A NUMERICAL APPROACH FOR OIL SLUG MOBILIZATION
IN A CAPILLARY TUBE
UNDER ABSENCE AND PRESENCE OF EXTERNAL EXCITATIONS

A Thesis
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In Partial Fulfillment of the Requirements
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By
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Harsh Dipak Joshi, candidate for the degree of Master of Applied Science in Industrial Systems Engineering, has presented a thesis titled, *A Numerical Approach for Oil Slug Mobilization in a Capillary Tube Under Absence and Presence of External Excitations*, in an oral examination held on April 22, 2013. The following committee members have found the thesis acceptable in form and content, and that the candidate demonstrated satisfactory knowledge of the subject material.

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This research attempts to study the mobilization of an oil drop, also known as a “slug”, via the aid of a water film. This differs from other work in this area wherein the oil drop is forced through the throat of a pore using external excitations. In the first simulation, an oil drop is placed in a small diameter tube filled with water and then flushed from the tube by injecting more water. The pressure difference between the inlet and outlet of the tube is then recorded. Unlike past research in this field, this work employs dimensionless numbers and sensitivity analysis to more accurately analyze the effects of different parameters on the pressure drop. The model in this thesis also likely represents a novel simulation in which vibrations are used to aid in the formation of the water film. Tests, that use these vibratory excitations, are carried out as part of a second series of simulations, in order to study and visualize how vibrations help in the movement of an oil slug. The results show that different vibrations produce varying results. An interesting relationship between pressure drop and the various types of vibrations, as well as water film length and pressure drop, has been found. Visual evidence of the formation of a water film due to vibrations is presented. All numerical simulations are verified using core-annular flow. The results of this research contribute to ideas already available in the field of enhanced oil recovery practice using vibro-seismic techniques. Ultimately, this work could lead to improved oil recovery methods.
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DEDICATION

This work is dedicated to my mother, Sandhya Joshi, and my father, Dipak Joshi.
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LIST OF ACRONYMS AND SYMBOLS

Acronyms

AFT average film thickness
CSF continuum surface force
LB lower bound
UB upper bound
VOF volume of fluid

Symbols

δ normalized film thickness
Δt time step, s
Δx element size, μm
ΔP pressure drop, Pa
μ_{oil} viscosity of oil, kg/m-s
μ_{water} viscosity of water, kg/m-s
\( \mu \)  \quad \text{dynamic viscosity}

\( \mu_{\text{non-wetting liquid}} \)  \quad \text{viscosity of the non-wetting liquid, kg/m-s}

\( \mu_{\text{wetting liquid}} \)  \quad \text{viscosity of the wetting liquid, kg/m-s}

\( \rho_{\text{oil}} \)  \quad \text{density of oil, kg/m}^3

\( \rho_{\text{water}} \)  \quad \text{density of water, kg/m}^3

\( \rho \)  \quad \text{density}

\( \rho_{\text{wetting liquid}} \)  \quad \text{density of the wetting liquid, kg/m}^3

\( \sigma_{\text{oil-water}} \)  \quad \text{oil-water interfacial tension, N/m}

\( \text{Re}_{\text{oil}} \)  \quad \text{Reynolds number based on the oil properties}

\( \text{Re}_{\text{water}} \)  \quad \text{Reynolds number based on the water properties}

\( A \)  \quad \text{amplitude of the excitation}

\( D \)  \quad \text{diameter of the tube, mm}

\( f \)  \quad \text{frequency of the excitation}

\( L_{\text{oil}} \)  \quad \text{length of the oil drop, mm}

\( L_{\text{water}} \)  \quad \text{length of the water region, mm}

\( M \)  \quad \text{viscosity ratio}

\( p \)  \quad \text{pressure}

\( r \)  \quad \text{radius of the tube, mm}

\( t \)  \quad \text{time, s}

\( V_{\text{fluid}} \)  \quad \text{fluid velocity in the element}
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1. INTRODUCTION

1.1 Background

Oil is one of the most important natural resources and is used in everyday life. Significant technological advancements have been developed to recover oil from underground oil reservoirs. These include Primary Recovery, with oil recovery under reservoir pressure, followed by Secondary Recovery, in which the reservoir is flooded with water to build pressure and recover the oil, and finally Tertiary or Enhanced Oil Recovery (EOR). Many different types of EOR techniques such as steam flooding, polymer flooding, and carbon dioxide flooding exist. Even though prominent technologies have been developed, on average, these technologies only produce 40% of the existing oil, while 60% remains unproduced due to a variety of limitations in EOR approaches [1].

An EOR technique using external stimulation such as vibration is a relatively new EOR method. Significant advances have been made in this field over the past few years with pore scale, macro scale, lab scale, and field scale studies. All studies have found that vibrations do tend to increase oil production and decrease the amount of oil left behind in a reservoir. Pore scale studies with vibrations usually utilize a curved capillary tube to mimic a pore throat and the trapping mechanism due to capillary pressure inside a reservoir pore. Then, they show how vibrations can provide the extra force needed to move the trapped oil drop through the throat/constriction of the pore. One aspect that is overlooked by all researchers, with the exception of
members of our research team, is how vibrations can help recover a trapped oil drop in a way which differs from overcoming the pore throat resistance. It is known that a high viscosity liquid such as oil, surrounded by a low viscosity liquid such as water, makes it easier to transport the oil [2] and reduces the pressure drop considerably, which in turn, reduces the amount of energy required to pump the oil. This means that if a water film can be established between the oil drop and the capillary tube wall, the pressure needed to mobilize the oil drop should decrease. Goldsmith [3] has shown the wetting film drainage procedures through experiments and [4] has also indirectly observed the water film drainage procedure. Following the water flooding procedure, some of the oil drops may become trapped in the throat of a pore, causing other oil drops behind it to become trapped in perhaps a straight part of the pore. The longer the oil slugs sit in the reservoir, the water film surrounding the oil slug will start to drain and eventually the oil slug will be touching the pore wall. This will lead to an increase in the pressure drop required to mobilize the trapped oil. Former research members [4, 5] have experimentally studied how the water film affects the pressure drop without any vibratory stimulation and have found that a water film does indeed reduce the pressure drop needed to mobilize the oil slug. Gang [4] has also studied experimentally the effect of vibratory stimulation on oil slug mobilization and found that vibrations do indeed help lower the pressure drop. He attributed this to the formation of a thin water film with the help of vibrations. The water film may or may not reduce the pressure drop significantly but it definitely is a by-product of the application of vibrations to an oil slug as shown by previous research members. The formation of the water film must be understood in order to completely understand why vibrations increase oil production.
1.2 Objectives

The main objectives of this research are given in what follows. First, the effect of water film on oil slug movement, or mobilization, in a capillary tube, via water injection, needs to be analyzed with a numerical approach because it will reveal further details about the flow than can be observed through experiments. Numerical simulation also filters out unwanted effects of the external environment such as temperature and the impurity of the liquids used in the experiment. The mesh verification for the thin film is carried out via simulating core-annular flows and comparing the obtained results with the published experimental results. The effect of the changing shape of the oil slug during mobilization upon the pressure drop vs. time is explained and is compared with experimentally obtained pressure drop vs. time. Effects of different parameters such as oil-water interfacial tension, water viscosity, contact angle, *Reynolds number*, and *capillary number* upon the pressure drop vs. time are analyzed. Both the Reynolds number and capillary numbers are defined in Section 2.7. Previous research members have not analyzed their results in terms of dimensionless numbers and thus, it is far more difficult to quantify the results.

The effects of external vibrations upon oil slug mobilization are studied through numerical simulations. The tube is first vibrated, followed by a water injection, and then, the pressure drop vs. time is recorded and compared with a similar case without external vibrations to observe if the vibrations have a positive impact. In order to reveal the effects of vibrations upon the mobilization of an oil slug, a series of simulations are performed under various parameters, such as different types of external vibrations and frequencies and the amplitude of the vibrations. Three different types of vibrations are employed: sine wave, square wave, and saw tooth wave.

It is expected that the results of this study will shed more light upon the impact of water film, vibrations and other parameters on the pressure drop. The study will not only deepen the understanding of oil slug mobilization in a capillary tube but
also contribute to other applications in the field of microscale multiphase flow.

1.3 Literature review

The following sections outline the existing works in these different areas.

1.3.1 Core-annular flow

A very important immiscible multiphase flow case regarding a non-wetting liquid, displacing a wetting liquid which subsequently leaves behind a thin film of wetting liquid is taken into consideration. This type of flow commonly occurs in petroleum reservoirs and is known as “drainage” in the field of Petroleum Engineering. Another name for this type of flow is “core-annular flow”. This flow mimics what happens in an ideal oil reservoir. A pore of the reservoir maybe filled with oil and then, water flooding is commenced in order to recover the oil from the reservoir. Water will enter the pore and displace a certain amount of oil while leaving behind oil in the form of a thin film. Since a single pore is being studied rather than the entire reservoir, this study can be considered a pore scale study.

A similar case of non-wetting gas phase/bubble displacing a wetting liquid has been widely studied and a number of different equations and correlations have been proposed to calculate the wetting film thickness left behind. One of the first equations to predict the wetting film thickness was proposed as \( \delta = 0.5Ca^{1/2} \) by [6] and valid for capillary numbers \( 10^{-4} \) to \( 10^{-2} \). The equation was developed based upon plotting the experimentally observed values on a graph for a range of capillary numbers. Bretherton [7] proposed an equation in the form \( \delta = 1.3Ca^{2/3} \), derived theoretically and the equation was tested by conducting experiments for a range of capillary numbers. The equation is valid for the range of capillary numbers: \( 10^{-6} \) to \( 10^{-2} \). The author could not match his proposed equation with the experimental values for the
entire range of capillary numbers tested. Gas-liquid experiments, conducted by [8], predicted the wetting film thickness to be dependent upon the gas bubble length and contradicted the conclusion of [7]. Chang [9] has offered an explanation for the discrepancy between the experimental data and the theoretical predictions based upon the presence of trace amounts of surfactants, giving rise to Marangoni surface flows in the fluids used. The author still could not fully explain the experimental and theoretical discrepancy as noted by [10]. Gas displacing wetting liquid experiments has also been carried out by [11, 12]. It is noted by the author of [7] that for values of capillary numbers larger than $3 \times 10^{-3}$, the results of [6, 7, 11, 12] are in broad agreement with the film thickness equation proposed by [6]. Below the capillary number of $3 \times 10^{-3}$, there is a wide spread disagreement and different equations with which to predict the wetting film thickness are proposed. In experiments intended to measure wetting film thickness, as conducted by [13], the film thickness values are over predicted due to the measurement techniques employed by the author. Authors of [14] have used computational Fluid Dynamics to calculate the thickness of wetting film in a gas displacing liquid flow. The simulation results of this study were based upon the same assumption made by [7]. Therefore, the simulation cannot confirm the results of [7] as could an actual experiment. Experiments of [15], in vertical tubes, also found discrepancies between the experimental and theoretical values obtained from equations of [6, 7], respectively. The theoretical values under predict the film thickness value. The author of [16] has used the Finite element method to confirm the prediction of [14], wherein finite Reynolds number (inertial) effects have a minor effect upon wetting film thickness. The author used significantly higher Reynolds numbers than the previous author. Taha [17] studied a 3D slug flow in square cross sectional vertical tubes, using a volume of fluid [18] (VOF) method. They employed a commercial software Fluent to simulate a single rising Taylor bubble in a flowing liquid through a vertical tube. They concluded that the Taylor bubble shape was
dependent upon the liquid viscosity and surface tension but it was independent of the bubble length. The thickness of the film surrounding the bubble was found to decrease as the surface tension increased. Akbar [19] employed the VOF method to study Taylor flow in round capillary tubes. The results of their study show good agreement with the results obtained by [20]. The author also proposed a correlation for the frictional pressure drop having a better fit than the one by [20].

No equations or correlations exist to deal with the case of a non-wetting liquid displacing a wetting liquid. Experimental data is also very scarce and controversial. The equation proposed by [7] is used occasionally to calculate the film thickness for liquid-liquid flow. However, this is incorrect because the original equation derived by [7] was obtained with the assumption of gas displacing liquid. The authors of [3, 21, 22] have conducted experiments to measure the wetting film thickness in a liquid-liquid flow which can be used to validate numerical simulation results. Hodges [23] has proposed an asymptotic theory by using a boundary-integral method for the wetting film but does not offer any testable prediction for film thickness. The latest experiment regarding wetting film thickness in an immiscible liquid-liquid flow was conducted by [10]. The author has tabulated his experimental data in the form of a table and also included in the table are the experimental values of [3, 21, 22]. A testable wetting film thickness equation has been derived from his theory which can be used to fit the experimental data. They have predicted a lower capillary number film thickness behavior to be proportional to $Ca^2$ and a higher capillary number film thickness to approach a constant value of 0.2.

