortcut at least once (Dubé & Robinson, 2010a; Robinson & Dubé, 2009a). In this study, of the 93 participants who used the associativity strategy at least once, four participants (4.0%) did not use the inversion shortcut at all. However, each of these four participants used negation at least once. Three of these four participants were in Grade 7 and one was in Grade 9. The four participants' frequency of associativity strategy use was 40%, 5%, 5%, and 5% whereas their frequency of negation strategy use was 10%, 50%, 50%, and 5%, respectively. In contrast, of the 119 participants who used the inversion shortcut at least once, 89 participants (74.8%) used the associativity strategy at least once. Of these 30 participants who did not use the associativity strategy, there were 11 in Grade 7, 11 in Grade 9, 6 in Grade 11, and 2 Adults.

The correlations between the use of the two conceptually-based strategies and the tendency for 96% of associativity strategy users to also be inversion shortcut users could suggest that developing the concept of associativity and applying it to problem solving requires some understanding of the inversion concept. In previous research, researchers have proposed that an understanding of the inversion concept enables an understanding of several mathematical concepts (Schnieder & Stern, 2010). In the present study, all participants who used the associativity strategy used either the inversion shortcut or the negation strategy to solve at least one problem. In contrast, 25% of the participants who used the inversion shortcut did not use the associativity strategy to solve even one problem. This could suggest that the procedural and conceptual knowledge required for the inversion shortcut is also required for associativity strategy use. For procedural knowledge, the inversion shortcut involves a logical deduction that the answer is the first number (i.e., insight problem solving; Siegler & Stern, 1998). Similarly, associativity strategy use involves a logical deduction that the answer is the same regardless of which operation is performed first. For conceptual knowledge, understanding the inverse relation between operations may be required to apply the associativity concept to three-term problems. The associativity concept can be used to solve three-term problems containing either a single operation (e.g., 3 + 27 + 25) or inversely related operations (e.g., 2 + 27 – 25) but the associativity concept cannot be used to solve three-term problems containing operations that must be performed in a particular order (e.g., 2 × 27 – 3). Therefore, associativity strategy use seems to require procedural and conceptual knowledge that is similar to inversion shortcut use.

Individual variability in strategy use. In previous research (Dubé & Robinson, 2010a, 2010b; Gilmore & Papadatou-Pastou, 2009; Robinson & Dubé, 2009a, 2009b), patterns of individual variability have been identified in children's and adults' inversion shortcut and associativity strategy use with some individuals using the conceptually-based strategies with markedly different frequencies than their peers. To investigate individual variability in strategy use, cluster analyses were performed on participants' frequency of inversion shortcut, left-to-right, and negation use on inversion problems as well as participants' frequency of associativity strategy and left-to-right use on associativity problems. Hierarchical clustering algorithms based on Euclidian distances were used to reduce the effect of outliers on cluster membership and maximize similarities among members in a cluster while minimizing similarities among members between clusters (DiStefano & Kamphaus, 2006). Subsequently, discriminant analyses were performed to aid in determining which cluster solution was the most parsimonious and best represented the data. A cluster solution of four relatively homogeneous groups of cases (see Table 5) was chosen in which the discriminant analysis successfully placed 98.1% of participants back into the four clusters (vs. 99.7%, 98.1%, and 98.1% for the two, three, and five cluster solutions, respectively). The clusters were labeled dual concept (DC, *n* = 63), inversion concept (IC, *n* = 29), negation concept (NGC, *n* = 35) and no concept (NOC, *n* = 33) based on the pattern of associated results for each cluster. The other cluster solutions were not chosen because they were not theoretically supported. The two cluster solution grouped inversion, negation, and no concept users together, which is grouping individuals who demonstrated an understanding of the inversion concept with individuals who did not demonstrate an understanding of the inversion concept. Similarly, the three cluster solution grouped the negation and no concept users. The five cluster solution identified a small subset (*n* = 14) of dual concept users who had slightly lower frequencies of inversion shortcut and associativity use but still used the conceptually-based strategies on the majority of problems.

Table 5. Strategy use (%) on Inversion and Associativity Problems by Cluster Membership

	Cluster			
	DC	IC	NGC	NOC
Strategy	*M* (*SE*)	*M* (*SE*)	*M* (*SE*)	*M* (*SE*)
Inversion problems				
Inversion shortcut	87.5 (1.4)	70.3 (2.1)	3.9 (1.9)	5.3 (2.0)
Negation	3.1 (1.4)	13.1 (2.1)	60.4 (1.9)	6.5 (2.0)
Left-to-Right	9.1 (1.7)	16.4 (2.6)	35.4 (2.3)	87.9 (2.4)
Associativity problems				
Associativity	80.6 (1.6)	5.1 (6.2)	1.7 (2.2)	6.2 (2.2)
Left-to-Right	19.0 (1.7)	94.7 (2.4)	98.0 (2.2)	93.5 (2.3)

The analyses yielded a pattern of individual variability unique to adolescence and early adulthood. Previous research has identified patterns of dual, inversion, and no concept users on addition and subtraction problems in Grades 2 through 4 (Robinson & Dubé, 2009a) and on multiplication and division problems in adulthood (Dubé & Robinson, 2010a). Also, previous research has identified a pattern of inversion, negation, and no concept users on multiplication and division problems in Grades 6 through 8 (Robinson & Dubé, 2009b). The present study identified a pattern of dual, inversion, negation, and no concept users on multiplication and division problems in adolescence and in early adulthood. For the development of conceptually-based strategy use on multiplication and division problems, this is the earliest occurrence of dual concept users. Also, this is the only occurrence of both dual and negation concept users in the same developmental period. However, it is possible that the pattern of dual, inversion, negation and no concept users exists in other developmental periods but was not investigated in previous studies (i.e., Dubé & Robinson, 2010a; Robinson & Dubé, 2009a, 2009b).

Individual variability in strategy use across development. To obtain a better understanding of how individual variability in conceptually-based strategy use develops, the proportion of participants in each cluster for each developmental period (see Figure 2) was analyzed using a chi-square analysis. To aid in the interpretation of the analysis, adjusted residual scores for each cell within the analysis were calculated to assess which cells significantly contributed to the rejection of the null hypothesis (see Appendix C). Adjusted residual scores are an index of the discrepancy between the observed and expected frequency in each cell and scores above two are understood as significantly contributing to the overall significance of the analysis (MacDonald & Gardner, 2000). The analysis indicated that the proportion of participants in each cluster was significantly different across development, $\chi^2(9, n = 160) = 40.07, p < .001$. Figure 2 illustrates how individual variability in strategy use changes across adolescence and into early adulthood. In Grade 7, the majority of participants belonged to the NGC cluster (DC = 12.5%, IC = 15.0%, NGC = 50.0%, NOC = 22.5%). In Grade 9, the majority of participants were split between the DC and IC clusters (DC = 35.0%, IC = 30.0%, NGC = 12.5%, NOC = 22.5%). In Grade 11, the DC cluster contained the greatest percentage of participants (DC = 45.0%%, IC = 15.0%, NGC = 15.0%, NOC = 25.0%). In Adults, the majority of participants belonged to the DC cluster (DC = 65.0%, IC = 12.5%, NGC = 10.0%, NOC = 12.5%). The proportion of Grade 7 participants in the NGC and DC clusters, Grade 9 participants in the IC cluster, and Adult participants in the DC and NGC clusters significantly contributed to the chi-square analysis, as indicated by their adjusted residual scores (respectively, 5.0, -4.0, 2.3, 3.8, and -2.1).

