MODELING FLOCCULATION IN SEDIMENTATION TANK WITH DEPTH-AVERAGED METHOD

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in Partial Fulfillment of the Requirements
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In
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By
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UNIVERSITY OF REGINA

FACULTY OF GRADUATE STUDIES AND RESEARCH

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Abstract

Sedimentation tanks are engineering constructions commonly used at water and wastewater treatment facilities, especially the rectangular sedimentation tanks. The construction, design, and maintenance of settling tanks greatly influence the accuracy of predictions of removal efficiency, thickness of sediment depositions, and particle size distributions in effluent. A one-dimensional mathematical model has been created to simulate the rectangular sedimentation tank and evaluate flocculation influence by using depth-averaged method. The model could be used for unsteady flow and non-uniform particles, and predict important information such as removal efficiency, sludge thickness, particle and sludge size distribution. By using the depth-averaged method with finite difference approximation, the flow governing equations, the sediment transport equations, and the equations which are used to calculate the floc settling velocities can be solved by using MATLAB program.

A comparison with the ideal model and Guo’s model showed that the flocculation model could be used for unsteady flow patterns, and that it is a comprehensive model inclusive of sedimentation tanks with fine particle flocculation. Furthermore, the model user can obtain the results for suspended sediment transport and the flocculation sedimentation tank at the same time to perform the comparison. This model can also provide large quantities of useful information regarding the tank, discrete particles, and floc, based on the desired removal efficiency, sediment concentration, sediment size in bottom sludge and effluent suspension, and sediment deposition thickness along the flow direction. The main objective of this research was to establish a full-scale numerical model to simulate rectangular sedimentation tank operation and evaluate its influence on flocculation.
The flocculation factor is a ratio between floc settling velocity and the corresponding discrete particle settling velocity, which was introduced to create the flocculation model. Using floc settling velocities, the flocculation sedimentation tank model reveals that the phenomenon of fine particle flocculation has a critical particle size, which means that particles greater than the critical particle size would be neglected during flocculation. This is consistent with the conclusion from Jianwei-Huang’s published paper in 1981 and other researchers’ investigations.

The derivation of governing equations, the numerical methods which were used to discretize the depth-averaged equations, the model setup procedure, and simulation results are discussed. The flocculation model is compared with experimental data and shown good agreement. At the end, the code for the numerical study has been included in the appendix.
Acknowledgement

I would like to express my sincere gratitude to my supervisor Dr. Yee-Chung Jin for the abundantly helpful, continuous support, and guidance. My sincere thanks also go to my co-supervisor Dr. Gang Zhao, for his encouragement, invaluable assistance, and immense knowledge. This research would not have been possible without their support.
Dedication

Thanks to my parents for their love and supporting me to come to Canada, my husband and parents in law take care of me during years, and relatives who endured this long process with me.
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### Notation

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>The cross-sectional area in the flow direction of the settling zone</td>
</tr>
<tr>
<td>$A'$</td>
<td>The surface area of the rectangular sedimentation tank</td>
</tr>
<tr>
<td>$a_i$</td>
<td>The empirical constant for the flocculation model</td>
</tr>
<tr>
<td>$B$</td>
<td>The width of the settling zone</td>
</tr>
<tr>
<td>$C$</td>
<td>The Chezy Coefficient</td>
</tr>
<tr>
<td>$C_{\mu}, C_1, \text{ and } C_2$</td>
<td>The standard constants of $\kappa - \varepsilon$ model</td>
</tr>
<tr>
<td>$c_0$</td>
<td>The inlet concentration</td>
</tr>
<tr>
<td>$c_i$</td>
<td>The sediment concentration for each groups in Lyn's model</td>
</tr>
<tr>
<td>$c_{sa}$</td>
<td>The raw water salinity</td>
</tr>
<tr>
<td>$c_{sap}$</td>
<td>The salinity at which its influence on the floc settling velocity is minimal</td>
</tr>
<tr>
<td>$c_{sa,min}$</td>
<td>The minimum threshold value of salinity for the raw water</td>
</tr>
<tr>
<td>$d$</td>
<td>The diameter of the particle</td>
</tr>
<tr>
<td>$d_0$</td>
<td>The critical diameter of suspended particles</td>
</tr>
<tr>
<td>$d_r$</td>
<td>A reference diameter</td>
</tr>
<tr>
<td>$dP%$</td>
<td>The proportion of particles with settling velocity equal to $\omega_1$</td>
</tr>
<tr>
<td>$F$</td>
<td>The flocculation factor</td>
</tr>
<tr>
<td>$F_{\text{floc}}$</td>
<td>The flocculation model</td>
</tr>
<tr>
<td>$f$</td>
<td>Darcy-Weisbach friction factor</td>
</tr>
<tr>
<td>$f_x$</td>
<td>The body force in the $x$ direction</td>
</tr>
<tr>
<td>$g$</td>
<td>Gravity acceleration</td>
</tr>
<tr>
<td>$H$</td>
<td>The surface elevation of the water</td>
</tr>
<tr>
<td>$H'$</td>
<td>The water depth in settling zone in Chapter 2</td>
</tr>
</tbody>
</table>
The depth of the water in the settling zone