1.3.2 Oil recovery with vibratory stimulation

Various authors have completed experimental, numerical, and theoretical work on an oil drop ganglion inside capillary tubes. Graham [24, 25] has shown that non-sinusoidal waves can achieve greater ganglion mobilization in narrow constricted tubes. Vigil [26]
has shown by using numerical simulations that vibrations can help entrapped oil ganglia move through a constriction. Li [27] has conducted experiments using a 2D artificial porous media, constructed by etching a glass plate. Initially, their micromodel was saturated with trichloroethylene [27] (TCE) and water was injected to displace the TCE. They have found that for a fixed acceleration amplitude, TCE is displaced far more quickly as the vibration frequency decreases. As for a fixed vibration frequency, more TCE is recovered as the acceleration amplitude is increased. They have also shown that the mobilization of oil is proportional to the amplitude and inversely proportional to the frequency of the waves. Beresnev [28] has completed theoretical analysis and shown the oil ganglia stuck in the pore throat and requiring a small push, maybe mobilized by an external vibration. However, in natural porous media, the threshold needed for different oil ganglia varies with the pore’s geometry. Due to the irregular characteristics of the reservoir pore space, the thresholds are hardly predictable. Due to the random nature of the porous media, it is difficult to predict the quantity of oil that can be recovered. Iassonov [29] found from the theoretical model that vibrations can significantly decrease the value of the minimum pressure gradient required to mobilize an entrapped oil ganglion and under constant pressure gradients, vibrations also tend to increase the average flow rate of the entrapped oil ganglion. They have also noted that the effects of vibrations are most pronounced in the zones of relatively low pressure gradients, and become smaller if the external pressure gradients are high.

Averbakh [30] has studied the motion of a liquid droplet inside a capillary tube under the influence of a static force and an acoustic field. The numerical results state that vibrations attenuate the influence of the capillary force and the droplet motion in a capillary of constant cross-section starts at a lower value of the static force. Pride [31] have done pore-scale numerical simulations using the lattice-boltzmann method in two dimensions. They have found that seismic stimulation will mobilize the
trapped oil, when two dimensionless criteria are met. The first condition is the static-force requirement wherein a seismic wave pushes on a trapped oil bubble, and the radius of curvature of the downstream meniscus of the bubble is reduced sufficiently to flow through the pore-throat constriction that is blocking its downstream progress. The second condition is the dynamic requirement that in a cycle of the time-harmonic stimulation, the meniscus must have enough time to advance through the constriction before the seismic force changes direction and begins to push the meniscus upstream.

Gang [4] has applied vibrations to an oil slug inside a capillary tube surrounded by water. He found that by fixing all the parameters, the pressure needed to mobilize the oil slug is reduced by increasing the frequency of the vibrations. The pressure is also found to decrease as the duration of vibratory stimulation is increased.

Oil recovery studies via vibrations on core samples have also been investigated by various authors. Kouznetsov [32] have studied the impact of vibro-energy on oil production at both laboratory and field scale. Their research found that in the presence of vibro-seismic sources, the rate of oil displaced by water is increased, and the amount of oil remaining decreases. They have suggested that the reason for increased oil productions is due to a reduction in oil/water interfacial tensions as well as an increase in the relative permeability of oil. Nikolaevskii [33] has attributed the positive effect of vibration upon the restoration of permeability for the dispersed oil phase. The restoration occurs by either drop clusterization or breakdown. The author also suggests the generation of high frequency ultrasonic waves by long-short-wave resonance may lead to the redistribution of oil and water in the pore space. Another conclusion reached by the author is that when surface-based vibrators are employed, the majority of the energy is used in generating waves that do not reach the oil bearing porous media. Naderi [34] has conducted various experiments to observe the effect of ultrasound on oil recovery. The author has employed various porous media models such as the core samples, 2D sand pack models and micro-
models. The core experiments showed that increasing ultrasonic radiation leads to further oil recovery by capillary imbibition. It was also observed that the effect of ultrasound on oil recovery is less significant for higher viscosity oil. The effect of ultrasound is more dominant for oil wet cases than for water-wet cases. The presence of initial water saturation in oil wet experiments also leads to higher oil recovery. The author’s 2D sand pack models have shown, through visualization, that the effect of ultrasound energy is more pronounced at lower injection rates. This is because as the injection rate is lowered, capillary forces become more dominant over the viscous forces. Micro-models experiments revealed similar results to the 2D sand pack models. The ultrasound was found to be more effective for lower injection rates, while for high injection rates, the ultrasound started to diminish and became less effective.

The use of seismic vibration to recover oil has been proposed as a low-cost method, with reports of several successful field results \[35\]. The key difficulty with the method is that the exact mechanism for recovery of oil is unknown, and consequently, it is difficult to predict the performance and design of the oil recovery project with vibrations \[35\]. Field experiments have been carried out employing actual earthquakes and a seismic vibrators to study the effects of vibrations upon oil productions by \[36\]. They found that vibrations have a positive impact upon oil recovery and oil production is increased near the residual oil saturation. Westermark \[37\] has also conducted field and lab experiments and found that downhole stimulation is limited in its effectiveness in oil recovery and surface vibro-seismic stimulation have the most success. According to the authors, lab tests have demonstrated different effects due to vibration on the flow of multi-phase fluids through porous media, although, it is still not clear which fluid and rock parameters are affected by the vibrations. An excellent literature review regarding the latest research in the field of oil recovery by seismic vibrations is given by \[35\]. The authors have also included a literature review on how to effectively deliver vibrational energy to a reservoir in order to increase oil
production.

1.3.3 Multiphase flow pressure drop

A pressure drop for multiphase flow is much harder to estimate than single phase flow due to the inherent nature and complexity of theoretical analysis of multiphase flow. Various correlations exist depending upon the nature of the multiphase flow such as liquid-liquid or liquid-gas flow. A multiphase flow pressure drop can be significantly different than a single phase pressure drop due to factors such as capillary pressure, number of dispersed liquid drops, and the three phase contact line. This research is focused upon the maximum pressure drop needed to mobilize an oil slug in a capillary tube. Thus, it is necessary to review the literature and attain the knowledge of what type of work has been done in the field of pressure drop for multiphase flow. A former research group member [4] has studied this problem. The author used a capillary tube filled with water and then placed an oil drop inside the tube and injected more water at a constant flow rate using a syringe pump. The author measured and recorded inlet and outlet pressure differential for the tube vs. time. The pressure vs. time graph showed three distinctive regions that were named: Stationary stage, Partially mobilized stage, and Completely mobilized stage. The author also noticed that the maximum pressure required to mobilize the oil slug increased the longer the oil slug was kept in the capillary tube. This was attributed to the fact that initially there is a thin water film between the oil slug and the capillary tube wall. After the oil slug is placed in the tube and a certain period of the time passes, the thin water film surrounding the oil slug begins to drain. The longer the oil slug is maintained in the tube, the thinner the water film will become. Drainage of the water film will increase the pressure drop because of the reduction of lower viscosity water between the tube wall and the oil slug. Dai [5] has shown that when an oil slug in a circular capillary tube, surrounded by water, is mobilized by constant water injection, the
An oil slug will experience three types of pressure drop: build-up, hold-up, and steady. According to their pressure vs. time graphs, it was noticed that hold-up differential pressure is greater than subsequent steady differential pressure. The pressure drop for an oil drop in water at steady state is much higher than a poiseuille pressure drop calculated by estimating the flow made up of chunks of water, oil, and water. Their study also showed the smaller the oil slug, the more pronounced the pressure drop, as compared to poiseuille pressure drop.

Adzima [38] has studied the flow of water drops in oil in rectangular channels. The author has found that the addition of water drops causes a substantially higher pressure drop than the flow of single phase oil at the same flow rate. This pressure drop is increased despite the fact that the viscosity of water drops is less than the viscosity of oil. The reasons behind this could be additional viscous stresses due to the modification of the velocity profile by water drops, or capillary pressure due to the drops and Marangoni stresses. They have also noted increases in pressure drop as the size of the drops are increased. Fuerstman [39] have derived a theoretical equation to predict the pressure drop between the inlet and outlet of a rectangular microchannel through which a flowing liquid carries bubbles. They have derived the total pressure drop across the channel by adding the pressure drop for regions which contain no bubbles, pressure drop across the gutter (the areas bounded by the curved body of the bubble and the corners of the channel), and pressure drop across the endcaps of the bubble. It must be noted that they have considered the bubbles to be inviscid and disregarded the small pressure drop caused by the gaseous bubble. According to their experiments, they conclude that pressure declines most rapidly in the region containing the endcaps and the number of bubbles in the channel influences the pressure drop most significantly. An electrical wheatstone bridge analogy is used by [40] to study two immiscible fluids through a network of microchannels.
flow inside the microchannel network is very sensitive to small differences in fluid resistance between various branches of network. The fluid resistance depends upon the viscosity ratio, processing conditions, and geometric resistance parameters of the various microchannels. Authors that have tried to obtain a theoretical equation for a multiphase flow pressure drop in a tube/channel have often used the pressure drop equation of [7] to account for a pressure drop due to bubble caps. This is because the pressure drop equation proposed by [7] estimated the pressure drop due to the bubble caps for an inviscid bubble. A single moving droplet in a rectangular microchannel is studied by [41] to observe the effects of drop size, droplet viscosity, and capillary numbers. The study found that as far as small droplets are concerned, excess pressure drop is independent of drop size and capillary numbers and is a weak function of the viscosity ratio of the two phases. As for large drops, where the droplet viscosity is less than the viscosity of the surrounding fluid, the pressure drop decreases with an increase in drop size and capillary numbers. The author has not considered case in which the droplet viscosity is higher than the viscosity of the surrounding liquid.

Various authors have attempted to discover equations to predict a pressure drop for multiphase gas-liquid flow in circular capillary tubes. Warnier [42] has improved upon a semiempirical equation of a previous author to predict a pressure drop across the inlet and outlet of a capillary tube. The pressure drop in the capillary tube would be due to the single phase slugs and the shape of the gas bubbles. Comparing the model predictions to the experiments, it is observed that their model generates improved pressure drop predictions than that of previous authors. Walsh [43] has come up with an empirical relationship by curve fitting their experimental data. The empirical relationship is in the form of a superposition of single phase poiseuille flow representing the single phase liquid slug flow and interfacial flow representing the bubble meniscus deformation. Walsh [43] that if a certain parameter in their paper is greater than 0.1, then, the effect of the slug length is not significant and the pressure
drop in the flow is dominated by the poiseuille flow. On the other hand, if it is less than 0.1, then the effect of the slug is significant and the pressure drop in the flow is dominated by the Taylor flow. Lee [44] has studied the pressure drop of two-phase plug flows in capillary tubes of different materials. The fluids used in the experiments are air and water. The focus of their experiments is to observe the difference between the pressure drop for wet-plug flow and dry-plug flow. Wet-plug flow occurs when the tube is hydrophilic and dry-plug flow occurs when the tube is hydrophobic. The pressure drop for a wet-plug flow increases with an increase in the superficial velocity of gas and liquid due to an increase in the frictional pressure drop. The pressure drop for a dry-plug flow increases with an increase in the superficial liquid velocity or a decrease in superficial gas velocity. This increase is due to an increase in the number of moving contact lines. Kreutzer [20] studied inertial and interfacial effects upon a pressure drop in gas-liquid two-phase Taylor flow using commercial computational fluid dynamics (CFD) software FIDAP. They found that at a high Reynolds number, fluid inertia has a significant impact upon the pressure drop. An excellent review of the dynamics of microfluidic droplets in square microchannels is conducted by [45]. The author has focused upon a review of formation, transport, and the merging of drops in microfluidics.

It can be observed that all pressure drop correlations and experiments in the literature focus on the case of a gas bubble moving with a surrounding liquid film. Certain authors have studied the pressure drop in accordance where there is a moving contact line but that is unrelated to our research. Dr. Liming Dai’s group has focused upon studying the case of a single oil slug initially touching the wall as the water film begins to develop. The basic reason for studying this type of flow is unrecovered oil, via water flooding, may have been left behind in the form of residual oil drops. Several oil drops may have formed complete water films around them and others may have formed a partial water film around them during the water flooding phase. Gang [4]
has shown experimental evidence of such water film drainage and has also shown that the pressure required to mobilize the oil slugs becomes lower in the presence of the water film. The author has failed to observe the effects of changing the interfacial tension, water viscosity, and velocity while keeping other parameters fixed. At the moment, it is difficult to find studies which look at predicting a pressure drop in a liquid-liquid slug flow.
2. NUMERICAL MODEL ESTABLISHMENT

2.1 Introduction

Water film development and drainage has been proposed by past researchers from our group via an indirect means of experimental pressure measurement. Gang [4] has shown several basic simulations to display water film development and drainage. More remains to be researched and the simulations in this thesis are focused at filling the gaps.

As mentioned in Chapter [1], the purpose of this research is to numerically study the development of a wetting film thickness as in the case of a core-annular flow and a water film in the case of a single oil drop flow. The effect of external excitation of water film formation around the oil drop is another area investigated in this thesis. To understand the process of mobilization of an oil drop, pressure drop across the tube, sensitivity analysis, and pictures of the change in the oil drop must to be thoroughly analyzed. The pressure drop is the only reliable piece of measurement obtained from the experiments. The pictures from the experiments fail to show the thin water film due to its very small thickness in order of microns.