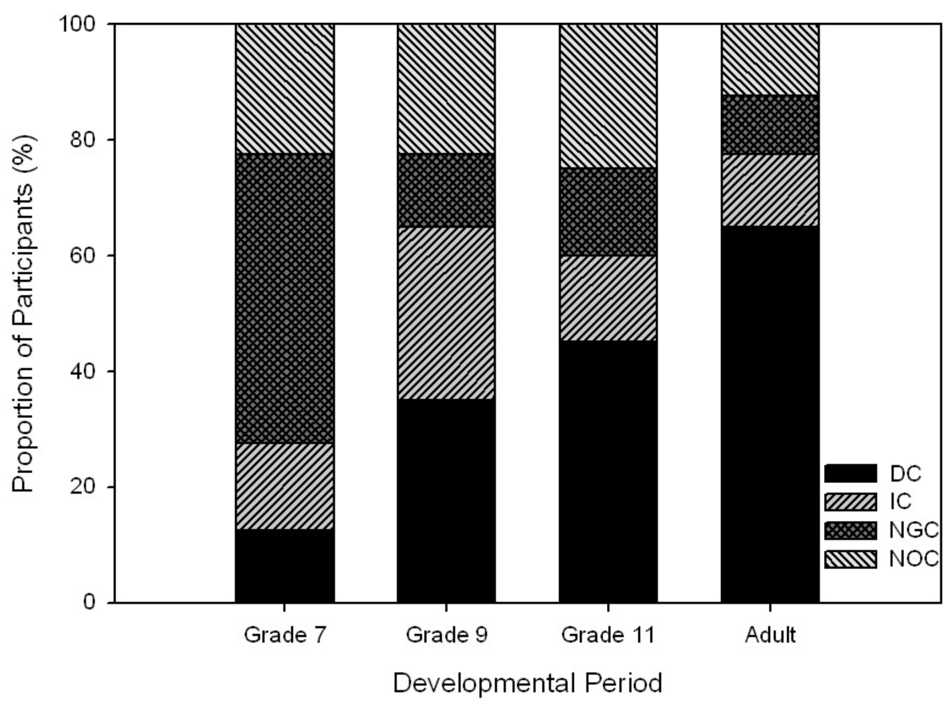


Figure 2. Proportion of participants (%) in the DC, IC, NGC, and NOC clusters by developmental period.

Analyzing cluster membership by developmental period yielded two unique findings pertaining to the development of individual variability in conceptually-based strategy use. One, there were a similar number of Grade 9 participants who use both strategies (DC = 35.0%), only the inversion shortcut (IC = 30.0%), or neither of the conceptually-based strategies (NGC + NOC = 35.0%). This suggests that Grade 9 is an important developmental period for the formation of the inversion and associativity concepts, because there is an almost equal opportunity to fully grasp both inversion and associativity, only grasp inversion, or neither. Two, the decrease in the NGC cluster accounts for most of the increase in the DC and IC clusters from Grade 7 to Grade 9. Compared to Grade 7, the proportion of participants in the NGC cluster decreased by 37.5%, the proportion of participants in the DC and IC clusters increased by 22.5% and 15%, respectively, and in the NOC cluster remained unchanged. This suggests Grade 7 participants who use the negation strategy to solve the majority of problems are more likely to either use the inversion shortcut and associativity strategy or to use only the inversion shortcut in Grade 9. This is further support for the hypothesis that the negation strategy precedes the use of the inversion shortcut. For Grade 11 participants, the decrease in the IC cluster accounted for most of the increase in the DC cluster, because the proportion of participants in the NOC cluster was relatively unchanged. For Adult participants, the decreases in the NOC cluster accounted for most of the increase in the

DC cluster. However, the study was not longitudinal so conclusions regarding cluster membership development are limited to inferences based on patterns in the data.

Interruption of Procedures Task

Accuracy and response latency data by number probe type. The interruption of procedures task was modeled after a task used by LeFevre et al. (1988) in which participants stated whether a number probe was present in a recently solved problem (e.g., 5 + 1, was 3 present?). LeFevre et al. found that participants were less accurate and slower when the number probe was the sum of the recently solved problem (e.g., 5 + 1, was 6 present?) than another unrelated number (5 + 1, was 9 present?). Similarly, participants in this study should be less accurate and slower when the number probe presented in the interruption of the procedures task was the product of the multiplication operation in the recently solved problem (3 × 7 ÷ 7, was 21 present?) than another unrelated number (3 × 7 ÷ 7, was 16 present?). Accuracies and response latencies of correct responses from the interruption of procedures task across the three number probe types (unrelated, product, operand) are presented in Figure 3. Latencies of correct responses were analyzed because they index the extent to which the product of the multiplication operation was active in memory during the interruption of procedures task despite the activation not resulting in an incorrect response (LeFevre et al., 1988). A slower correct response on the interruption of procedures task would indicate that the product of the multiplication operation was more active in memory and caused greater interference with the interruption of procedures task. Furthermore, only trials in which the mathematical problem was correctly solved were included in the analyses of response latencies. It is not known whether the product of the multiplication operation was activated for trials on which the mathematical problem was incorrectly solved. Two repeated measure ANOVAs performed on the percentage of correct responses and response latencies of correct responses with number probe type as the within subject variable suggest that the pattern of data found by LeFevre et al. (1988) was replicated in the present study. Participants were less accurate and slower when the number probe was the product of the multiplication operation (M = 92.1%, M = 1350 ms) than when it was an unrelated number (M = 96.0%, M = 1271ms), $F(2, 318) = 8.52$, $MSE = 160.72$, $p < .001$, $\eta_p^2 = .05$, HSD = 3.4, $F(2, 302) = 6.01$, $MSE = 40127.72$, $p = .003$, $\eta_p^2 = .04$, HSD = 53, respectively.

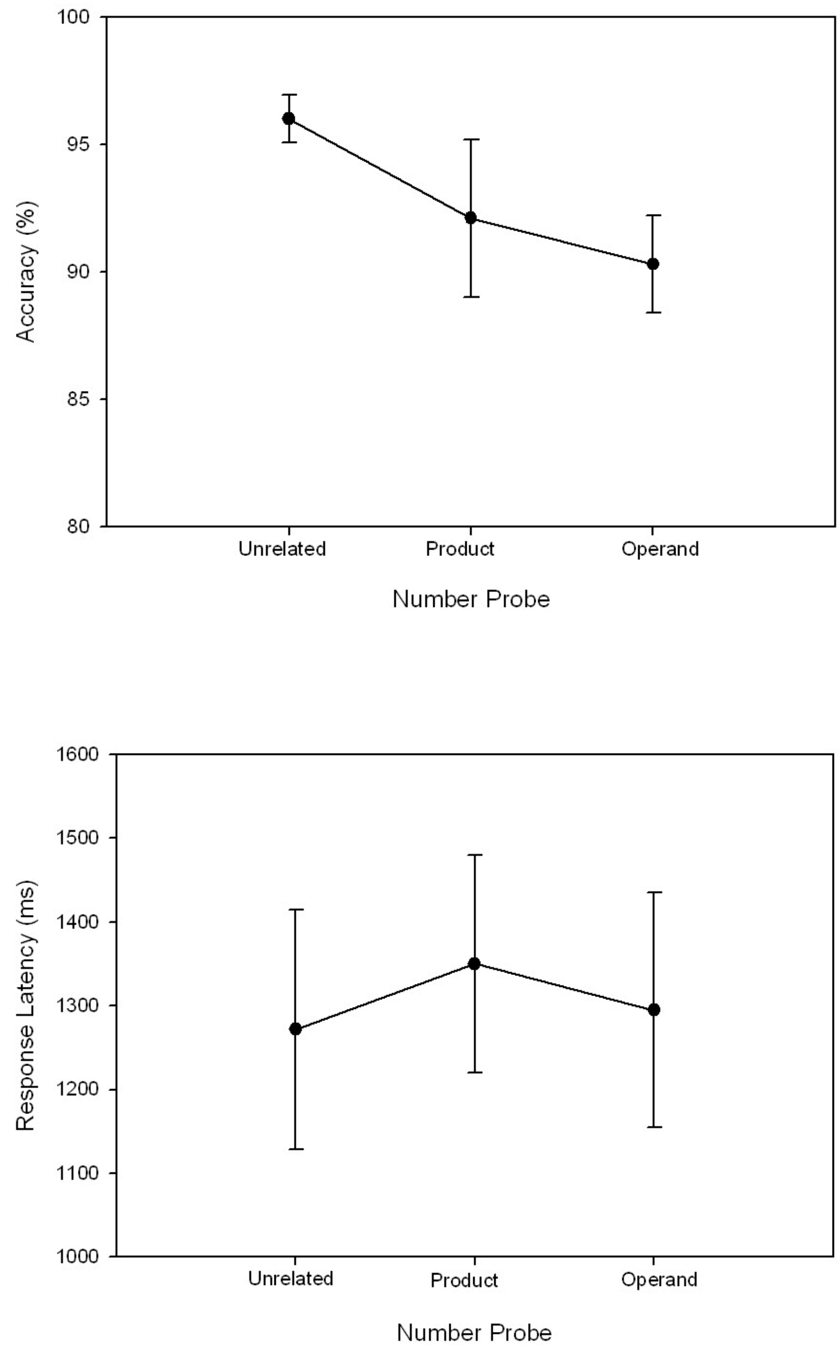


Figure 3. Percentage of accurate responses and response latencies (ms) of correct responses by number probe type. Error bars are 95% confidence intervals.