The subscript of space scale when construct the model

The superscript of time scale when construct the model

Which is equal to \[ \left( \frac{1 + k_1 s_p^a}{1 - k_2 s_p} \right) \]

A coefficient reflects the capacity of sediment transported by a certain flow

Coefficients used to calculate flocculation correction factor

Correction factors, which indicate the influence of particle size, sediment concentration, salinity, and turbulence intensity

An empirical coefficient

A dimensionless coefficient

The length of the rectangular sedimentation tank

A coefficient, suggested value as 0.92

The number of particle groups

Manning roughness coefficient

An empirical exponent with value between 1.8 to 2.0

An empirical exponent ranging between 1 and 2 with a mean value of 1.3

An empirical exponent

Empirical coefficients

The wetted perimeter

The proportion of particles with a settling velocity smaller than the minimum settling velocity \( \omega_0 \)

Pressure of the flow
\( p_b \) Water pressure at the flow bottom

\( p_H \) Water pressure at the flow surface

\( \bar{p} \) The depth-averaged flow pressure

\( P_k \) The rate of production of turbulent kinetic energy which due to mean velocity gradients

\( Q \) The inflow rate

\( q \) The discharge per unit width

\( q_v \) The overflow rate (surface loading rate)

\( R \) The hydraulic radius

\( R_l H \) Richardson

\( R_n \) Truncation Error or Lagrange Remainder

\( r \) An empirical exponent ranging between 3 and 5

\( s \) The time-averaged suspended sediment concentration in water

\( s_b \) and \( s_H \) Suspended sediment concentration at the bottom of the tank and the surface of the flow

\( \bar{s} \) The depth-averaged sediment concentration of the cross-section

\( s^* \) The mean sediment carrying capacity

\( s_b^* \) The sediment carrying capacity near the bed of the rectangular sedimentation tank

\( s_p \) The sediment concentration related to the properties of water-sediment mixture, its value varies from 1.5 to 15 kg/m$^3$

\( T \) Detention time

\( t \) Time scale (sedimentation tank operation time)

\( t_s \) Settling time
\( \Delta t \) Time interval

\( U \) The nominal tank velocity

\( u \) The longitudinal flow velocity

\( u_H \) and \( u_b \) Horizontal flow velocity at water surface and the tank bottom

\( \bar{u} \) The depth-averaged horizontal flow velocity

\( u_s \) The shear velocity

\( V \) The volume of the rectangular sedimentation tank

\( V_{sc} \) The scour velocity for the rectangular sedimentation tank

\( v \) The flow velocity in the \( y \)-direction in a two-dimensional turbulent model

\( v' \) The suspended particles horizontal velocity

\( w \) The vertical flow velocity

\( w_H \) and \( w_b \) Vertical flow velocity at water surface and the tank bottom

\( x \) and \( z \) The horizontal (flow direction) and vertical coordinates, respectively

\( \Delta x \) Space interval

\( y \) The horizontal coordinate (perpendicular to flow direction) in Cartesian Coordinates

\( z_b \) The total elevation of the tank bottom including the thickness of the sludge

\( z_g \) The elevation of the tank bottom

\( z_s \) The thickness of the sludge which accumulated at the bottom of the tank

\( \tan \alpha \) \( \tan \alpha = \frac{\partial z_g}{\partial x} \) is the bottom slope of the tank
\( \beta, \beta_1 \) and \( \beta_2 \)  
Ratio coefficients

\( \varepsilon \)  
Dissipation rate of turbulent energy

\( \varepsilon_x \) and \( \varepsilon_z \)  
The diffusion coefficients in \( x \) and \( z \) direction

\( \left( \varepsilon_x \frac{\partial x}{\partial z} \right)_b \)  
The quantity of sediments re-suspended from the tank bottom to the flow very near the bed of the tank due to diffusion

\( \left( \varepsilon_x \frac{\partial x}{\partial z} \right)_H \)  
The quantity of sediments re-suspended from the tank bottom to the surface of the flow

\( \gamma \)  
The proportionality constant relating concentration to density differences

\( \kappa \)  
The turbulent kinetic energy per unit mass

\( \lambda' \)  
The drag coefficient

\( \mu \) and \( \nu \)  
The dynamic and kinematic viscosities of water

\( \mu_{t1} \) and \( \mu_{t2} \)  
Dynamic viscosities of the water at the temperature \( t_1 \) and \( t_2 \), respectively

\( \nu_{eff} \)  
The effective viscosity

\( \nu_t \)  
The turbulent viscosity

\( \rho \)  
The mass density of the fluid

\( \rho_1 \)  
The mass density of the particle

\( \rho_d \)  
The density of the sediment

\( \sigma_x \)  
The Reynolds turbulent normal stress assigned along the \( x \)-direction