This is where the use of numerical simulations to compliment the experimental results become a powerful tool. An extremely thin water film can be easily viewed
in simulations without having to employ expensive cameras and visualization equipment. The entire flow can be viewed at different instances in time and thus, help us understand, in depth, the process of oil slug mobilization. Numerical simulations are also powerful tools when it comes to sensitivity analysis in which one parameter varies while fixing all other parameters. Repeating the same thing in experiments would be extremely difficult because it is very difficult to obtain liquid samples that only possess a different viscosity or density while other parameters remain the same. Variation in parameters such as temperature, tube diameter, and purity of fluids maybe difficult to control within the experiments and they can inadvertently contaminate the results. In numerical simulations, all the above mentioned variations can be eliminated and the results would then represent ideal test conditions.

Multiphase flow is inherently computationally expensive and the utilization of a powerful computer is needed to obtain good quantity results in a sound and timely manner. The advancement of computer technologies such as parallel computing and efficient codes has made it more possible than ever before to simulate multiphase flows.

Very popular and powerful CFD software known as ANSYS Fluent is used in this thesis research. Fluent can simulate a wide variety of flows including turbulence, multiphase flow, heat transfer, and acoustics. The Fluent code has been verified by years of industry and academic research and test cases. Taha [17] and [19] have employed this software in their multiphase gas-liquid studies. Preprocessing of the simulation is conducted using software known as ANSYS Workbench. Preprocessing of the simulation consists of generating a high quality mesh because the accuracy of the results is dependent upon the quality of the mesh. Post processing can be completed in Fluent but ANSYS CFD-Post has been chosen for this purpose. CFD-Post has far more post processing capability than Fluent. Post processing includes generating graphs, images at some instances in time, and animation clips.
2.2 Volume of fluid method

Various multiphase models are available in Fluent such as Eulerian, Mixture, and VOF. Choosing the correct model is critical to obtain accurate results in the least amount of time. Since two immiscible liquids are being employed and there is interest in the interface between the two liquids with surface tensions effects, the very popular VOF model is chosen according to [46]. The VOF method has been developed to track the fluid-fluid interface. The movement of the interface between the two fluids is tracked based upon the volume fraction $c_1$. The value of $c_1$ varies between zero and one. $c_1$ is zero for cells in which there is no fluid1 and $C_1$ is one for cells in which there is only fluid 1 [18]. Cells containing both fluid 1 and fluid 2 would contain the free surface and have $c_1$ values between zero and one. The VOF method requires only one set of mass and momentum equations for all fluids, thus, making the computation simpler. All equations employed in the VOF method are shown below. The VOF method is used to track the interface between two immiscible liquids so that a wetting film thickness can be measured and compared to published results.

The continuity/conservation of mass is given by

$$\nabla \cdot \mathbf{u} = 0,$$

(2.1)

while the momentum/Navier-Stokes equations are defined as

$$\rho \left( \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} \right) = -\nabla p + \nabla \cdot (\mu \mathbf{S}) + \mathbf{F},$$

(2.2)

$$S_{ij} = 0.5 \left( \frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j} \right),$$

(2.3)
and, finally, the volume fraction is given by

\[
\frac{\partial C}{\partial t} + u \cdot \nabla c = 0,
\]

(2.4)

\[
\rho = c\rho_1 + (1 - c)\rho_2,
\]

(2.5)

\[
\mu = c\mu_1 + (1 - c)\mu_2,
\]

(2.6)

where \(u\) denotes the velocity field, \(p\) is the pressure, \(\rho\) is the density, \(\mu\) is the dynamic viscosity, \(c\) is the volume fraction field, \(S\) is the rate of strain tensor, and \(F\) is the source term, which in this case, would be the interfacial tensions force.

Several of the major assumptions used for the simulations in this thesis are that the Laminar flow, incompressible liquid, and the gravity effects are negligible. The gravity effects are negligible because the small diameter tube employed in the simulation leads to a small bond number [10]. The governing equations of this flow are the conservation of mass and momentum, along with the volume fraction equation as written below from [47].

\section{2.3 Geometry and meshing}

The experimental work being done by our research group utilizes a straight capillary tube and for that reason, a straight capillary tube is also used in the simulations of the present research. The geometry used in all the simulations in this thesis is a 5 mm long tube with an inner radius of 0.25 mm, as shown in Figure 2.1 (page 20). The reason for choosing a small length and diameter is because a typical microflow is studied in the tubes with a diameter in the micrometer range [48]. Another reason is that a short tube with small diameter requires less meshing and thus less computational power.
is required. This geometry is meshed using ANSYS-Workbench. The cylindrical capillary tube is modeled by a two-dimensional axisymmetric option in Fluent which will result in Fluent solving the governing equations in the cylindrical coordinates \[46\]. Use of the axisymmetric option enables the employment and meshing of only half of the actual flow domain. This results in a mesh with fewer elements and saves computation time. It is a known fact in CFD that meshing in the region of interest should be finer than other region to obtain an accurate solution. Following those guidelines, two different meshes were used in the beginning and the most accurate out of the two meshes was used for rest of the research. Both meshes are made finer near the wall to capture the thin film which is the focus of this research. Mesh 1 has 102000 quadrilateral elements and Mesh 2 has 136000 quadrilateral elements. Mesh 2 is finer than Mesh 1 near the wall. Thin film flow experiments of \[10\] are chosen for comparing the results of the two different meshes. Simulations at two different capillary numbers are run with Mesh 1 and Mesh 2 and the results are compared with experiments results of Table III in \[10\] as shown in Table 2.1. It can be seen that the film thickness obtained with Mesh 2 is much closer to the experimental value than the film thickness obtained with Mesh 1. It will be seen in chapter 3 that Mesh 2 indeed generates accurate results and the results match well with the experimental values. For Mesh 2, as shown in Figure 2.2 the 0.01 mm of length near the wall is meshed finer and the rest 0.24 mm length away from the wall is meshed coarse. The quadrilateral elements in the fine region has a width of 1 \(\mu\)m and a length of 2.9412 \(\mu\)m. The quadrilateral elements in the coarse region has a width of 4.8 \(\mu\)m and a length of 2.9412 \(\mu\)m. Furthermore, the quality of the mesh was judged by three different metrics in the Ansys Workbench: element quality, skewness, and orthogonal quality. It is ensured that the five element across the film rule by \[49\] is satisfied for both meshes. Another important assumption for all the simulations is that the tube is assumed to be a rigid body and no deformation is considered.
<table>
<thead>
<tr>
<th></th>
<th>Ca</th>
<th>Re</th>
<th>Experimental value</th>
<th>Simulation value</th>
<th>Relative error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mesh 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S 1</td>
<td>0.00096</td>
<td>49.86</td>
<td>0.023</td>
<td>0.0142</td>
<td>38.26</td>
</tr>
<tr>
<td>S 2</td>
<td>0.26</td>
<td>0.94</td>
<td>0.30</td>
<td>0.2551</td>
<td>14.97</td>
</tr>
<tr>
<td>Mesh 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S 1</td>
<td>0.00096</td>
<td>49.86</td>
<td>0.023</td>
<td>0.019</td>
<td>17.39</td>
</tr>
<tr>
<td>S 2</td>
<td>0.26</td>
<td>0.94</td>
<td>0.30</td>
<td>0.2610</td>
<td>13</td>
</tr>
</tbody>
</table>

Table 2.1: Grid test for two different meshes to determine the most accurate mesh, where S is simulation.

Figure 2.1: The dimensions of the tube employed in the simulation.

Figure 2.2: Longitudinal view of Mesh 2 with fine mesh near the wall to capture the thin films accurately.
2.4 Solver options

The multiphase volume-of-fluid model in Fluent 13 is selected for the simulation of the multiphase flow. The implicit body force option is turned on to stabilize the solution, as recommended in Fluent 13 user’s guide \[46\]. A first order, Non-iterative time advancement method is selected for the transient formulation because it saves computational time according to Fluent 13.0 user’s guide \[46\]. The other solution methods were chosen according to \[49\]. Pressure-velocity coupling is obtained by employing the PISO method. Green-Gauss node based is chosen for the gradient. PRESTO! is used for pressure discretization. As for the momentum equation, QUICK is chosen and for the volume fraction equation, explicit Geo-Reconstruct is chosen. The explicit Geo-Reconstruct scheme works by using a piecewise-linear approach to represent the interface \[50\]. The pressure and Momentum correction tolerances for Non-iterative solver controls are set to 0.01 and 0.001, respectively. It was ensured that the absolute values of residuals achieved were sufficiently low $O(10^{-7})$ for $x$ and $y$ velocities and $O(10^{-10})$ for continuity. Fluent employs the continuum surface force \[51\] (CSF) model to model the surface tension effects.

The Courant number \[46\] (Co), which is important for stability of the fluid flow, is set to 0.25. The time-step in the simulation is then determined by using the variable time stepping method determined on the set courant number, the mesh density and the cell velocity. The Courant number is given as

$$Co = V_{\text{fluid}} \cdot \left( \frac{\Delta t}{\Delta x} \right), \quad (2.7)$$

where $\Delta x$ is the element size, $\Delta t$ is the time step, and $V_{\text{fluid}}$ is the fluid velocity in the element. The VOF courant number for the VOF equation and the global courant number for the remaining transport equations are both set to 0.25.
2.5 Boundary and initial conditions for different simulations

The meshing and solver options for the three different simulations conducted in this paper are the same but they have different boundary conditions. A detailed explanation and treatment of the boundary conditions for each case is presented in the subsequent subsections.

2.5.1 Boundary and initial conditions for core-annular flow

As observed in [Figure 2.3] the left end of the tube has a velocity-inlet condition where a fixed velocity is applied. The right end has an applied pressure-outlet condition and a gauge pressure of 0 Pa is set, meaning atmospheric pressure. At the wall, there is an applied no slip condition. The contact angle for all simulations of core-annular flow is set to 180°. In core-annular flow, the initial condition is that the tube is initially filled with one liquid (viscous wetting liquid) and the second liquid (less viscous non-wetting liquid) is injected to displace the initial liquid. To do this in Fluent, the volume fraction of the entire tube is patched such that Liquid 1 initially occupies the tube. Liquid 2 is injected from the inlet and it displaces Liquid 1 from the tube. A certain amount of Liquid 1 is left behind in the tube in the form of a thin film which is a point of interest in this study. Simulations are run until Liquid 2 invades Liquid 1 such that there is a constant film thickness, several millimeters behind the meniscus, separating the two liquids, as shown in [Figure 2.4].

2.5.2 Boundary and initial conditions for single oil slug flow

The boundary conditions are a velocity inlet for the inlet, pressure outlet for the outlet, and a non-slip condition at the wall. The contact angle for all the simulations is set at 180°, unless stated otherwise. The main difference between setting up this
Figure 2.3: The geometry used in the simulations along with the applied boundary conditions. The figure depicts only half of the whole tube.

Figure 2.4: The geometry of the core-annular flow.
Figure 2.5: The initial shape of the oil drop inside the capillary tube.

type of flow, compared to a core-annular flow, is in patching the volume fraction. In experiments of this type of flow, the tube is filled with water and then, an oil slug is injected into the tube. In the simulation, the initial condition is defined by patching an oil slug volume fraction, with a shape as shown in Figure 2.5 without any water film surrounding it. The actual shape of the patched oil volume fraction region could be anything. If we patch it in the shape of a square and run the simulation without any velocity, Fluent will automatically simulate the realistic shape of the oil slug based upon the interfacial tension and contact angle inputs. The inlet velocity will inject pure water to displace the oil slug from the capillary tube. In the case of a simulation that starts without any water film surrounding the oil slug, the simulation is stopped when the entire oil slug becomes covered by the water film. Upon reaching this state, no further change in the oil shape takes place.
2.5.3 Boundary and initial conditions for single oil slug flow with external excitation

The boundary conditions are velocity inlet for the inlet, pressure outlet for the outlet, and a no slip boundary condition at the wall. The contact angle for all the simulations is set at 180°. For the initial condition, an oil slug volume fraction is patched inside the capillary tube without a surrounding water film as done in Section 2.5.2. External excitation such as a sine wave, a square wave, and a saw tooth wave are applied as the moving wall boundary condition by using a user defined function (UDF) in Fluent. No water is injected during the period in which external vibrations are turned on. The vibrations are stopped after the oil drop is vibrated for about three periods and then the water is injected through the tube inlet. It is found that the duration of vibration has no impact on the water film formation and pressure drop profile. Hence, it is safe to stop the vibrations after about three periods or earlier. The oil slug should be vibrated for at least two periods so to make sure proper water film formation. It is not possible to give the real time in seconds for the completion of three periods due to the computational speed being dependent on the machine being employed. The pressure difference between the inlet and outlet of the tube, as a function of time, are recorded and compared to the simulations in which no external vibrations are applied. The amplitude and frequency of the external vibrations are varied by changing the actual values in the UDF code.