Replicating the pattern found in LeFevre et al. (1988) has important implications for interpreting the effect that strategy use from the mathematical problem solving task had on performance on the interruption of procedures task. The task used by LeFevre et al. used two-term problems (e.g., 5 + 1) and the use of three-term problems in this study may have affected whether the product of the multiplication operation interfered with the interruption of procedures task. There was considerable variability in both the accuracy and response latency data, as indicated by the large confidence intervals, suggesting that performance on the interruption of procedures task was affected by another factor.

Effect of strategy use from the mathematical problem solving task on the interruption of procedures task. A source of variability in the accuracy and response latencies for the interruption of procedures task could be the strategy used in the recently solved mathematical problem. If the strategy used in the recently solved problem influenced whether the product of the multiplication operation was activated in memory then participants' responses on the interruption of procedures task would be affected. An effect of strategy use on the interruption of procedures task could provide evidence for Siegler and Araya's (2005) hypothesis that conceptually-based strategies interrupt the activation of computational strategies.

Evidence for the interruption of the left-to-right computational strategies during conceptually-based strategy use would be found if the accuracy and speed of rejecting the product of the multiplication operation (3 × 7 ÷ 7, was 21 present?) was different for trials following conceptually-based strategy use than for trials following left-to-right computational strategy use. The percentage of accurate responses and response latency of correct responses on the three number probe types following correct strategy use on inversion and associativity problems are presented in Tables 6 and 7, respectively.

Table 6. Percentage of Accurate Responses to the Interruption of Procedures Task by Strategy Use on Inversion and Associativity Problems and Number Probe Type.

	Number Probe		
	Unrelated	Product	Operand
Strategy	*M* (*SE*)	*M* (*SE*)	*M* (*SE*)
Inversion Problems			
Inversion shortcut	99.7 (1.0)	98.8 (1.0)	97.8 (0.7)
Left-to-Right	98.6 (1.3)	81.2 (1.3)	92.9 (0.9)
Negation	99.2 (1.7)	88.4 (1.7)	96.8 (1.1)
Associativity Problems			
Associativity strategy	93.4 (1.9)	98.0 (1.8)	88.6 (1.3)
Left-to-Right	93.5 (1.3)	89.3 (1.4)	83.0 (1.1)

Table 7. Response Latencies (ms) of Correct Responses on the Interruption of Procedures Task by Strategy Use on Inversion and Associativity Problems and Number Probe Type.

	Number Probe		
	Unrelated	Product	Operand
Strategy	*M* (*SE*)	*M* (*SE*)	*M* (*SE*)
Inversion Problems			
Inversion shortcut	1347 (60)	1365 (58)	1256 (42)
Left-to-Right	1509 (79)	1463 (59)	1509 (57)
Negation	1455 (104)	1634 (86)	1283 (67)
Associativity Problems			
Associativity strategy	1510 (77)	1393 (79)	1475 (56)
Left-to-Right	1329 (115)	1582 (60)	1525 (45)

To determine whether conceptually-based strategy use interrupts computational strategies, accuracies and response latencies for responses to product number probes were analyzed collapsed across participants. For responses to product number probes following inversion problems, two 3-level (problem solving strategy: inversion shortcut, left-to-right, negation) one-way Welch's variance weighted ANOVAs were used to analyze the accuracy and response latency data. Welch's variance weighted ANOVAs and Games-Howell post hoc tests were used due to the differences in the frequency of strategy use. Welch's weighted ANOVA is more robust when sample sizes are unequal and the Games-

Howell post hoc test was designed for unequal sample sizes (Kulinskaya, Staudte, & Gao, 2003). For responses to product number probes following associativity problems, two independent-sample t-tests (associativity strategy vs. left-to-right strategy) were used to analyze the accuracy and response latency data. Originally, the analyses included problem size but it did not affect the pattern of accuracies and response latencies so the data were collapsed across this factor in all analyses.

For responses to product number probes following inversion problems, there was a main effect of strategy use on accuracy and response latencies, $F(2, 246.073) = 28.01$, $MSE = 58.74$, $p < .001$, $\eta^2 = .08$, $F(2, 322.62) = 3.39$, $MSE = 790669278.34$, $p = .035$, $\eta^2 = .01$, respectively (see Tables 8 and 9). Responses to product number probes were more accurate following inversion shortcut use than following negation strategy use ($p = .001$) or left-to-right strategy use ($p < .001$), which did not significantly differ from each other ($p = .138$). Responses to product number probes were faster following inversion shortcut use than following negation strategy use ($p = .027$) but not following left-to-right use ($p = .466$), which did not significantly differ from each other ($p = .235$). For responses to product number probes following associativity problems, responses were more accurate and faster following associativity strategy use than responses following left-to-right strategy use, $t(673) = -4.34$, $p <.001$, $t(625) = 1.92$, $p = .05$, respectively.

The results from the interruption of procedures task suggest that strategy use on the inversion and associativity problems affected responses to the interruption of procedures task. For both inversion and associativity problems, when participants used the left-to-right strategy to solve the mathematical problem they were more likely to incorrectly state that the product of the multiplication operation was present in the mathematical problem than when they used conceptually-based strategies. Also, when participants used the negation strategy or the left-to-right strategy to solve inversion and associativity problems, respectively, they were slower to correctly state that the product of the multiplication operation was not present in the mathematical problem than when they used the inversion shortcut or the associativity strategy to solve inversion and associativity problems, respectively. As in LeFevre et al. (1988), increased errors and slower response latencies likely occurred because the product of the multiplication operation interfered with the response to the interruption of procedures task. Therefore, the data from the interruption of procedures task suggests that there was less interference following inversion shortcut and associativity strategy use than following negation and left-to-right strategy use because participants produced faster responses and fewer errors on the interruption of procedures task following the use of conceptually-based strategies than following the use of negation and the left-to-right strategy.

Effect of cluster membership on the interruption of procedures task. Given that strategy use on the mathematical problem solving task affected performance on the interruption of procedures task, it should follow that individual variability in strategy use on the mathematical problem solving task affected performance on the interruption of procedures task. Participants who reported using conceptually-based strategies on the majority of problems (i.e., DC and IC cluster) should respond to the product number probes more accurately and quickly than participants who reported using computational strategies on the majority of problems (NOC cluster, see Table 8).

Table 8. Percentage of Accurate Responses and Response Latencies (ms) of Correct Responses by Cluster Membership on the Three Number Probe Types.

	Number Probe		
	Unrelated	Product	Operand
Strategy	*M* (*SE*)	*M* (*SE*)	*M* (*SE*)
Accuracy			
DC	96.7 (0.7)	97.9 (1.0)	93.2 (1.0)
IC	96.5 (1.0)	96.2 (1.5)	92.2 (2.7)
NGC	94.9 (1.3)	89.7 (3.7)	89.1 (1.5)
NOC	95.5 (1.0)	79.7 (5.8)	84.1 (3.1)
Response Latency			
DC	1337 (170)	1368 (154)	1346 (164)
IC	1151 (32)	1180 (50)	1177 (47)
NGC	1325 (71)	1412 (75)	1287 (56)
NOC	1263 (50)	1395 (75)	1351 (66)

Two 4-level (cluster: DC, IC, NGC, NOC) one-way ANOVAs were conducted on the percentage of accurate responses and response latencies of correct responses for product number probes. Due to the differences in sample sizes in each cluster, Welch's variance weighted ANOVAs and Games-Howell post hoc tests were used. There was a main effect of cluster membership on accuracy and response latencies, $F(3, 64.23)$ =

4.50, $MSE = 65819.38$, $p = .006$, $\eta^2 = .12$, $F(3, 78.77) = 3.33$, $MSE = 105113152.72$, $p = .026$, $\eta^2 = .01$, respectively. For responses to the product number probe, participants in the DC cluster were more accurate than participants in the NOC cluster ($p = .020$), who were also less accurate than participants in the IC cluster ($p = .045$). No other differences in accuracy among clusters were significant. Also, there was a trend for participants in the IC cluster to be faster than participants in the NGC ($p = .058$) and NOC ($p = .066$) clusters. No other differences in response latencies among clusters were significant.