2.6 Cluster specifications

Multiphase simulations are inherently computationally expensive. To speed up the computations, a high speed cluster has been employed. The cluster has 16 compute nodes. Each node has two processors with 4 cores each. Each compute node has Dell PowerEdge M610 blades with 2 Intel E5540 CPU’s (processors) running at 2.5GHz.
For this research, 8 parallel processing licenses were purchased from ANSYS to be able to use the cluster.

### 2.7 Dimensionless numbers

Dimensionless parameters are used in this thesis to help quantify the effects of difference parameters such as velocity, viscosity, interfacial tensions, tube diameter, and density. The four main dimensionless parameters used are the capillary number, the Reynolds number, the Weber number, and the viscosity ratio. The capillary number, Reynolds number, and Weber number are based upon the properties of the wetting liquid. The capillary number, Reynolds number, and Weber number are connected with each other and two of the numbers should be sufficient to determine the third number. This means only two of the numbers can be varied independently. The viscosity ratio is defined as the viscosity of the annular liquid, divided by the viscosity of the core liquid, which is surrounded by the annular liquid. These equations are defined below. The capillary number \[52\] (Ca) is given as

\[
Ca = V_{\text{inlet}} \cdot \frac{\mu_{\text{wetting liquid}}}{\sigma_{(\text{oil-water})}}, \quad (2.8)
\]

where \(V_{\text{inlet}}\) is the velocity at the inlet of the tube, \(\mu_{\text{wetting liquid}}\) is the viscosity of the wetting liquid (occupies the annulus of the tube), and \(\sigma_{(\text{oil-water})}\) is the oil-water interfacial tension, while the Reynolds number \[52\] (Re) is given as

\[
Re = \frac{\rho_{\text{wetting liquid}} \cdot V_{\text{inlet}} \cdot D}{\mu_{\text{wetting liquid}}}, \quad (2.9)
\]
where \( \rho_{\text{wetting liquid}} \) is the density of the wetting liquid, and \( D \) is the diameter of the tube. The Weber number \( \text{53} \) (We) is given as

\[
\text{We} = \frac{\rho_{\text{wetting liquid}} \cdot V_{\text{inlet}}^2 \cdot D}{\sigma_{\text{oil-water}}} = \text{Ca} \cdot \text{Re}.
\] (2.10)

Finally, the viscosity ratio \( \text{10} \) (\( M \)) is given as

\[
M = \frac{\mu_{\text{wetting liquid}}}{\mu_{\text{non-wetting liquid}}},
\] (2.11)

where \( \mu_{\text{non-wetting liquid}} \) is the viscosity of the non-wetting liquid (occupies the core of the tube).

The VOF method in Fluent diverges and produces erroneous results when parameters outside its capability range are chosen. An example of a situation that would lead to divergence would be when different liquids with significant differences in their property, are employed in the simulation. The parameters in this thesis such as Reynolds number, capillary number, viscosity ratio, velocity, and interfacial tension are chosen such that to avoid the divergence problem. An appropriate and fine meshing is required to capture the correct thickness of the interface. Too coarse of a mesh leads to a smudged interface, whereas too fine of a mesh leads to extreme computation times. In this thesis, a good balanced mesh is obtained by conducting a grid-independence study.

\[\text{2.8 Conclusion}\]

The present chapter discusses details that are necessary to understand and form a backbone for the rest of the simulations discussed in this thesis. The reason for choosing the VOF method, and how the method works, is briefly presented. The geometry and meshing used in all the simulations in this thesis are shown. The
solver options that are selected inside the Fluent, to obtain accurate results, are also mentioned. Boundary conditions for different simulations, along with the setup of each simulation are presented in separate subsections. Finally, the definitions of important dimensionless numbers such as Reynolds number, capillary number, and viscosity ratio are given.
3. NUMERICAL SIMULATION OF CORE ANNULAR FLOW

3.1 Introduction

A very important case of an immiscible two phase flow of a non-wetting liquid, displacing a wetting liquid, which subsequently leaves behind a thin film of wetting liquid, is analyzed in this chapter. This type of flow commonly occurs in petroleum reservoirs and it is known as drainage in the realm of Petroleum Engineering. Another name for this type of flow is core-annular flow. This flow mimics what happens in an ideal oil reservoir. A pore of the reservoir maybe filled with oil and then water flooding is commenced to recover the oil from the reservoir. The water will enter the pore and displace an amount of oil while leaving behind some oil in the form of a thin film. Since a single pore is being studied rather than the entire reservoir, this study can be considered a pore scale study. A small diameter cylindrical tube is used to simulate the displacement of one liquid by a second immiscible liquid. The displaced liquid leaves a thin film behind just as an oil film would be left behind in the case of an oil recovery by water flooding. The simulations are conducted to match the wetting film thickness obtained from using the volume-of-fluid method and comparing them to the latest experimentally published values by [10]. As mentioned in Chapter [1], theoretical equations do not exist for this type of flow and hence, simulation and
experiments are the only way to obtain reliable and sound results. This study also helps establish a benchmark mesh for subsequent research due to similar types of flow conditions, including the presence of a wetting film.

In this portion of the research, the impact of the capillary number, Reynolds number, and viscosity ratio on the residual wetting film thickness is studied.

### 3.2 Immiscible fluid interface

It is a known fact that the interface between two immiscible liquids is very thin and on the order of an angstrom \[54\]. Also, no gradual change exists from one liquid to the other. The volume fraction changes of the liquids are sudden at the interface. The VOF method cannot capture the sharp interface and hence, the resulting interface will appear smeared rather than sharp. The smeared interface is very thin but not as thin as it would be in reality. This is why an upper and lower bound for the wetting film thickness is reported in Table 3.2 (page 34). The wetting film thickness can be anywhere between the upper and lower bound because of the smearing effect. The smearing effect is shown in Figure 3.1. The lower and upper bound can be averaged and reported as one value and this approach is used in Table 3.3 (page 36) and Table 3.4 (page 38).
3.3 Capillary number and film thickness

The wetting film thickness is proportional to the tube radius as observed from Brether-\text{ton’s equation in [7]. In order to match experimental results which may employ tubes of different diameters with simulations, a normalized film thickness is used. The normalized film is dimensionless and is obtained by dividing the actual film thickness, obtained from the simulation, by the radius of the tube employed in the simulation. The normalized film thickness will be denoted by the symbol $\delta$. Despite Brehter-\text{ton’s Equation not being valid for the entire range of capillary numbers as proposed by the author, it is generally believed to be accurate in the capillary range of $10^{-5}$.
to $10^{-2}$ \[10\]. The equation, specifically Bretherton’s equation, is stated below, once again, for convenience, namely

$$
\delta = 1.3Ca^{2/3}.
$$

Table 3.1: Properties of the two immiscible liquids used in the simulations.

<table>
<thead>
<tr>
<th>Liquid</th>
<th>Density (kg/m$^3$)</th>
<th>Viscosity (kg/(m · s))</th>
<th>Surface Tension (N/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>998.2</td>
<td>0.001003</td>
<td>0.003375 – 0.1215</td>
</tr>
<tr>
<td>Oil</td>
<td>860</td>
<td>0.04 – 0.148</td>
<td>0.003375 – 0.1215</td>
</tr>
</tbody>
</table>

In the first case, a wetting film thickness, at a given capillary number is of interest. The capillary number and Reynolds number are calculated based upon the properties of the wetting liquid, which in this case is oil. A small inlet velocity can create a long computation time with which to obtain the solution because the flow velocity is very slow. Large inlet velocity can lead to a high courant number, resulting in a smaller time step and longer computation time. Hence, water properties are held constant and water-oil interfacial tension and oil viscosity is adjusted in order to obtain the proper inlet velocity, as shown in Table 3.1. These calculations are done based upon the equation for the capillary number. The capillary number is fixed because it is the input, the oil viscosity and water-oil interfacial tension are chosen to represent different types of oil, and velocity is the unknown parameter that is being calculated (output). If the velocity is too small or too big then the oil viscosity and water-oil interfacial tension are changed and velocity calculated once again. This is done until a correct velocity is found for that simulation. Proper care is taken to ensure that the Reynolds number is kept small ($< 2$ for all cases) to avoid any inertial effect affecting the film thickness. The viscosity ratio is kept constant in all cases to ensure that it does not affect the wetting film thickness. Flow is simulated at various capillary numbers and constant viscosity ratios to verify its effect upon the residual wetting film thickness. The mesh is chosen according to \[49\] so at least five elements exist.
across the wetting film. The range of capillary numbers simulated is from 0.00096 to 2. The studied flow geometry is shown in Figure 2.4 (page 23).

The simulation results are summarized in Table 3.2. The experimental results, originally taken from four different papers, have been tabulated by [10] has tabulated into one giant table. Table 3.2 shows the lower and upper bound film thickness obtained from the simulations for each capillary number and compared them to the published experimental results and to Equation 3.1 for small capillary numbers. Different papers give film thicknesses for different values of capillary numbers and occasionally more than one paper has reported a film thickness for the same capillary number. Entries with dashes indicate unavailable data for that capillary number. Table 3.2 results are depicted in a graphical form in Figure 3.2. According to Figure 3.2, simulation and experimental results are quite close to each other.

Figure 3.2: Ca vs. normalized film thickness. Comparison of the film thickness values from the simulation with that of published experimental values for the capillary number range of 0.00096 to 2.
<table>
<thead>
<tr>
<th>Ca</th>
<th>LB (δ)</th>
<th>UB (δ)</th>
<th>21 (δ)</th>
<th>22 (δ)</th>
<th>3 (δ)</th>
<th>10 (δ)</th>
<th>Equation 3.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00096</td>
<td>0.0145</td>
<td>0.0183</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>0.023</td>
<td>0.013</td>
</tr>
<tr>
<td>0.01</td>
<td>0.044</td>
<td>0.076</td>
<td>0.059</td>
<td>—</td>
<td>—</td>
<td>0.061</td>
<td>0.062</td>
</tr>
<tr>
<td>0.022</td>
<td>0.078</td>
<td>0.112</td>
<td>0.067</td>
<td>0.088</td>
<td>—</td>
<td>—</td>
<td></td>
</tr>
<tr>
<td>0.033</td>
<td>0.100</td>
<td>0.131</td>
<td>—</td>
<td>—</td>
<td>0.093</td>
<td>—</td>
<td></td>
</tr>
<tr>
<td>0.048</td>
<td>0.121</td>
<td>0.158</td>
<td>—</td>
<td>0.130</td>
<td>—</td>
<td>—</td>
<td></td>
</tr>
<tr>
<td>0.05</td>
<td>0.123</td>
<td>0.164</td>
<td>—</td>
<td>0.120</td>
<td>—</td>
<td>—</td>
<td></td>
</tr>
<tr>
<td>0.072</td>
<td>0.151</td>
<td>0.188</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>0.130</td>
<td>—</td>
</tr>
<tr>
<td>0.075</td>
<td>0.158</td>
<td>0.189</td>
<td>—</td>
<td>0.150</td>
<td>—</td>
<td>—</td>
<td></td>
</tr>
<tr>
<td>0.082</td>
<td>0.160</td>
<td>0.200</td>
<td>—</td>
<td>0.170</td>
<td>0.140</td>
<td>—</td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>0.179</td>
<td>0.212</td>
<td>—</td>
<td>0.170</td>
<td>—</td>
<td>—</td>
<td></td>
</tr>
<tr>
<td>0.17</td>
<td>0.220</td>
<td>0.262</td>
<td>—</td>
<td>0.210</td>
<td>—</td>
<td>—</td>
<td></td>
</tr>
<tr>
<td>0.19</td>
<td>0.234</td>
<td>0.266</td>
<td>—</td>
<td>0.240</td>
<td>—</td>
<td>—</td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>0.236</td>
<td>0.269</td>
<td>—</td>
<td>0.240</td>
<td>—</td>
<td>0.210</td>
<td>—</td>
</tr>
<tr>
<td>0.23</td>
<td>0.245</td>
<td>0.284</td>
<td>—</td>
<td>0.230</td>
<td>—</td>
<td>—</td>
<td></td>
</tr>
<tr>
<td>0.25</td>
<td>0.254</td>
<td>0.286</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>0.210</td>
<td>—</td>
</tr>
<tr>
<td>0.26</td>
<td>0.255</td>
<td>0.289</td>
<td>—</td>
<td>0.300</td>
<td>—</td>
<td>—</td>
<td></td>
</tr>
<tr>
<td>0.29</td>
<td>0.260</td>
<td>0.302</td>
<td>—</td>
<td>0.240</td>
<td>—</td>
<td>—</td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>0.295</td>
<td>0.336</td>
<td>—</td>
<td>0.270</td>
<td>—</td>
<td>—</td>
<td></td>
</tr>
<tr>
<td>0.88</td>
<td>0.323</td>
<td>0.360</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>0.270</td>
<td>—</td>
</tr>
<tr>
<td>1</td>
<td>0.331</td>
<td>0.363</td>
<td>—</td>
<td>0.320</td>
<td>—</td>
<td>0.270</td>
<td>—</td>
</tr>
<tr>
<td>2</td>
<td>0.351</td>
<td>0.382</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>

Table 3.2: Comparison of the film thickness values from the simulation with that of published experimental values. A dash is used to represent no value in that paper at that capillary number, where LB is the lower bound, UB is the upper bound, and δ is the normalized film thickness.
Experimental values from two different researches can differ somewhat, perhaps due to measurement errors. This difference can be depicted by the purple and orange dots representing reference [22] and [10], respectively in Figure 3.2 (page 33). Our simulation results appear to match more closely to reference [22] data than reference [10].

The authors of [10] have proposed from their theory that as capillary numbers become larger, the film thickness will level off at a constant value of $\delta = 0.2$. Their experiments have found a leveled off value of approximately 0.27. The author of [22] found the leveled off value to be approximately 0.32. According to the simulation herein, this value seems to be about 0.36 and the leveling off phenomena can be seen in Figure 3.2 (page 33).

### 3.4 Reynolds number effect

The small discrepancy between this simulation and the published experimental values can be explained by considering the Reynolds number effect. The simulations have been run at the same capillary numbers stated in the published values but capillary numbers depend upon velocity, viscosity, and surface tensions. Therefore, various combinations of velocity, viscosity, and surface tensions can offer the give same capillary number. However, this could affect the film thickness because the Reynolds number also involves velocity and viscosity. Hence, the same capillary number can have a different Reynolds number. It has been shown by [14] and [16] that the Reynolds number can have an effect upon the film thickness and since the author has only matched the capillary numbers and not the Reynolds number, it could very well be the reason for the small discrepancy. This is perhaps the reason why there is some error between the simulation results and that of [10]. The authors of [10] have not reported the velocity or flow rate used in the experiment. Hence, it is not possible
Table 3.3: Effect of Reynolds number on the film thickness at a constant capillary number and viscosity ratio, where C is case, where S is the simulation, $\rho_{\text{oil}}$ is the density of oil, $\mu_{\text{oil}}$ is the viscosity of oil, and AFT is the average film thickness.

<table>
<thead>
<tr>
<th></th>
<th>$\rho_{\text{oil}}$ (kg/m$^3$)</th>
<th>$\mu_{\text{oil}}$ (kg/m·s)</th>
<th>$\sigma_{\text{oil-water}}$ (N/m)</th>
<th>Ca</th>
<th>Re</th>
<th>AFT ((\delta))</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td>C 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S 1</td>
<td>860</td>
<td>0.001003</td>
<td>0.1215</td>
<td>0.00096</td>
<td>49.86</td>
<td>0.0160</td>
<td>1</td>
</tr>
<tr>
<td>S 2</td>
<td>860</td>
<td>0.001003</td>
<td>0.003375</td>
<td>0.00096</td>
<td>1.74</td>
<td>0.0164</td>
<td>1</td>
</tr>
<tr>
<td>C 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S 1</td>
<td>860</td>
<td>0.001003</td>
<td>0.003375</td>
<td>0.01</td>
<td>14.43</td>
<td>0.0577</td>
<td>1</td>
</tr>
<tr>
<td>S 2</td>
<td>860</td>
<td>0.001003</td>
<td>0.00675</td>
<td>0.01</td>
<td>28.85</td>
<td>0.0582</td>
<td>1</td>
</tr>
<tr>
<td>S 3</td>
<td>860</td>
<td>0.001003</td>
<td>0.01519</td>
<td>0.01</td>
<td>64.93</td>
<td>0.0588</td>
<td>1</td>
</tr>
</tbody>
</table>

to calculate their Reynolds number and compare it with the once in this chapter. Fig. 2 in [16] and Fig. 10 in [14] shows that for capillary number less than or equal to 0.01, the Reynolds number has very little effect in the range of 0-70 and the film thickness increases with increasing Reynolds number but the increase is very small and it almost looks as if the film thickness is independent of the Reynolds number.

To test if this is indeed the case and to see the Reynolds number effect, two different simulations were run. Water density and viscosity are kept constant, as shown in Table 3.1 (page 32) and oil properties and interfacial tension are changed to generate different capillary and Reynolds numbers. In Table 3.3, the results of two different cases involving the same capillary number, at different Reynolds numbers, are shown. Case 1 is for capillary number 0.00096 and Case 2 is for capillary number 0.01. The viscosity ratio for all simulations is kept constant at 1 to mitigate its effect. According to the table, it can be observed that for any given capillary number, the film thickness becomes thicker as the Reynolds number is increased but the increase is very small. This is exactly the prediction of the authors of [14,16]. Although [14,16] have obtained their results as for a gas bubble displacing a liquid, according to the above table, a similar trend seems plausible for the flow of liquid displacing another liquid.
3.5 Viscosity ratio effect

According to [10], there is no noticeable change in the film thickness with a changing viscosity ratio. However, [55] predicts a thinning of the film thickness with an increase in the viscosity ratio. The conflicting piece of information between the two papers is the disagreement between the effects of the viscosity ratio on wetting film thickness. Hence, the authors would like to predict how the viscosity ratio affects the film thickness from simulations run by the authors. The authors of [55] have said that the thinning of film thickness with an increase of viscosity ratio becomes more apparent at Ca > 1, based upon the simulations. At lower capillary numbers, the effect is still there but it is difficult to detect because of experimental uncertainty and could be the reason why it is not observed by some authors. It is further stated that numerous capillary number experiments cannot be carried out because of the limitation of the experimental procedures.

The authors of this thesis have run three different simulations for capillary number 0.2 and 2 at different viscosity ratios and kept the Reynolds number at less than two to minimize the inertial effects. The Reynolds number cannot be kept constant because it also involves the changing parameter of viscosity. Table 3.4 shows the parameters used in the three simulations. The highest viscosity ratio which can be used is about 148. Simulations employing viscosity ratios higher than 148 produce incorrect results and thus, it is one of the shortcomings of the VOF method. According to the table, the film thicknesses for all three cases of capillary number 0.2 are extremely close to each other and no definitive trend in the film thickness is apparent with an increasing viscosity ratio. A similar phenomenon is noticed for the capillary number of 2. The change in film thickness between the viscosity ratios of 39.88 and 73.78 is much greater than the change between 73.78 and 99.70. The results of our team appear to agree with the conclusion of [10] (i.e. the film thickness is independent of the viscosity ratio). Table 3.5 shows the result of a simulation in which the core fluid
### Table 3.4: Effect of viscosity ratio on the film thickness at a fixed capillary number and low Reynolds number.

<table>
<thead>
<tr>
<th></th>
<th>( \rho_{\text{oil}} ) (kg/m(^3))</th>
<th>( \mu_{\text{oil}} ) (kg/m ( \cdot ) s)</th>
<th>( \sigma_{(\text{oil-water})} ) (N/m)</th>
<th>Ca</th>
<th>Re</th>
<th>AFT</th>
<th>( M )</th>
</tr>
</thead>
<tbody>
<tr>
<td>C 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S 1</td>
<td>860</td>
<td>0.04</td>
<td>0.027</td>
<td>0.2</td>
<td>1.45</td>
<td>0.2524</td>
<td>39.88</td>
</tr>
<tr>
<td>S 2</td>
<td>860</td>
<td>0.074</td>
<td>0.0675</td>
<td>0.2</td>
<td>1.06</td>
<td>0.2519</td>
<td>73.78</td>
</tr>
<tr>
<td>S 3</td>
<td>860</td>
<td>0.148</td>
<td>0.10125</td>
<td>0.2</td>
<td>0.40</td>
<td>0.2521</td>
<td>147.56</td>
</tr>
<tr>
<td>C 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S 1</td>
<td>860</td>
<td>0.04</td>
<td>0.003375</td>
<td>2</td>
<td>1.81</td>
<td>0.3663</td>
<td>39.88</td>
</tr>
<tr>
<td>S 2</td>
<td>860</td>
<td>0.074</td>
<td>0.00675</td>
<td>2</td>
<td>1.06</td>
<td>0.3575</td>
<td>73.78</td>
</tr>
<tr>
<td>S 3</td>
<td>860</td>
<td>0.1</td>
<td>0.00675</td>
<td>2</td>
<td>0.58</td>
<td>0.3561</td>
<td>99.70</td>
</tr>
</tbody>
</table>

Table 3.5: Wetting film thickness for a case with viscous core liquid displacing a less viscous liquid, where \( \rho_{\text{water}} \) is the density of water, \( \mu_{\text{water}} \) is the viscosity of water.

<table>
<thead>
<tr>
<th></th>
<th>( \rho_{\text{water}} ) (kg/m(^3))</th>
<th>( \mu_{\text{water}} ) (kg/m ( \cdot ) s)</th>
<th>( \sigma_{(\text{oil-water})} ) (N/m)</th>
<th>Ca</th>
<th>Re</th>
<th>AFT</th>
<th>( M )</th>
</tr>
</thead>
<tbody>
<tr>
<td>C 3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S 1</td>
<td>998.2</td>
<td>0.001003</td>
<td>0.135</td>
<td>0.00096</td>
<td>64.30</td>
<td>0.0260</td>
<td>0.0251</td>
</tr>
</tbody>
</table>

is more viscous than the annular fluid. This will give a viscosity ratio of less than one according to Equation 2.11. Comparing Simulation 1, Case 3, of Table 3.5 with that of Simulation 2, Case 2, of Table 3.3 (page 36), it is observed that they are almost identical except for the Reynolds number and viscosity ratios. The difference in the Reynolds number is very small and its effect will be negligible. The viscosity ratios are very different, along with the thickness of the film. From this, it appears if the film thickness does indeed increase with a decreasing viscosity ratio, as concluded by [55]. The results of the viscosity ratio effect upon film thickness are conflicting and to confirm or rebuke the conclusion of [55], simulations at higher capillary numbers and viscosity ratio must be conducted.
4. NUMERICAL SIMULATION OF A SINGLE OIL SLUG FLOW

4.1 Introduction

This research is undertaken to further deepen understanding and knowledge of the behavior of mobilization of an oil slug in a water filled capillary tube. Mobilization is interpreted as no further change in the oil slug shape and the pressure drop vs. time graph becomes a flat line. As mentioned in Section 1.3.3 most two phase flows involve gas bubbles flowing within a surrounding thin liquid film. No other authors, other than the authors of this study and former research members have studied the transient pressure drop in a multiphase flow and how different parameters affect it. This study takes the perspective that oil slugs are drops of oil left behind in a pore by a secondary recovery process such as water flooding. To be able to find a way to recover this left over oil with enhanced oil recovery (EOR) methods, it is necessary to have a complete understanding of the mobilization process of an oil slug in a tube. One way to obtain this understanding is by conducting experiments and numerical simulations. The advantages of numerical simulations are obvious, as mentioned in Chapter 2. In this section, an oil slug is placed in a capillary tube, filled with water. Initially, no water film exists between the oil slug and the capillary tube. A constant flow rate is applied until the oil slug is mobilized (completely surrounded by a water
The pressure drop between the inlet and outlet of the tube is monitored with respect to time and to observe the maximum pressure needed to mobilize the oil slug and the behavior of the pressure drop upon completion of the mobilization process. In Chapter 5, it will be shown that simulations, where the oil slug is mobilized with water injection, help to establish a benchmark wherein the pressure drop of simulations with vibrations are compared.

The main goal of this chapter is to obtain the maximum pressure drop needed to mobilize the oil slug and the steady state pressure drop upon a complete formation of the water film. Pressure drop, along with the shape of the oil slug is studied to see how the oil slug is mobilized and its impact on the variation of the pressure drop. A sensitivity analysis is conducted by changing the surface tension, the velocity, and changing the water viscosity, respectively, while keeping other parameters constant. The impact of changing the Reynolds number and the capillary number on a pressure drop is presented. Theoretical equations will be derived and compared with the simulation results.