Individual variability in strategy use on the mathematical problem solving task was reflected in participants' performance on the interruption of procedures task. Participants who used conceptually-based strategies had less difficulty rejecting probes corresponding to the product of the multiplication operation as not being present in the recently solved problem than participants who used strategies that required the product of the multiplication operation to be activated during problem solving. This suggests that the product of the multiplication operation was less activated in memory during the interruption of procedures task for users of conceptually-based strategies than for non-users and negation strategy users. These results further support the hypothesis that conceptually-based strategy use interrupts or prevents the activation of computational strategies.

General Discussion

The study had two goals. The first goal was to map the developmental trajectory of inversion shortcut and associativity strategy use on multiplication and division problems across adolescence and into early adulthood. The second goal was to determine whether empirical evidence could be found for the interruption of procedures mechanism proposed in SCADS*. The results of the study suggest that an understanding of inversion and associativity begins to mature in early adolescence and continues to develop into early adulthood. Further, the results of the study suggest that inversion shortcut and associativity strategy use involve the interruption of computational strategies. These results have implications for understanding the development of conceptual mathematical knowledge and the process of applying conceptual mathematical knowledge to problem solving.

Development of Conceptually-Based Strategy Use

The frequency of strategy use across adolescence and into early adulthood identified in this study provides new information on the development of the inversion and associativity concepts. In the present study, inversion shortcut and associativity strategy use increased across adolescence and into early adulthood. Specifically, inversion shortcut use significantly increased between Grades 7 and 9 whereas associativity strategy use increased between Grades 7 and 11. This is the first study to identify an increase in inversion shortcut and associativity strategy use on multiplication and division problems across development. In previous research, inversion shortcut and associativity strategy use on multiplication and division problems did not increase as a function of age across Grades 6, 7, and 8 (Robinson & Dubé, 2009c, 2011; Robinson et al., 2006). Furthermore, the present study identifies when the frequency and pattern of inversion shortcut and associativity strategy use reaches adult levels. In previous research, adults used the inversion shortcut on the majority of problems, the inversion shortcut was used more frequently than the associativity strategy, and the associativity strategy was used on just under half of associativity problems (Dubé & Robinson, 2010a; Robinson & Ninowksi, 2003). In the present study, the inversion shortcut was used to solve the majority of inversion problems by Grade 9 but the frequency of associativity strategy use did not approach adult levels until early adulthood.

Integration of conceptual knowledge. Individuals' understanding of inversion and associativity continues to develop into adulthood. This is surprising considering that the concepts of inversion and associativity are necessary to develop a complete understanding of the additive composition of numbers, reasoning involving part-whole relations, and commutativity (Bisanz, Watchorn, Piatt, & Sherman, 2009; Lai, Baroody, & Johnson, 2008). However, this prolonged development does not mean that children do not possess some understanding of the inversion and associativity concepts. Preschool children apply the inversion concept to solve concrete addition and subtraction inversion problems (Bryant et al., 1999; Rasmussen et al., 2003; Sherman & Bisanz, 2007) and approximately 18% of children across Grades 2, 3, and 4 use the inversion shortcut and the associativity strategy to solve the majority of addition and subtraction inversion and associativity problems (Robinson & Dubé, 2009a). Alternatively, this prolonged development could suggest that the inversion and associativity concepts are continually integrated across development into an individual's understanding of mathematical operations.

As individuals learn the mathematical concepts and procedures required for multiplication and division they may have to integrate their existing knowledge of inversion and associativity into their understanding of multiplication and division. The process of integrating conceptual knowledge of mathematics into newly learned mathematical knowledge has been proposed as the means by which conceptual and procedural mathematical knowledge iteratively develop (Rittle-Johnson, Star, & Durkin, 2009). As individuals learn new mathematical procedures and concepts, their previous procedural and conceptual knowledge should be used as a scaffold to acquire and interpret new information (Ma, 1999; Rittle-Johnson et al., 2001; Schneider & Stern, 2010). Furthermore, the notion that inversion and associativity need to be integrated over time supports Baroody's (1994) hypothesis that individuals construct abstract representations of mathematical concepts (e..g, inversion and associativity schemas) which strengthen over time and are eventually applicable to multiple problems regardless of problem characteristics.

However, previous research has suggested that individuals' understanding of inversion and associativity for addition and subtraction does not transfer to their understanding of inversion and associativity for multiplication and division. For example, Robinson et al.'s (2006) study of Grade 6 and 8 children's inversion shortcut and associativity strategy use found that conceptually-based strategy use on addition and subtraction problems did not promote conceptually-based strategy use on multiplication and division problems. Specifically, participants solved either a set of addition and subtraction problems and then a set of multiplication and division problems or solved the problems in the reverse order. If use of conceptually-based strategies on one problem type promotes use of conceptually-based strategies on the other problem type then participants' frequency of conceptually-based strategy use for a given problem type should have been higher in the second set than in the first set (Robinson et al., 2006). This was not the case and the researchers concluded that individuals' understanding of inversion and associativity does not transfer between operations. One interpretation of these results is that there are separate inversion and associativity concepts for addition and subtraction and multiplication and division (Robinson & Dubé, 2009b).

Alternatively, the results from Robinson et al. (2006) do not necessarily mean that there are multiple inversion and associativity concepts. Indeed, children and adults who use the inversion shortcut on one problem type are more likely to use it on the other problem type (Robinson & Ninowksi, 2003; Robinson et al., 2006). Thus, individuals who understand the concepts of inversion and associativity can apply their understanding to solve different problem types. This suggests that there is one inversion concept and one associativity concept, which can be applied to both addition and subtraction and multiplication and division. The results of the present study suggest that the process of integrating the general concepts of inversion and associativity into one's understanding of multiplication and division is prolonged and only becomes strong in mid to late adolescence.

Factors affecting the adoption of conceptually-based strategies. The development of inversion shortcut and associativity strategy use identified in this study is in line with previous observations that individuals' understanding of the associativity concept develops after their understanding of the inversion concept. In previous research, children's and adults' frequency of inversion shortcut use is higher than their frequency of associativity strategy use (Robinson & Dubé, 2009a, 2009b, 2011; Robinson & Ninowski, 2003). In the present study, participants' frequency of inversion shortcut use was higher than their frequency of associativity strategy use and significant increases in inversion shortcut use occurred earlier than significant increases in associativity strategy use. This suggests that individuals' understanding of associativity is not as fully developed as their understanding of inversion across adolescence and into early adulthood. Alternatively, early adolescents may not judge the increased accuracy and speed of the associativity strategy as a sufficient reason to switch away from the well practiced left-to-right computational strategy and adopt the associativity strategy. In the present study, the left-to-right strategy was substantially slower than the associativity strategy for participants in Grade 7 but the speed of the left-to-right strategy substantially increased in Grade 9 (see Figure 1 bottom). This increased speed may have contributed to the prolonged use of the left-to-right strategy on associativity problems. Another possibility is that the associativity strategy may not be as easy to discover as the inversion shortcut.

The inversion shortcut may have been easier to discover than the associativity strategy due to the use of the intermediate negation strategy. In Grade 7, participants used the inversion shortcut and negation strategy with equal frequency to jointly solve the majority of inversion problems. After Grade 7, participants used the inversion shortcut to solve the majority of inversion problems while using the negation strategy to solve fewer than 20% of inversion problems. This increase in inversion shortcut use and decrease in negation strategy use suggests that negation strategy use is a precursor to increased inversion shortcut use for multiplication and division problems.

This pattern of inversion shortcut and negation strategy use is also consistent with previous research. In Siegler and Stern's (1998) microgenetic study of children's inversion shortcut use on addition and subtraction problems, more than 80% of the children used the negation strategy before discovering the inversion shortcut strategy. Similarly, in Robinson and Dubé's (2009) microgenetic study of Grade 6 children's inversion shortcut use on multiplication and division problems, 73% of the children used the negation strategy to solve at least one of the three problems preceding their discovery of the inversion shortcut. The negation strategy precedes the inversion shortcut because the negation strategy is generated by simplifying problem solving (i.e., completing the multiplication operation then realizing that the division operation does not need to be calculated; Bisanz & LeFevre, 1990; Siegler & Stern, 1998) and further simplification results in the inversion shortcut (i.e., realizing that the multiplication operation does not need to be calculated). Thus, previous studies identified negation strategy use as a precursor to inversion shortcut use within individual strategy discovery. Further, the present study suggests that negation strategy use is also a precursor to inversion shortcut use across development. Therefore, the use of the negation strategy may partially account for why the frequency of inversion shortcut use increases before the frequency of associativity strategy use because an intermediary strategy does not precede the associativity strategy.

Increases in mathematical knowledge. The results from the accuracy, solution latency, and strategy execution data suggest that procedural, factual, and conceptual knowledge of mathematics improved across adolescence and into early adulthood. The improvement in accuracies and solution latencies between Grade 7 and Grade 9 reflects an increase in all three types of mathematical knowledge. The average solution latencies on inversion and associativity problems in Grade 9 are faster than the average execution speed of the negation and left-to-right strategies. This suggests that the improved solution latencies between Grades 7 and 9 are partly due to a switch to the faster inversion shortcut and associativity strategy. Also, the speed of executing the left-to-right strategy increased between Grades 7 and 9, which suggests that participants in Grade 9 had better procedural and/or factual knowledge than participants in Grade 7. Finally, the speed of executing the inversion shortcut and associativity strategy gradually improved across development, which suggests that participants had a better understanding of the underlying mathematical concepts. This could also indicate that participants' confidence criteria for using the inversion shortcut and associativity strategy decreased. Lower confidence criteria indicate that participants are more certain that using a given strategy will result in the correct answer (Siegler, 1988a). Therefore, the improved accuracy and speed of the left-to-right and conceptually-based strategies indicates that both fundamental computational skills and conceptual understanding of abstract mathematical concepts increase across adolescence and into early adulthood.

Interdependence of conceptual knowledge. The results suggest that there is a medium to strong positive relationship between inversion shortcut and associativity strategy use across adolescence and into early adulthood. Moreover, all participants who used the associativity strategy also used either the inversion shortcut or the negation strategy but some participants who used the inversion shortcut did not use the associativity strategy. This pattern of strategy use gives some insight into the direction of the relationship between inversion shortcut and associativity strategy use. Taken together, these results provide further support that the use of the inversion shortcut typically develops before the use of the associativity strategy and suggests that understanding the inversion concept aids in developing an understanding of the associativity concept.

Understanding the development of the relationship between the inversion and associativity concepts is important for two reasons. The first is that it may provide information on how conceptual knowledge develops. Possessing deep conceptual knowledge of mathematics means not only understanding the principles underlying mathematics but also understanding how these principles interrelate (Schneider & Stern, 2010; Stern, 1992). If understanding the inversion concept aids understanding the associativity concept then studying this developmental process is tantamount to studying the formation of deep conceptual knowledge. Typically, research has focused on the iterative development between procedural and conceptual knowledge (Bisanz et al., 2009). Little research has been conducted on how existing conceptual knowledge aids the development and understanding of new conceptual knowledge. The present study has identified a developmental period (i.e., adolescence and early adulthood) and process (i.e., conceptual knowledge of inversion aiding conceptual knowledge of associativity) that future research could investigate to better understand how existing conceptual knowledge aids new conceptual knowledge. For example, a microgenetic study of how the inversion shortcut and associativity strategy develop across Grade 9 could provide detailed information on how individuals use their knowledge of one concept to generate and/or understand the other concept.

Secondly, understanding how individuals' knowledge of the inversion concept aids their understanding of the associativity concept may provide useful information for mathematical pedagogy. Elementary mathematics instruction should capitalize on students' existing conceptual understanding of mathematics to teach new mathematical procedures and concepts (Ma, 1999). However, little research has been conducted on how individuals use their existing conceptual knowledge to learn new mathematical concepts. Studying the development of inversion shortcut and associativity strategy use across adolescence and into adulthood may be a window into this process.

Individual Variability in Strategy Use

The results of the study identified a pattern of individual variability in inversion and associativity strategy use that is unique to the developmental period from adolescence and into early adulthood. Overall, there were clusters of individuals who used both concepts, only the inversion concept, the negation strategy, or the left-to-right strategy to solve the majority of the inversion and associativity problems. Also, individuals who used the inversion shortcut and associativity strategy were significantly more accurate and faster problem solvers than participants who used the negation and/or left-to-right strategy. Identifying this unique pattern of individual variability is important for understanding the overall development of the inversion and associativity concepts.

Importance of identifying individual variability. Previous research has identified patterns of individual variability in children's and adults' inversion and associativity strategy use (Dubé & Robinson, 2010a; Robinson & Dubé, 2009a, 2009b) and the present study expands this body of research by detailing individual variability in a previously understudied developmental period (i.e., adolescence). Combining the results of the present study with previous research suggests that individual variability in conceptually-based strategy use exists throughout development. Identifying individual variability is critical for gaining a complete understanding of cognitive development, because not investigating individual variability can often lead to incorrect conclusions about how change occurs (Siegler, 2007). In terms of inversion shortcut and associativity strategy use, solely investigating the frequency of conceptually-based strategy use in each developmental period would depict the development of these concepts as a gradual increase in understanding. In contrast, the patterns of individual variability identified by researchers suggest that, across development, there are individuals who have a better understanding of the inversion and associativity concepts than some of their same age and older peers. Thus, the present study further highlights the importance of investigating individual variability.

Existence of variability throughout development. The existence of individual variability in inversion shortcut and associativity strategy use throughout development also highlights the importance of investigating why some individuals demonstrate a better understanding of these concepts than their peers. If individual variability exists throughout development then understanding development requires researchers to identify the factors responsible for the individual variability and to detail how these factors affect development. Few studies have identified sources of individual differences in conceptually-based strategy use (Schneider & Stern, 2009). For example, there is no evidence that individual differences in inversion shortcut use are predicted by individual differences in procedural or factual mathematical knowledge (Gilmore & Papadatou-Pastou, 2009). One study found that individual differences in analogical reasoning ability and central executive functioning predict individual differences in inversion shortcut use (Dubé & Robinson, 2010b). Currently, it is not known whether analogical reasoning ability and executive functioning affect the development of conceptually-based strategies. Why the pattern of individual variability in inversion shortcut use and associativity strategy use changes across development is not well understood.

The present study details how individual variability in conceptually-based strategy use changes across adolescence and into early adulthood. In previous studies, cluster membership was unaffected by the age of participants (Robinson & Dubé, 2009a, 2009b). In the present study, the proportion of participants in each cluster varied depending on the developmental period. The majority of Grade 7 participants were in the NGC cluster, the majority of Grade 9 participants were split between the DC and IC clusters, and the majority of Grade 11 and Adult participants were in the DC cluster (see

Figure 2). The change in cluster membership between Grade 7 and Grade 9 is further support that the negation strategy precedes inversion shortcut use. Future longitudinal research is needed to confirm this finding. The majority of participants in Grade 7 used the negation strategy whereas the majority of participants in Grade 9 used either both conceptually-based strategies or only the inversion shortcut. The pattern of strategy use in Grade 9 might be particularly important for understanding the development of inversion shortcut and associativity strategy use. Participants in Grade 9 were just as likely to use both conceptually-based strategies, only the inversion shortcut, or neither of these strategies during problem solving. Currently, researchers have not investigated how cluster membership changes across development because in previous studies cluster membership did not differ among different developmental periods. Given the results of the present study, future research should investigate the change in conceptually-based strategy use that occurs across Grades 9 and 10 to investigate how and why cluster membership changes.