4.2 Pressure drop and oil slug shape change

In this section, the change in the pressure drop vs. time graphs will be correlated to the shape of the oil slug. The results show that whenever there is a change in the shape of the pressure drop vs. time graph, there will be a corresponding change in the shape of the oil slug. Figure 4.1 (page 42) shows the pressure drop vs. time between the inlet and outlet of the capillary tube. This pressure drop graph is taken from a simulation with capillary number 0.00192 and Reynolds number 69.67. According to this figure, the mobilization of the oil slug occurs at approximately 0.015s. Figure 4.2 - Figure 4.9 (pages 43 - 47) show the oil slug shape at different times. Figure 4.2 (page 43) shows the oil slug shape just before the maximum pressure value is reached.
on pressure drop graph. Figure 4.3 (page 44) shows the oil slug shape just after
the maximum pressure value. In both figures, the rear of the slug is deformed and
almost identical. The only noticeable difference is at the front end of the oil slug.
At $t = 0.00067184s$, no water film exists between the tube wall and the oil slug. At
$t = 0.00100184s$, the water film formation started at the front end of the oil slug and
this is the reason the pressure decreased following the attainment of the maximum
pressure. Figure 4.4 - Figure 4.6 (pages 44 - 45) shows the oil slug shape on the linear
portion of Figure 4.1. The only difference in the three images is the amount of oil slug
covered by the water film. Therefore, it can be concluded that in the linear region of
the pressure drop graph the oil slug shape remains the same except for an increase
in the length of the water film. According to the pressure drop graph, it is observed
that at approximately $t = 0.11s$, the pressure drop graph changes in shape and there
is a significant decrease in the pressure drop. In observing Figure 4.7 (page 46), some
light will be shed on why the pressure drop graph goes from linear to experiencing
a big decrease in the pressure drop. The image shows that the water film formation
is still taking place but the rear meniscus of the oil slug has changed its shape. The
rear meniscus used to be flat but now has become a little more rounded. This would
explain the reduction in pressure drop because if the front and back meniscus have
the same shape, then there would be no capillary pressure. The net capillary pressure
is due to the difference in the shape of the front and the rear meniscus. A transition
of the rear meniscus from flat to hemispherical occurs from a higher pressure drop to
lower pressure drop between the inlet and outlet of the tube. A reduction in pressure
drop can be significant and it will be discussed in a later section. The final shape of
the oil slug, after mobilization, depends upon the capillary number. At low capillary
numbers, the difference in the shape of the rear meniscus and the front meniscus
is very small but will still result in a capillary pressure drop. At higher capillary
numbers, the difference in the shape of the rear and front meniscus increases and

41
results in an increase capillary pressure drop. In the ranges of capillary numbers, studied in this chapter, the difference in a steady state oil slug, at different capillary numbers, is negligible. Similar shape changes in the oil slug can be observed with an increasing Reynolds number.

Another point worth mentioning is that of the oil slug breakage in a capillary tube, [4] has noted that the oil slug will break during the mobilization process if the injection velocity of water is too high. That fact is not solely based on our simulations. In the first simulation, the velocity is low but the capillary number is high and in the second simulation, the velocity is high but the capillary number is low. What is observed is that the oil slug breaks at a high capillary number and stays intact at a low capillary number. Thus, it can be concluded that it is the capillary number, and not the inlet velocity that will determine whether or not oil slug breakage will occur as shown in Figure 4.10 (page 48) and Figure 4.11 (page 49). Inlet velocity is one of the parameters that is used in calculating the capillary number, as shown in

Figure 4.1: Pressure drop vs. time for Ca = 0.00192 and Re = 69.67.
Figure 4.2: Shape of the oil slug at $t = 0.00067184$s (original in color), where $t$ is the time.
Figure 4.3: Shape of the oil slug at $t = 0.00100184s$ (original in color).

Figure 4.4: Shape of the oil slug at $t = 0.00600184s$ (original in color).
Figure 4.5: Shape of the oil slug at $t = 0.00800184s$ (original in color).

Figure 4.6: Shape of the oil slug at $t = 0.0106018s$ (original in color).
Figure 4.7: Shape of the oil slug at $t = 0.0110018s$ (original in color).

Figure 4.8: Shape of the oil slug at $t = 0.0140018s$ (original in color).
Equation 2.8. The breakage of the oil slug also depends upon the length of the liquid slug. A short liquid slug will stay intact even at high capillary numbers because the oil slug will be quickly covered by a water film and this will allow the injecting water to travel through the film rather than penetrate through the oil slug. Figure 4.12 (page 50) shows the intact oil slug at capillary number 0.007, after the mobilization. A long oil slug with an initial water film will not break the oil slug because the water will simply pass through the water film region. A shorter oil slug can stay intact at high capillary numbers and a longer oil slug can break even at low capillary numbers.
Figure 4.10: Shape of the oil slug at $Ca = 0.05$ and velocity 0.17 m/s (original in color).
Figure 4.11: Shape of the oil slug at capillary number 0.00192 and velocity 0.21057 m/s (original in color).
Figure 4.12: Shape of the oil slug at capillary number 0.07430 and velocity 0.2 m/s (original in color).
4.3 Sensitivity analysis of various parameters

4.3.1 Changing the interfacial tension and capillary number

Interfacial tension is an important factor in a two phase immiscible flow. An interfacial tension is a property of two liquids, such as air and water, and it only changes if one of the liquids is changed. Changing the interfacial tension while holding other parameters constant amounts to changing the types of liquids employed in the simulation. The importance of the changing interfacial tension is that it will also change the capillary number because interfacial tension is one of the parameters needed to calculate the capillary number. Interfacial tension does not affect the Reynolds number, hence, we can observe the sole effect of interfacial tension/capillary number on the pressure drop, as shown in Figure 4.13 and Figure 4.14 (page 53). According to both figures, it can be observed that the maximum pressure drop and steady state pressure drop for all simulated capillary numbers appears to slowly increase with decreasing capillary number. However, Figure 4.13 shows a lower capillary number leads to a lower pressure drop value for the linear region because, at lower capillary numbers, the difference between the radius of curvature between the front and back meniscus is small compared with the difference at high capillary numbers. This would lead to a lower capillary pressure and overall a smaller pressure drop. In Figure 4.13, it can also be observed that for lower capillary numbers, the pressure drop difference between the end of the linear line and the beginning of the straight line (steady state) is smaller than for a high capillary number. The answer for this can found in the previous section where the rear meniscus of the oil slug changes back to a hemispherical shape as the oil slug is gradually covered by a water film. At a low capillary number, the initial rear meniscus deformation is small to begin with and hence, the deformation needed to achieve the hemispherical shape is smaller and hence, the pressure drop is lower. At steady state, the lower capillary number experiences a higher pressure
Figure 4.13: Pressure drop vs. time for varying capillary numbers obtained by changing interfacial tension and fixed Reynolds number (original in color).

drop because a lower capillary number leads to a thinner water film (as observed in Table 3.2) and a higher pressure drop.

4.3.2 Changing water viscosity

The water viscosity in a petroleum reservoir also experiences a slight change due to the high pressure and temperature. The water viscosity in a reservoir varies between 0.5 cp to 1 cp. Simulations with varying water viscosity plus other fixed parameters are run to quantify and understand the effect of water viscosity on a pressure drop. The range of water viscosity employed in the simulations is between 0.5 cp to 1 cp. A lower water viscosity would reduce the pressure drop because it would reduce the shear stress in a single phase flow. The same effect for a two phase flow can be expected because in the part of the tube containing only water, the pressure drop will
Figure 4.14: Pressure drop vs. capillary number. The graph shows both the maximum pressure drop and the steady state pressure drop (original in color).
Figure 4.15: Pressure drop vs. time for varying water viscosity (original in color).

decrease and in the part of the tube containing an oil slug surrounded by a water film, less viscous water will also reduce the pressure drop. Figure 4.15 shows the pressure drop vs. time for changing water viscosity. The graph also shows the capillary number and Reynolds number for each simulation. The capillary number and Reynolds number vary because both are calculated using the water viscosity. As expected, the simulation with the lowest water viscosity has the lowest maximum pressure drop value and the simulation with highest viscosity has the highest maximum pressure drop value and all other simulations fall between those two values. It can be noted that a low viscosity leads to a low pressure drop throughout the entire simulation, including before and after the mobilization. A high viscosity leads to a high pressure drop throughout the entire simulation. Figure 4.16 shows how maximum and steady state pressure increases with an increasing water viscosity.
Figure 4.16: Pressure drop vs. water viscosity. The graph shows both the maximum pressure drop and the steady state pressure drop (original in color).
Figure 4.17: Pressure drop vs. time for varying inlet velocity (original in color).

4.3.3 Changing velocity

The impact of an inlet velocity on the pressure drop must to be quantified. It is worthwhile to mention that changing velocity will lead to varying capillary and Reynolds number because both are calculated based on the velocity. It is expected that an oil slug in a simulation with a higher velocity will be mobilized faster than an oil slug in a simulation with a lower velocity. According to Figure 4.17, it can be noted that a simulation with a high velocity has the highest maximum pressure and a simulation with low velocity has the lowest maximum pressure. As expected, higher velocity leads to faster mobilization as demonstrated by shifting to the left of the transition region between the partially mobilized state and the completely mobilized state. The graph also shows that at lower velocity, the slope of the partially mobilized state is very gradual compared to the slope for high velocity. Figure 4.18 shows that both the maximum and steady state pressure drop increase with increasing inlet velocity.
Figure 4.18: Pressure drop vs. inlet velocity. The graph shows both the maximum pressure drop and the steady state pressure drop (original in color).
4.3.4 Changing the Reynolds number

In this section, simulations with fixed capillary numbers but a varying Reynolds number have been conducted to study its effect. This approach is different from the changing water viscosity and inlet velocity in that only the Reynolds number is changed while the capillary number is fixed. Figure 4.19 - Figure 4.26 (pages 59 - 66) show both the pressure drop vs. time and pressure drop vs. Reynolds number at different capillary numbers. Observing the pressure difference vs. Reynolds number figures, it can be observed that at low capillary numbers, both the maximum and steady state pressure drop increase with the increasing Reynolds number and reaches a maximum value and then starts to decrease with an increasing Reynolds number. As the capillary number increases, this phenomena ceases to occur and the maximum pressure drop and steady state pressure drop both increase with an increasing Reynolds number in the Reynolds number range tried in these simulations. According to the the pressure difference vs. time, the same phenomena as viewed with the changing of the inlet velocity can be observed but the effect is not as pronounced. The oil slug in the simulation with the higher Reynolds number mobilizes faster than the oil slug in the simulation with a lower Reynolds number.

According to all the figures regarding pressure drop vs Reynolds number, even though the trend of maximum pressure drop and steady state pressure drop is similar, the slope of the maximum pressure drop is greater than the slope of the steady state pressure drop. This means that the maximum pressure drop is more sensitive to change in the Reynolds number than the steady state pressure drop.

The shape of the pressure drop vs time, as obtained in the experiments, can be predicted based upon the simulation results by looking at the green line in Figure 4.19. It is known from the graphs regarding pressure data of other simulations that as the capillary and Reynolds number become smaller, the nonlinear portion declines and the linear region becomes flatter with a smaller slope. Thus, at a low Reynolds number
and capillary number, as in the experiment, it can be expected that the nonlinear portion will be very small and the linear portion will have a small slope, as viewed in Figure 4.28 (page 68).

4.4 Verification of the numerical results

4.4.1 Verification with indirect experimental results

As mentioned earlier in Chapter 2, the geometry and meshing used in all the simulations in this thesis is identical. In this chapter, the type of flow studied is slightly different from core-annular flow (Chapter 3) in that there is an oil slug surrounded by water in a tube, compared to the tube initially filled with Liquid 1 and Liquid 2, injected to displace Liquid 1. However, in both cases, the object of interest is the
Figure 4.20: Pressure drop vs. Reynolds number at Ca = 0.00096. The graph shows both the maximum pressure drop and the steady state pressure drop (original in color).
Figure 4.21: Pressure drop vs. time for varying Reynolds number with Ca = 0.00140 (original in color).
Figure 4.22: Pressure drop vs. Reynolds number at Ca = 0.00140. The graph shows both the maximum pressure drop and the steady state pressure drop (original in color).
Figure 4.23: Pressure drop vs. time for varying Reynolds number with Ca = 0.00172 (original in color).
Figure 4.24: Pressure drop vs. Reynolds number at Ca = 0.00172. The graph shows both the maximum pressure drop and the steady state pressure drop (original in color).
Figure 4.25: Pressure drop vs. time graph for varying Reynolds number with Ca = 0.00192 (original in color).
Figure 4.26: Pressure drop vs. Reynolds number at Ca = 0.00192. The graph shows both the maximum pressure drop and the steady state pressure drop (original in color).
Figure 4.27: Typical pressure profile obtained from mobilization experiments (taken from [4]).

formation of a thin liquid film. As mentioned in Chapter [1] studies regarding oil slug mobilization with no initial presence of wetting film have not been found other than those performed by former research members. A quantitative matching of the simulation and experimental results is not possible due to the employment of different parameters. An indirect way to verify the results is needed since there are no studies looking at this type of flow. That being said, the mesh and time step is deemed accurate based upon Chapter [3] results which compare the numerical results with the published experimental results.