The pattern of individual variability also suggests that changes between Grade 7 and Grade 9 and between Grade 9 and Grade 11 affect individuals' inversion shortcut and associativity strategy use. One source of change could be mathematics instruction. However, instruction does not explain the individual differences present within each developmental period. Another possible source of change could be individual differences in executive functioning. There is evidence that inversion shortcut use is predicted by executive functioning (Dubé & Robinson, 2010b), which increases across adolescence due to the maturation of the prefrontal cortex (Toga et al., 2006). Also, there are individual differences in the rate of maturation (Sowell et al., 2003). Thus, grade differences in conceptually-based strategy use found in this study could be explained by the maturation of executive functioning. Further evidence for this hypothesis could be found in future research with adult participants who have diminished executive functioning, such as individuals with dementia. If dementia sufferers have lower frequency of conceptually-based strategy use than healthy age-matched adults then this would suggest that executive functioning is involved in conceptually-based strategy use. Currently, no research has investigated the frequency of inversion shortcut or associativity strategy use in older adults or adults with dementia. How executive functioning enables inversion shortcut and associativity strategy use is not known but the results of the present study indicate that the interruption of procedures mechanism may be involved.

Interruption of Procedures Mechanism

Siegler and Araya (2005) proposed that computational strategies must be interrupted in order for the inversion shortcut to be selected. Previously, researchers had not tested this hypothesis experimentally. In the present study, the results from the interruption of procedures task indicate that the product of the multiplication operation was less active in memory following inversion shortcut and associativity strategy use than following left-to-right strategy use. Therefore, the results of this study provide the first evidence that the use of conceptually-based strategies interrupts or prevents the activation of procedural strategies.

Interruption of procedural strategies. How conceptually-based strategies interrupt procedural strategies has not been proposed. However, evaluating existing strategy selection models could serve as a starting point for understanding this process. When solving a problem such as 4 × 4, the answer can either be directly retrieved from memory or a backup strategy can be implemented (Siegler & Shipley, 1995). Backup strategies involve decomposing a problem into smaller retrieval sequences (Siegler, 1988b). For example, solving the problem 4 × 4 using the backup strategy of repeated addition can be completed by retrieving the answer to 4 + 4 = 8, then retrieving the answer to 8 + 4 = 12, and then retrieving the answer to 12 + 4 = 16. When an individual uses retrieval to answer the problem 4 x 4, the activation for retrieving the number 16 races against a backup strategy. Specifically, the retrieval of the number 16 races against the first step of the smaller retrieval sequence (i.e., 4 + 4; Siegler & Shipley, 1995). Retrieval occurs if the associated strength between the problem 4 × 4 and the answer 16 is higher than the associated strength between the first step of the smaller retrieval sequence and its answer (i.e., between 4 + 4 and 8; Shrager & Siegler, 1998). Retrieval could also win at a later part in the sequence (e.g., retrieving 4 × 4 = 16 while performing the second step of the backup strategy). Moreover, not only is there a race between retrieval and backup strategies but there is inhibition of backup strategies. Using retrieval inhibits/decreases the associated strength of responses for backup strategies (i.e., retrieving 8 x 4 = 32 inhibits 8 + 4; Bajic & Rickard, 2009). Given this model for how strategy selection occurs when using retrieval or procedural strategies, the next step is to extend this model to conceptually-based strategies.

Strategy selection for conceptually-based strategies can be conceptualized as acting like retrieval in the previous model. For example, when solving the inversion problem 3 × 7 ÷ 7, inversion shortcut use (i.e., 'retrieving'/deduce the logical fact that the answer is just the first number) races against the execution of procedural strategies (e.g., retrieving 3 × 7 = 21, 21 ÷ 7 = 3). The inversion shortcut is executed if the associated strength between the problem 3 × 7 ÷ 7 and the inversion shortcut is higher than the associated strength between the first step of the procedural strategy and its answer (i.e., between 3 × 7 and 21). The inversion shortcut could also win at a later part in the sequence (e.g., 'retrieving'/deducing the logical fact that the answer is just the first number while performing the second step of the backup strategy). Executing the inversion shortcut after performing the first step of the procedural strategy results in the negation strategy (Bisanz & LeFevre, 1990). Moreover, the results of the present study suggest that the use of conceptually-based strategies interrupts or prevents the activation of procedural strategies. This is similar to the way that retrieving 8 x 4 = 32 inhibits backup strategies and decreases the associated strength of responses for backup strategies.

This framework for understanding the selection process for conceptually-based strategies has two implications for mathematical cognition. First, the execution of a conceptually-based strategy operates in memory similarly to the retrieval of an arithmetic fact; both result in the inhibition of backup strategies. This is counter-intuitive because inhibition during retrieval is a decrease in activation likelihood of a node in memory resulting from another node being activated (Bjork, 1989) and concepts like inversion and associativity are not typically thought of as a single node in memory that can be specifically retrieved (LeFevre et al., 2006; Stern 1992).

One possible explanation is that the process of applying conceptual knowledge to problem solving results in the proceduralization of mathematical concepts. Schneider and Stern (2010) have proposed that mathematical concepts exist at a declarative level (i.e., general and abstract) and that they must be proceduralized in order for them to be used in a specific problem solving situation. Proceduralization is the transformation of declarative knowledge into productions that can be retrieved into working memory instead of the abstract declarative knowledge (Anderson, 1982). In terms of inversion shortcut and associativity strategy use, the proceduralization of the inversion and associativity concepts may produce simple productions which are retrieved into working memory instead of the abstract concepts themselves. The retrieval of these single productions is more similar to the retrieval of mathematical facts and might explain how conceptually-based strategy use results in the interruption of procedural strategies. During conceptually-based strategy use, the productions for the inversion shortcut or associativity strategy may race against and inhibit backup strategies.

Another explanation of how conceptually-based strategy use results in the interruption of backup strategies is that retrieval structures activate several nodes in memory that form conceptual mathematical knowledge. Ericsson and Kintsch (1995) proposed that retrieval structures are a system of retrieval cues in which information held in working memory retrieves interconnected information held in long-term memory. Analogously, Cowan (2001) proposed that contextual factors of a stimulus (e.g., operations in a mathematical problem) can be used to increase the activation of interconnected information stored in long-term memory, which is “chunked” and entered into the focus of attention. It is possible that retrieval structures act as a bridge between an attended to mathematical problem and interrelated mathematical principles stored in long-term memory. This may allow individuals to apply their conceptual knowledge of mathematics to problem solving by using a single retrieval structure, which activates the interrelated principles that construct the inversion or associativity concept. From this perspective, inversion or associativity retrieval structures are what enter into the strategy selection process and interrupt backup strategies.

The existence of inversion and associativity productions or retrieval structures could account for some of the developmental trends found in inversion shortcut and associativity strategy use. The lower frequency of conceptually-based strategy use on multiplication and division problems as compared to addition and subtraction problems could be the result of separate productions or retrieval structures being required depending on the mathematical operations involved. Also, individual differences in the existence of inversion and associativity productions or retrieval structures could account for individual differences in strategy use. The majority of children evaluate the inversion shortcut and associativity strategy to be good strategies and to be better than the left-to-right strategy (Robinson & Dubé, 2009a, 2009b). However, a child's ability to evaluate conceptually-based strategies does not predict their frequency of conceptually-based strategy use. Perhaps the majority of children possess the declarative knowledge required to evaluate conceptually-based strategies but lack either the specific productions or retrieval structures required to use conceptually-based strategies.

Furthermore, the existence of productions or retrieval structures fits within both theories of inversion shortcut use. The inclusion of productions or retrieval structures into Baroody's theory would combine current models of skill acquisition (e.g., Anderson, 2010) and the interaction between working memory and long-term memory (e.g., Ericsson & Kintsch, 1995) with Baroody's theory of how schemas direct problem solving behaviour. Within Baroody's (1994) theory, the inversion schema is an abstract understanding of the inversion concept that is applied to problem solving. The creation of productions or retrieval structures for different problems might describe how individuals apply their general knowledge of inversion to all problems. Within the SCADS* model, the existence of productions or retrieval structures could account for how conceptually-based strategies interrupt backup strategies in a manner similar to how retrieval of an arithmetic fact inhibits the activation of backup strategies. It is not known whether productions or retrieval structures account for how conceptually-based strategies interrupt backup strategies. However, the present study has provided the first evidence that conceptually-based strategies interrupt backup strategies and future research is required to determine how this process occurs.