Another verification via indirect experimental results is done by comparing Figure 4.2 - Figure 4.9 (pages 43 - 47) to that of Fig. 8 in [56]. The parameters used by the author in the experiments along with the capillary number are very similar to the parameters of the simulation shown in Figure 4.2 - Figure 4.9 (pages 43 - 47). The similar oil slug shape at similar capillary number and parameters serves as another indirect verification of the numerical simulations conducted in this thesis.
4.4.2 Verification with the group’s experimental results

Another criterion to verify the simulation results will be to compare the shapes of the oil slug and the pressure profile obtained from simulations to that in the experiments. Dr. Liming Dai’s research group has done similar experimental studies that can be used for qualitative comparison but not for quantitative comparison with the simulation results. The values within the pressure graph, obtained from the experiment, will not match the values obtained from simulation due to the reason mentioned in previous section, but the trend and shape of both pressure profiles should be similar. Former research member [4] has divided the pressure graph, obtained from the experiments, into three specific stages, namely stationary stage, partially mobilized stage, and completely mobilized stage. The image is reproduced in Figure 4.27 for the sake of convenience. Figure 4.28 shows an actual pressure profile from an experiment and it looks very much like the pressure profile shown in Figure 4.27. Comparing these graphs with that shown in Figure 4.1 (page 42), they are similar. There is an
increase in the pressure profile until a maximum value is reached known as the *stationary stage*. The partially mobilized stage in the pressure profile, obtained from the simulation, consists of a linear portion representing a water film formation, followed by a nonlinear portion, due to a reduction in capillary pressure. The pressure profile eventually becomes a horizontal flat line after the water film formation is complete and is known as the *completely mobilized stage*. As can be observed, the trends in the experimental and simulation pressure profile are similar and the differences can be attributed to the use of different parameters.

The tube length, velocity, tube diameter, oil slug length in the experiments and the simulations are different. In the simulations, the tube length, tube diameter, and oil slug length are much shorter while the velocity is much higher. The reason behind this is to save computation times since thin film multiphase flow are inherently computationally expensive. Employing a shorter tube and diameter reduces the number of meshing elements needed. A shorter oil slug reduces the time it takes to be surrounded by a water film and hence, reduces the simulation time. A faster velocity speeds up the water film forming process and leads to a reduction in the total simulation time needed. A sensitivity analysis of various parameters sheds light on how an increase or decrease in one of the parameters affects the pressure drop. During the water film formation, the pressure drop decreases linearly with time, and how this slope changes with different parameters must be understood. Increasing water viscosity leads to an increase in the maximum pressure required to move an oil slug and an increase in the steady state pressure drop. The slope of the linear pressure drop region remains more or less constant for the range of water viscosity employed. Increasing inlet velocity also increases the maximum pressure drop required to move an oil slug, as well as the steady state pressure drop. The slope of the linear pressure drop region decreases as the inlet velocity is decreased. As for an increasing Reynolds number at a high capillary number, the maximum pressure drop and steady state
pressure drop increase. As for increasing the Reynolds number at low capillary numbers, the maximum pressure drop and steady state pressure drop reach a maximum value and then start to decrease. Just as with the changing inlet velocity, the slope of the linear portion of the pressure drop region decreases with increasing the Reynolds number. Changing the capillary number at a fixed Reynolds number does not appear to significantly affect the maximum pressure drop and steady state pressure drop. The slope of the linear portion of the pressure drop seems to be approximately the same for all capillary numbers.

4.4.3 Verification with theoretical pressure drop equations

The theoretical maximum pressure drop is obtained by single phase poiseuille’s equation for oil and water and the pressure drop due to the curvature of the meniscus given by Bretheron [7]. The total maximum pressure drop is calculated by Equation 4.1. The assumption made in deriving this equation is that the maximum pressure drop which happens just before water film formation is due to three different individual pressure drops: Pressure drop due to water, pressure drop due to oil, and pressure drop due to the curvature of the meniscus of the oil slug. It is also assumed that the oil slug is touching the wall and no water film exists between the oil slug and tube wall. The previous assumptions are justifiable based upon Figure 4.1 - Figure 4.9 (pages 42 - 47). As seen in the previous sections, the theoretical maximum pressure drop is always below the maximum pressure drop obtained from the simulations. One reason for this is that the deformation of the rear meniscus just before water film formation is large and this means a higher capillary pressure drop. Bretherton’s pressure drop
is not sufficient and the maximum pressure drop is always under predicted.

\[
\Delta P = 4.52 \left( \frac{\sigma_{\text{oil-water}}}{r} \right) \left( \frac{3 \mu_{\text{water}} V_{\text{inlet}}}{\sigma_{\text{oil-water}}} \right)^{\frac{2}{3}} + 0.5 \left( \frac{64}{\text{Re}_{\text{oil}}} \right) \left( \frac{L_{\text{oil}}}{D} \right) \rho_{\text{oil}} V_{\text{inlet}}^2 + 0.5 \left( \frac{64}{\text{Re}_{\text{water}}} \right) \left( \frac{L_{\text{water}}}{D} \right) \rho_{\text{water}} V_{\text{inlet}}^2
\]  

(4.1)

where \(\Delta P\) is the pressure drop, \(r\) is the radius of the tube, \(\text{Re}_{\text{oil}}\) is the Reynolds number based on the oil properties, \(L_{\text{oil}}\) is the length of the oil drop, \(\text{Re}_{\text{water}}\) is the Reynolds number based on the water properties, \(L_{\text{water}}\) is the length of the water region.

The total steady state pressure drop is calculated by \textbf{Equation 4.2}. The steady state pressure drop is reached when the entire oil slug is covered by the water film. The assumption made in deriving this equation is that since the oil slug is surrounded by the water film, the oil slug is treated as if it were also water. This assumption would not have been valid if the oil was in contact with the tube wall because this would cause much higher pressure drop due to the higher viscosity of oil. The assumption of ignoring the viscosity of oil slug will slightly underestimate the pressure drop but it will simplify the calculations considerably. Hence, the total pressure drop is due to pressure drop due to single phase water and pressure drop due to the curvature of the meniscus of the oil slug as given by Bretherton’s equation.

\[
\Delta P = 4.52 \left( \frac{\sigma_{\text{oil-water}}}{r} \right) \left( \frac{3 \mu_{\text{water}} V_{\text{inlet}}}{\sigma_{\text{oil-water}}} \right)^{\frac{2}{3}} + 0.5 \left( \frac{64}{\text{Re}_{\text{water}}} \right) \left( \frac{L_{\text{water}}}{D} \right) \rho_{\text{water}} V_{\text{inlet}}^2
\]  

(4.2)

As seen in the figures in the previous sections, the theoretical equations derived in
this section are in good agreement with the pressure drop obtained from the simulation. The theoretical pressure drop underestimates the actual pressure drop obtained from the simulation due to the above mentioned assumptions. It is seen from the figures in the previous sections of this chapter that the theoretical pressure drop closely follow the trend of the pressure drop obtained from the simulations. These shows that the pressure drop obtained from the simulations can be predicted based on the theoretical equations by adding a safety factor. The theoretical pressure drop also serves to validate the numerical simulations.
5. NUMERICAL SIMULATION
   OF A SINGLE OIL SLUG FLOW
   WITH EXTERNAL EXCITATION

5.1 Introduction

EOR by vibratory stimulation is a little understood method and research in this field remains ongoing. Important studies and advances made in this area are highlighted in Chapter 1. A past member of our research group [4] has shown that vibratory stimulations have a positive effect upon oil slug mobilization. It was also shown that the pressure drop needed to mobilize an oil slug with vibratory stimulation is between the pressure drop needed to move the oil slug after it has been inside a tube for 24 hours and the pressure drop needed to move the oil slug as soon as it is placed in the tube. According to this information, it was hypothesized that the vibrations help form a thin water film around the tube. Visual proof of this idea was not presented due to the lack of high quality expensive visualization equipment. It is also seen from the previous chapter that a formation of a water film around the oil slug leads to a significant decrease in the pressure drop and it is expected that vibrations will help form a thin water film around the oil slug which will help lower the pressure drop needed to push the oil slug out of the tube [4].
In this study, the interest is in understanding how external excitations aid in the mobilization of the oil slug at a pore scale. Simulations are carried out to mobilize trapped oil slugs in a capillary tube of 0.5 mm in diameter and 5 mm in length filled with water. The capillary number is fixed at 0.00192 and Reynolds number is fixed at 25.72 for all the simulations. Excitations in form of different types of waves such as square wave, sine wave, and saw tooth wave have been employed to see which one has the most significant effect on the mobilization. The amplitudes and the frequency of the waves are varied to study their impact. Effects of external excitation is measured and evaluated in the following manner: External excitation is applied to the tube containing an oil slug surrounded by water without any external water injection. Following the vibration, water injection is then applied and pressure vs. time is monitored to see the variation in the pressure drop due to external excitations.

5.2 Establishing upper and lower bound pressure drop limits

In the previous chapter, all the simulations were conducted with no initial water film between the oil slug and the tube wall to simulate the oil droplet in pore. In order to quantify the effect of vibrations, a pressure drop profile obtained from a simulation with external excitation needs to be compared with the pressure drop profile obtained from a simulation without external excitations. All the parameters between the two simulations should be same except for the presence and absence of the external excitations. All the simulations with external excitations in this chapter are conducted with Ca = 0.00192 and Re = 25.72 to ensure a multiphase flow that is dominated by surface tension effects rather than viscous or inertial effects. Figure 5.1 shows the pressure profile for Ca = 0.00192 and Re = 25.72 in the absence of external excitation. The maximum pressure drop is 538 Pa (A) and the steady state pressure
Figure 5.1: Pressure drop vs. time for Ca = 0.00192, Re = 25.72 with no initial water film. (A) represents the maximum pressure drop and (B) represents the steady state pressure drop. (original in color).

The upper bound for all simulations with vibrations should be smaller or equal to the upper bound (A) in Figure 5.1 due to the presence of the water film. The steady state pressure drop (B) for all vibration simulations will be same as the lower bound in Figure 5.1 because the steady state pressure drop is unaffected by vibrations.

The properties of the liquids employed in all the simulations for this chapter are given in Table 5.1.

<table>
<thead>
<tr>
<th>Fluid</th>
<th>Density (kg/m³)</th>
<th>Viscosity (kg/(m · s))</th>
<th>Surface Tension (N/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>998.2</td>
<td>0.001003</td>
<td>0.027</td>
</tr>
<tr>
<td>Oil</td>
<td>860</td>
<td>0.04</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.1: Properties of the two immiscible liquids used in the simulations.
5.3 Effect of external excitations on water film development

In this section, the qualitative effect of external excitation on an oil slug will be shown via pictures obtained from post processing of the simulation data. Figure 5.2 - Figure 5.6 (pages 77 - 80) show an oil slug under the effect of square wave external excitation, at different times. It can be observed that the vibrations cause the oil to move back and forth in a simple harmonic motion in the absence of any water injection. The back and forth motion of the oil slug leads to the constant formation and drainage of the water film on the opposite end of the oil slug. The water film initially starts forming from one end of the oil slug, reaches its maximum length, and then begins to drain from that end and starts forming on the opposite end. Formation and drainage of water film on both sides of the oil slug is a completion of one cycle of vibration. All vibrations of different types have the same type of effect on the oil slug and the only difference would be in the length of the water film established.

The three different types of vibrations employed in this section are: the square wave, the sine wave, and the saw tooth wave. It is important to study which wave, all things being equal, has the highest impact in terms of water film development. The effect can be qualitatively studied by looking at the figures of water film development around the oil slug. Figure 5.7 - Figure 5.9 (pages 81 - 82) show water film development around the oil slug for the square wave, sine wave, and saw tooth wave at an amplitude of 0.04026 mm and a frequency of 20 Hz and Figure 5.10 - Figure 5.12 (pages 83 - 84) show the water film development around the oil slug for the square wave, sine wave, and saw tooth wave at an amplitude of 0.04026 mm and a frequency of 40 Hz. From the visual inspection of the pictures and from Table 5.2 it can be noted that at any given frequency, the water film around the oil slug vibrated with the square wave being the greatest, followed by the sine wave and finally the
Figure 5.2: Square wave, $A = 0.04026$ mm, $f = 20$ Hz, $t = 0.109908$s (original in color), where $A$ is the amplitude of the excitation, and $f$ is the frequency of the excitation.
Figure 5.3: Square wave, $A = 0.04026$ mm, $f = 20$ Hz, $t = 0.115733$s (original in color).
Figure 5.4: Square wave, \( A = 0.04026 \) mm, \( f = 20 \text{ Hz} \), \( t = 0.125378 \text{s} \) (original in color).
Figure 5.5: Square wave, $A = 0.04026$ mm, $f = 20$ Hz, $t = 0.136077$s (original in color).

Figure 5.6: Square wave, $A = 0.04026$ mm, $f = 20$ Hz, $t = 0.150045$s (original in color).
sawtooth wave. The rear meniscus of the oil slug for the square wave experiences the maximum deformation followed by the sine wave and then the sawtooth wave. It can also be observed that as frequency is increased, the water film surrounding the oil slug decreases for all types of waves. As frequency increases, the deformation of the rear meniscus of the oil slug also decreases.

5.4 Effect of external excitations on a pressure drop

In the previous section, it was noted that applying external excitation to an oil slug, with no initial water film, helps form a water film around the oil slug. It was also observed that different amplitudes and frequencies lead to different amounts of water film surrounding the oil slug. In this section, the effect of vibration upon the pressure drop, due to water film formation is analyzed. Specifically, the impact of amplitude
Figure 5.8: Sine wave, $A = 0.04026$ mm, $f = 20$ Hz (original in color).

Figure 5.9: Square wave, $A = 0.04026$ mm, $f = 20$ Hz (original in color).
Figure 5.10: Sawtooth wave, $A = 0.04026$ mm, $f = 40$ Hz (original in color).