Role of the interruption of procedures mechanism in the development of conceptually-based strategies. Combining participants' strategy use, performance on the interruption of procedures task, and results from previous research provides information on the development of conceptually-based strategies. In the present study, the results suggest that negation strategy use precedes inversion shortcut use and that conceptually-based strategy use matures across adolescence and into early adulthood. Also, the results suggest that backup strategies are more highly activated in memory during negation strategy and left-to-right strategy use than during inversion shortcut use. In previous research, children who used the inversion shortcut scored higher on measures of executive functioning than children who used the negation or left-to-right strategy (Dubé

& Robinson, 2010b). Also, there are significant increases in executive functioning across adolescence and into early adulthood (Toga et al., 2006). Taken together, these results suggest that conceptually-based strategy use increases across adolescence and into early adulthood because maturation of executive functioning across adolescence improves the ability to interrupt backup strategies. This would explain why increases in inversion shortcut use co-occur with decreases in negation strategy use and why associativity strategy use is negatively correlated with negation strategy use. From this perspective, adolescents who use the negation strategy understand the inversion concept but have difficulty interrupting backup strategies at the onset of problem solving, which decreases the likelihood of using the inversion shortcut or the associativity strategy. Currently, models of conceptually-based strategy use do not consider the role of executive functioning on conceptually-based strategy use. Future research is needed to incorporate the role of the interruption of procedures mechanism into models of conceptually-based strategy use while accounting for how executive functioning affects the process of applying conceptual knowledge to problem solving.

Conclusions

The results of the present study contribute to a considerable body of research on inversion shortcut and associativity strategy use. In particular, there are three key conclusions that augment current understanding of conceptual mathematical knowledge and its application to problem solving.

Maturation in Adolescence

Individuals' understanding of the inversion and associativity concepts matures across adolescence and early adulthood. This conclusion supports a fundamental hypothesis about the nature of conceptually-based strategy use; the type of reasoning required to apply the inversion and associativity concept to problem solving is related to the type of reasoning required for scientific thought (Piaget, 1978). Previous research indicates that scientific reasoning matures across adolescence and into early adulthood (Amsel & Brock, 1996) and the present study suggests that inversion shortcut and associativity strategy use similarly matures across adolescence and into early adulthood. Furthermore, the maturation of inversion shortcut use, associativity strategy use, and scientific reasoning skills in adolescence are preceded by very young children exhibiting naive reasoning abilities in each of the domains. Preschool children exhibit a naive conceptual understanding of inversion and associativity (Bryant et al., 1999; Klein & Bisanz, 2000; Rasmussen et al., 2003; Sherman & Bisanz, 2007) and naive scientific reasoning skills (e.g., evidence evaluation skills, Koerber, Sodain, Thoermer, & Nett, 2005). Also, children have difficulty applying their naive understanding of the inversion concept to formal three-term algebraic problems (Baroody & Lai, 2007; Canobi, 2005; Gilmore, 2006; Gilmore & Spelke, 2008) and have difficulty applying their naive scientific reasoning skills to formal scientific experiments (Koerber et al., 2005). Thus, conceptually-based strategy use and scientific reasoning skills both maturing across adolescence and early adulthood is further evidence of the relationship between these two domains of reasoning and demonstrates the domain-general nature of mathematical problem solving ability. This does not mean that the reasoning underlying inversion shortcut and associativity strategy use is the same as scientific reasoning. Rather, it demonstrates that the factors affecting the development of mathematical reasoning likely affect the development of scientific reasoning (e.g., increases in working memory capacity and abstract reasoning skills). Researchers investigating mathematical cognition need to look beyond the domain of mathematics to fully understand how mathematical problem solving develops.

Individual Variability

Previous research has demonstrated that identifying individual variability is essential to understanding mathematical problem solving skills. For example, researchers identified individual differences in children's and adults' use of retrieval and backup strategies for solving addition and subtraction problems. There are individuals who are proficient at retrieval and use retrieval during problem solving, proficient at retrieval but use backup strategies during problem solving, and not proficient at retrieval but use retrieval during problem solving (Hetch, 2006; Siegler, 1988). By identifying these individuals, researchers concluded that children's and adults' strategy use is affected by their confidence criteria for a given strategy. Thus, identifying individual variability was essential for understanding the development of addition and subtraction problem solving and for identifying a factor affecting strategy choice (i.e., confidence criteria).

Similarly, the present study demonstrates that identifying individual variability is essential for understanding the development of conceptually-based strategy use. There is individual variability in inversion shortcut and associativity strategy use across childhood and adulthood (Dubé & Robinson, 2010a; Gilmore & Papadatou-Pastou, 2009; Robinson & Dubé, 2009a, 2009b). Also, the present study has concluded that individual variability in inversion shortcut and associativity strategy use also characterizes development across adolescence and into early adulthood. Having identified variability in conceptually-based strategy use across development, researchers can now confidently state that the development of conceptually-based strategy use is characterized by variability and not simple linear increases in strategy use.

Moreover, investigating why individual variability in inversion shortcut and associativity strategy use occurs has identified a factor that may affect other types of mathematical problem solving. The present study suggests that individuals who use the inversion shortcut or associativity strategy are able to interrupt their automatic tendency to use procedurally-based strategies. This result has potential applications beyond inversion shortcut or associativity strategy use. The interruption of procedures mechanism may also be involved in other types of mathematical problem solving. For example, children have difficulty solving problems that require a conceptual understanding of equivalence (e.g., 3 + 5 + __ = 17; McNeil, 2007). Typically, children incorrectly solve an equivalence problem by adding all numbers in a problem instead of applying their understanding of equivalence to solve the problem (i.e., the sum all strategy, 3 + 5 + _ = 17, 3 + 5 = 8, 8 + 17 = 25; Sherman & Bisanz, 2009). It is not known whether children have difficulty interrupting their tendency to sum all of the digits in the problem. Therefore, identifying the role of the interruption of procedures mechanism in the present study could lead researchers to investigate the role the interruption of procedures mechanism plays in other types of mathematical problem solving.

Models of Mathematical Knowledge

The results of the present study suggest that the application of the inversion and associativity concept to problem solving involves the interruption of procedural strategies. This result has the potential to improve understanding of how factual, procedural, and conceptual knowledge of mathematics interact during problem solving to generate conceptually-based strategies. Bisanz and LeFevre (1990) studied the inversion concept to identify the different types of mathematical knowledge and understand how the different types of knowledge interacted and developed. Building on their work, researchers have proposed that the three knowledge types iteratively develop within individuals with increases in one knowledge type leading to increases in the other knowledge types (Rittle-Johnson et al., 2001). Despite this strong theoretical model, several studies have failed to identify a link between individuals' procedural and factual knowledge of mathematics and their ability to use conceptually-based strategies (Bryant et al., 1999; Gilmore & Papadatou-Pastou, 2009; Robinson & Dubé, 2009a, 2009b). Moreover, Canobi (2005) suggests that teaching children conceptually-based strategies (i.e., procedural knowledge) does not increase their conceptual knowledge. Taken together, this suggests that how factual, procedural, and conceptual knowledge generate conceptually-based strategies is not fully understood. However, researchers have found a link between conceptually-based strategy use and general cognitive skills (Dubé & Robinson 2010b; Rasmussen et al., 2005). This link suggests that factors outside of mathematical knowledge affect conceptually-based strategy use.

The results of the current study suggest a new type of interplay between conceptual and procedural knowledge during problem solving, a competitive one. The application of conceptual knowledge during problem solving may require the interruption of procedural knowledge. The idea that conceptually-based strategies are produced from a competition between procedural and conceptual knowledge seems different from the idea that conceptually-based strategies are produced from the iterative interaction between procedural and conceptual knowledge. How this competition is resolved is not fully understood but it may be linked to the idea of flexible mathematical knowledge. Flexibility in problem solving occurs when the most adaptive problem solving method is chosen, regardless of whether the problem solving is based on procedural or conceptual knowledge (Newton et al., 2010). Thus, the use of conceptually-based strategies may be the product of flexibly choosing shortcuts over procedurally-based strategies.

Further study is required to understand how the types of mathematical knowledge interact during conceptually-based strategy use and how general cognitive abilities enable conceptually-based strategy use. Conceptually-based strategy use may require either the proceduralizing of conceptual knowledge or the use of retrieval structures. How mathematical concepts are proceduralized is not understood (Schneider & Stern, 2010). Also, it is not known whether a proceduralized concept falls under the definition of procedural or conceptual knowledge. Similarly, how retrieval structures activate mathematical concepts has not been investigated and it is unclear which type of mathematical knowledge best encapsulates this process. Alternatively, a fourth type of mathematical knowledge may need to be defined that highlights the role general cognitive abilities play in the application of conceptual knowledge to problem solving. For example, the existence of the interruption of procedures mechanism demonstrates the importance of understanding how general cognitive abilities, such as central executive functioning, facilitate the application of mathematical knowledge to problem solving. An individual who understands the inversion or associativity concept but lacks the executive functioning required to interrupt procedural strategies may be less likely to use conceptually-based strategies during problem solving. Therefore, future models of mathematical knowledge need to account for how factual, procedural, and conceptual knowledge interact to generate conceptually-based strategies and account for the role general cognitive abilities play in this process.

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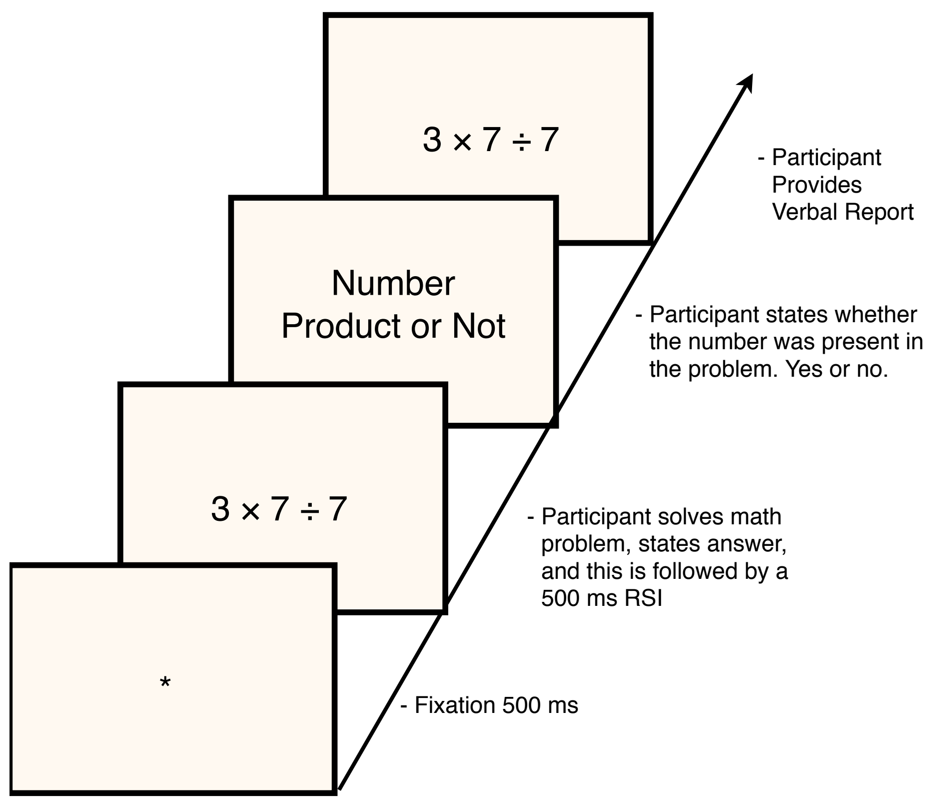
Appendix A

Inversion	Associativity
Small	
3 × 4 ÷ 4	3 × 4 ÷ 2
7 × 2 ÷ 2	2 × 8 ÷ 4
4 × 3 ÷ 3	3 x 8 ÷ 2
8 × 3 ÷ 3	4 x 6 ÷ 3
4 × 2 ÷ 2	2 x 9 ÷ 3
3 × 8 ÷ 8	4 x 6 ÷ 2
4 × 5 ÷ 5	5 x 4 ÷ 2
2 × 9 ÷ 9	2 x 10 ÷ 5
2 x 11 ÷ 11	2 x 12 ÷ 6
2 x 12 ÷ 12	2 x 12 ÷ 4
Large	
5 × 9 ÷ 9	4 × 9 ÷ 3
6 × 8 ÷ 8	5 x 12 ÷ 6
7 × 6 ÷ 6	7 x 9 ÷ 3
3 × 9 ÷ 9	4 x 8 ÷ 2
8 × 4 ÷ 4	12 x 10 ÷ 2
5 × 8 ÷ 8	9 x 6 ÷ 2
6 × 5 ÷ 5	3 x 12 ÷ 4
9 × 7 ÷ 7	8 x 10 ÷ 5
7 x 12 ÷ 12	9 x 12 ÷ 4
5 x 10 ÷ 10	7 x 10 ÷ 5

Appendix B

Trial Event Sequence



Appendix C

The number of participants in each cluster by developmental period with adjusted residual scores.

	Cluster			
Developmental Period	DC	IC	NGC	NOC
Grade 7				
Observed	5	6	20	9
Adjusted Residual	-4	-0.6	5	0.3
Grade 9				
Observed	14	12	5	9
Adjusted Residual	-0.7	2.3	-1.7	0.3
Grade 11				
Observed	18	6	6	10
Adjusted Residual	0.8	-0.6	-1.2	0.8
Adult				
Observed	26	5	4	5
Adjusted Residual	3.8	-1.1	-2.1	-1.5

Note. The sign on the adjusted residual score indicates the direction of the difference between the observed and expected frequencies in each cell. Values greater than two, regardless of sign, are considered as significantly contributing to the significance of the chi-square analysis.

Appendix D

UNIVERSITY OF
REGINA

OFFICE OF RESEARCH SERVICES

MEMORANDUM

DATE: October 1, 2009

TO: Adam Dubé
Psychology, Campion

FROM: Dr. Bruce Plouffe
Chair, Research Ethics Board

Re: **Conceptually-Based Strategy Use: Investigating Development Across Adolescence Into Early-Adulthood and Underlying Mechanisms (File # 16S0910)**

Please be advised that the University of Regina Research Ethics Board has reviewed your proposal and found it to be:

☑ 1. APPROVED AS SUBMITTED. Only applicants with this designation have ethical approval to proceed with their research as described in their applications. For research lasting more than one year (Section 1F). **ETHICAL APPROVAL MUST BE RENEWED BY SUBMITTING A BRIEF STATUS REPORT EVERY TWELVE MONTHS.** Approval will be revoked unless a satisfactory status report is received. Any substantive changes in methodology or instrumentation must also be approved prior to their implementation.

☐ 2. ACCEPTABLE SUBJECT TO MINOR CHANGES AND PRECAUTIONS (SEE ATTACHED). Changes must be submitted to the REB and approved prior to beginning research. Please submit a supplementary memo addressing the concerns to the Chair of the REB.** Do not submit a new application. Once changes are deemed acceptable, ethical approval will be granted.

☐ 3. ACCEPTABLE SUBJECT TO CHANGES AND PRECAUTIONS (SEE ATTACHED). Changes must be submitted to the REB and approved prior to beginning research. Please submit a supplementary memo addressing the concerns to the Chair of the REB.** Do not submit a new application. Once changes are deemed acceptable, ethical approval will be granted.

☐ 4. UNACCEPTABLE AS SUBMITTED. The proposal requires substantial additions or redesign. Please contact the Chair of the REB for advice on how the project proposal might be revised.

Dr. Bruce Plouffe

cc: Dr. Katherine Robinson – Psychology, Campion

** supplementary memo should be forwarded to the Chair of the Research Ethics Board at the Office of Research Services (Research and Innovation Centre, Room 109) or by e-mail to research.ethics@uregina.ca

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