Figure 5.11: Sine wave, $A = 0.04026$ mm, $f = 40$ Hz (original in color).
Figure 5.12: Square wave, $A = 0.04026$ mm, $f = 40$ Hz (original in color).

<table>
<thead>
<tr>
<th>Wave Type</th>
<th>$A$ (mm)</th>
<th>$f$ (Hz)</th>
<th>Water film length (mm)</th>
<th>Max pressure drop (Pa)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Square Wave</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S 1</td>
<td>0.04026</td>
<td>10</td>
<td>1.5</td>
<td>120</td>
</tr>
<tr>
<td>S 2</td>
<td>0.04026</td>
<td>20</td>
<td>0.001141</td>
<td>422</td>
</tr>
<tr>
<td>S 3</td>
<td>0.04026</td>
<td>30</td>
<td>0.000907075</td>
<td>460</td>
</tr>
<tr>
<td>S 4</td>
<td>0.04026</td>
<td>40</td>
<td>0.000673</td>
<td>487</td>
</tr>
<tr>
<td><strong>Sine Wave</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S 1</td>
<td>0.04026</td>
<td>20</td>
<td>0.000831</td>
<td>460</td>
</tr>
<tr>
<td>S 2</td>
<td>0.04026</td>
<td>30</td>
<td>0.00068246</td>
<td>484</td>
</tr>
<tr>
<td>S 3</td>
<td>0.04026</td>
<td>40</td>
<td>0.000534</td>
<td>503</td>
</tr>
<tr>
<td><strong>Sawtooth Wave</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S 1</td>
<td>0.04026</td>
<td>20</td>
<td>0.000439</td>
<td>518</td>
</tr>
<tr>
<td>S 2</td>
<td>0.04026</td>
<td>30</td>
<td>0.0003922</td>
<td>522</td>
</tr>
<tr>
<td>S 3</td>
<td>0.04026</td>
<td>40</td>
<td>0.000345</td>
<td>532</td>
</tr>
<tr>
<td><strong>No External Excitation</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S 1</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>538</td>
</tr>
</tbody>
</table>

Table 5.2: Approximate maximum water film length surrounding the oil slug for different types of waves at a fixed amplitude and varying frequencies.
at constant frequency and the impact of frequency at constant amplitude will be analyzed. The pressure drop for different waves at the same amplitude and frequency are compared to observe what type of vibrations lead to the least pressure drop required to mobilize the oil slug.

Figure 5.13 shows pressure drop vs. amplitude for fixed frequency of a sine wave. The graph depicts both maximum pressure drop and steady state pressure drop obtained from the simulations. It can be observed that as the amplitude is increased, the maximum pressure drop needed to mobilize the oil slug decreases. The steady state pressure drop stays constant and its numerical value is equal to 110 Pa, as obtained in Figure 5.1 (page 75), without vibration. Figure 5.14 (page 87) shows pressure drop vs. frequency for the fixed amplitude of a sine wave. As noted, as frequency is increased, the maximum pressure drop increases while the steady state pressure drop remains constant for all frequencies. According to the graph, the maximum pressure drop at a frequency of 10 Hz, is almost equal to the steady state pressure drop at a frequency of 10 Hz. The reason for this is that by applying a frequency of 10 Hz leads to the entire oil slug being covered by the water film and this would lead to the same pressure drop value as the pressure drop at steady state. From the Table 5.2 it can be observed that any type of vibration at any frequency and amplitude leads to a lower maximum pressure drop than the case without any external excitation.

5.5 Effect of different types of waves on pressure drop

The previous section analyzed the effect of different amplitudes and frequencies of a square wave upon maximum pressure drop. In this section, different types of waves are compared with square waves at the same frequency and amplitude to see how the maximum pressure drop changes with different types of vibrations. The three waves,
Figure 5.13: Pressure drop vs. square wave amplitude, at a frequency of 20 Hz (original in color).
Figure 5.14: Pressure drop vs. square wave frequency, at an amplitude of 0.04026 mm (original in color).
Figure 5.15: Pressure drop vs. wave frequency, for Square wave, Sine wave, and Sawtooth wave. The amplitude for all the waves is fixed at 0.04026 mm (original in color).

used in this section are the square wave, sawtooth wave, and sine wave. Figure 5.15 shows the maximum pressure drop and steady state pressure drop for different vibrations at a fixed amplitude and two different frequencies. As noted for all vibrations, the maximum pressure increases with an increase in frequency as viewed for the square waves in Section 5.4. Also, the square wave leads to the highest reduction in maximum pressure drop, followed by the sine wave and the sawtooth wave, respectively. This is due to the fact that square waves form the longest water film around an oil slug, followed by sine waves and sawtooth waves, respectively, as observed in Section 5.3. These results can also be observed in Table 5.2 (page 84), which also shows that the higher the water film length, the lower the maximum pressure drop and the smaller the water film length, the higher the maximum pressure drop required to mobilize the oil slug.
It is demonstrated in the numerical simulation results that under the influence of external vibratory stimulation, a thin water film is formed around the oil slug. It is this thin water film that reduces the maximum pressure drop needed to move the oil slug in the subsequent water injection step. Different types of waves, along with frequencies and amplitudes are found to have a definitive affect upon the formation of a water film, as well as the maximum pressure drop required to move the oil slug.
6. CONCLUSION

6.1 Summary

A CFD software Fluent is employed to study an unsteady, two phase, laminar, thin film, immiscible flow at a pore scale. Three basic simulation types have been carried out in this thesis: Core-annular flow (one liquid displaces the liquid that initially occupied the pore), static oil slug flow (mobilization of the oil slug with a water injection), and vibratory oil slug flow (mobilization of the oil slug flow with external excitation). All three sets of simulations involve the formation of a thin film which is a very computationally expensive process.

The following findings can be drawn from the research conducted in this thesis.

Core-annular flow is simulated using the VOF method in a commercial CFD package, known as “Fluent”. The motivation of this study was to compare the wetting film thickness, obtained from the simulation, with that of the latest published experimental studies. The range of simulated capillary numbers are 0.00096 to 2. According to the simulation data, the residual film thickness increases with the capillary number and levels off to a value of 0.36 which is very close to the experimentally obtained value of 0.33. The trend of capillaryumber vs. film thickness also follows very closely to that of the experimental data trend. A good match with the experimental results also verifies the meshing used in the simulations. As for the range of viscosity ratios studied, the film thickness appears to be independent of the viscosity ratio.
The effect of the Reynolds number upon the film thickness, as described by [14,16], also seems plausible for the liquid-liquid flow from our simulations. It is found that the volume of fluid method is capable of performing thin film multiphase simulations and generating reliable results. A good understanding of a pore-scale water flooding mechanism is obtained.

An oil slug flow in a capillary tube representing a single pore is of serious importance because it provides us with insight as to how an oil slug is pushed out of the pores of a reservoir. From the simulations, it is found that as water is injected, a thin water film starts forming from the front of the oil slug and the oil slug shape deforms and becomes more elongated at the front and flatter at the rear. The oil slug shape changes throughout the mobilization process which is completed when the entire oil slug is surrounded by a water film. The pressure drop vs. time graph also shows distinctive behaviour and changes in correspondence with the change in the shape of the oil slug. The pressure drop is high before the formation of water film and becomes much smaller upon the mobilization of the oil slug. The oil slug simulations in the tube filled with water have shown what the previous research members hypothesized based upon an indirect pressure drop measurement wherein a water film does indeed reduce the pressure needed to mobilize the oil slug. The three distinct regions, mentioned by the previous researchers according to experimental results, are also observed in the numerical pressure drop profile. Sensitivity analysis with which to study the effects of water viscosity, interfacial tension, inlet velocity, and Reynolds number is conducted. Changing the interfacial tension amounts to changing the capillary number and does not appear to have a very strong effect in influencing the maximum pressure drop in the range of capillary numbers studied. However, low capillary numbers lead to the lowest pressure drop for the entire simulation time after the maximum pressure is reached. Changing water viscosity leads to the lowest pressure drop for the entire simulation for low water viscosity and the highest pressure
drop for the entire simulation for high water viscosity. Changing the inlet velocity leads to a higher pressure drop throughout the entire simulation and leads to faster oil slug mobilization. Increasing the Reynolds number at low capillary numbers leads to an increase in steady and maximum pressure drop, then a maximum value is reached and the pressure drop begins to decline. Increasing the Reynolds number at a high capillary number leads to an increase in the steady and maximum pressure drop and no decrease takes place.

Simulations with external excitations are conducted in the last part of this research. Three different types of waves: square wave, sine wave, and sawtooth wave were utilized in this research as external excitations. It is found that any type of external excitation helps form a thin water film around an oil slug. At a fixed amplitude and frequency, the length of this water film is greatest for a square wave, followed by a sine wave and lastly, a sawtooth wave, respectively. It is noted for all three different waves, an increase in amplitude, at a fixed frequency, will result in a longer water film and a decrease in amplitude will result in a shorter water film. A similar statement can be made with respect to frequency at a fixed amplitude in that a higher frequency will reduce the length of the water film and a lower frequency will increase the length of the water film. The effect of the increase/decreases in water film length has a direct correlation with the maximum pressure drop, as observed from the Pressure drop Vs. Time. It is found that increasing the amplitude will lower the maximum pressure drop and increasing the frequency increases the maximum pressure drop. At fixed amplitude and frequency, square wave excitation will lead to the lowest maximum pressure value, followed by the sine wave and the sawtooth wave, respectively. As observed, higher amplitude and lower frequency are beneficial in generating a longer water film and in reducing the maximum pressure needed to mobilize the oil slug. It is also found that the square wave outperforms the sine wave and sawtooth wave in terms of water film development, which in turn leads to a lower maximum pressure
6.2 FUTURE WORK

6.2.1 Experimental work on core samples

The research in this thesis focuses upon oil slug mobilization at a pore scale by employing a single capillary tube. In order to form a realistic prediction of the effects of vibrations on trapped oil slugs and oil recovery, it is necessary to employ core samples collected from an oil reservoir. Experiments conducted with cores would be more realistic as they would have multiple pores with different diameters and lengths and the pore morphology is random. An experimental setup of cores would be similar to what is used in core flooding experiments. A core sample is initially saturated with oil and then water is injected to displace the oil while applying the vibratory stimulation. The amount of oil recovered can be compared to the initial amount of oil in the core and this will determine how much oil is recovered. The oil recovered via vibratory stimulation should be compared to the oil recovered via water flooding (no vibratory stimulation) to see how much more oil can be recovered via vibrations. Vibrations of different types, frequency, and amplitude should be applied to the core. Thus, a relationship between vibratory stimulation at different frequencies and amplitudes and the increase in oil production percentage can be established.

6.2.2 Additional numerical simulations

The current numerical model has provided valuable information regarding the relationship between a pressure drop and the formation of a water film, the effects of different parameters on a pressure drop, and the effects of vibratory stimulation on a pressure drop. More research remains to be undertaken, as described below.

A new capillary tube model, with a more complex geometry to represent a real pore
with a varying and uneven diameter, is needed in order to understand how effective water film formation is for the oil drops that are trapped due to constrictions in the pore throat. Vibrations should make it easier to push the trapped oil out of the pore throat by water film formation and as an external force but its effect needs to be quantified.

Effects of other parameters such as oil slug length, oil slug viscosity, and tube diameter have not been studied in this thesis due to time constraints. These are important factors that could impact the maximum pressure drop needed to mobilize the oil slug in simulations, with and without vibrations. Hence, simulations to study the affect of these parameters on the pressure drop should be conducted.

All simulations with external excitation in this thesis are first vibrated, followed by water injection. Simultaneous water injection and vibration has not been attempted because it requires a longer tube for the oil slug to reach a steady state and simulating a longer tube is more computationally expensive. Hence, an optimum meshing with faster computational capability should be developed for simultaneous vibration and water injection to see if the results are different from those obtained in this thesis.

Simulations with multiple slugs in the capillary tube, without any initial water film thickness, should be run. This will show how mobilization occurs when there is more than one oil slug present. A water film might develop simultaneously for either slugs, or one at a time, or perhaps the slugs might join together to form one giant oil slug. Simulations with more than one oil slug should also be run with external excitation.

Vibrations without water injection simply move the oil slug back and forth in a simple harmonic motion. In this thesis, the authors have started the water injection when the oil slug was covered by the maximum amount of water film on the right side (the flow of the oil slug is from left to right during water injection). It is not clear if the maximum pressure drop or the entire pressure drop vs. time graph would
be affected had the injection been started when the water film covered the left and right ends of the oil slug while the middle part remained in contact with the tube wall. These phenomena should be explored in far more detail.

6.2.3 Power requirement estimate

Energy required to recover an oil slug with external excitations is an important parameter. Any project starts with the estimate of economical feasibility before it can actually be undertaken. It is very important to produce the economical analysis for oil recovery with external excitations and compare it with other EOR methods to judge whether or not the external excitations approach can be undertaken at a larger scale.
BIBLIOGRAPHY