\( \sigma_{xb} \) and \( \sigma_{xH} \)  
The Reynolds turbulent normal stress in the \( x \)-direction at the bottom of the tank and the surface of the flow
\( \sigma_\kappa, \sigma_c, \text{and} \ \sigma_\varepsilon \)  The turbulent Schmidt numbers for \( \kappa, c \) and \( \varepsilon \) (dimensionless number)

\( \bar{\sigma}_x \)  The depth-averaged Reynolds turbulence normal stress

\( \tau_b \)  Shear stress

\( \tau_p \)  Bed shear stress

\( \tau_{xb} \text{ and } \tau_{xH} \)  The Reynolds turbulent shear stress in the \( x \)-direction at the tank bottom and the surface of the flow

\( \tau_{zx} \)  The Reynolds turbulent shear stress assigned at the bottom of the flow

\( \omega \)  The settling velocity

\( \omega_0 \)  The minimum settling velocity

\( \omega' \)  The particles’ free falling velocity

\( \omega_b \text{ and } \omega_H \)  The particles’ falling velocities at the bottom of the tank and the surface of the flow

\( \omega_{bs} \)  The quantity of sediments settled to the bottom of the tank due to gravity per unit time

\( \omega_f \)  The settling velocity of floc

\( \omega_{fp} \)  The settling velocity for each floc group

\( \omega_{hs} \)  The quantity of sediments settled to the surface of the flow per unit time

\( \omega_{si} \)  The settling velocity for each group of particles

\( \zeta_p \)  The particle size distribution

\( \zeta_p^* \)  The percentage of the group of particles in the sediment-carrying capacity

XVI
Chapter 1 Introduction

1.1 Background: Open-channel Flow

1.1.1 Open-channel Flow

Open-channel flow describes flow which is not entirely closed by solid boundaries. A surface not adjacent to a solid boundary is called a free surface. The free surface of an open-channel flow has the boundary conditions of pressure equals to the atmospheric pressure and the shear stress equals to zero. The main driving force is the fluid weight-gravity force as there are no pressure forces driving flow through the channel. Most open-channel flows involve fluids with a fairly low viscosity and relatively large characteristic lengths (Chanson 2004).

Open-channel flow has been well researched both in theoretical and practical terms for centuries because of its important applications in human society, such as sedimentation tanks, canals, partially full sewers, etc. Many natural systems also exhibit open-channel flows such as estuaries, rivers, and streams. Open-channel flow systems perform many functions specific to agriculture, industry, and municipal water management. Therefore, it is necessary to learn the characteristics of open-channel flow in order to reduce the risk and raise the efficiency of open-channel flows and open-channel constructions.

In general, gravity-driven flow and the existence of a free surface are the two defining characteristics of open-channel flow compared to closed conduits, which is why open-channel flow is also called free surface flow. A sedimentation tank is a practical engineering application of open-channel flow in water and wastewater treatment. This is particularly true when the suspended sediment concentrations are low, and the presence of the particles do not affect the mechanics of the flow pattern (Imam et al. 1983). The derivation of the governing equations for sedimentation tanks is based
on the open-channel flow boundary conditions with the addition of some carefully constructed assumptions. Correlating full-scale lab experiments to the results generated from numerical models is an effective method for developing accurate and predictive modeling tools.

1.1.2 Modeling Open-channel Flow

Numerical analyses of open-channel flows have been in development for many decades. Researchers often focus on describing the characteristics of open-channel flows by using several governing equations. When conducting open-channel flow simulations, there are many important factors need to take into account, such as water depth, discharge rate, and velocity (Jain and Jain 2001). With the development of hydraulic simulations, computational fluid dynamics (CFD) became a very useful tool for modeling open-channel flow. Through the use of CFD, many hydraulic models have been developed, and the study of open-channel flow has been opened up to the field of computer science (Fischer-Antze et al. 2001). In most cases, open-channel flow is treated as a one-dimensional fluid flow since the depth of the water is always much smaller than the length of the flow path. This assumption can also be used when setting up a sedimentation tank model (Katopodes 1984).

Depending on how the flow velocity at a given point changes with respect to time, open-channel flow can be divided into steady flow and unsteady flow (Chaudhry 1993). In this research, steady and unsteady flows were considered for both derivate and discretize governing equations for flow pattern. During the derivation of the governing equations, the depth-averaged method was used to simplify the two-dimensional Reynolds mass conservation equation and momentum equation into one-dimension, which helped to reduce the complexity of the governing equation with little loss in accuracy (Jin and Steffler 1993). By using the depth-averaged method there are
two important assumptions: a uniform vertical velocity distribution and a hydrostatic pressure distribution (Steffler and Blackburn 2002). With known water depth at the boundaries, the depth at each point or node will be known after solving the governing equations.

1.2 Background: Sedimentation Tanks

1.2.1 Sedimentation Tanks

In the past decades, the transportation and accumulation of pollutants were essential topics of engineering studies, and sedimentation control played an important role in water and wastewater management (García 1999). The sedimentation tank is one of the most important and widely used operations in water and sewage treatment facilities. Sedimentation uses the force of gravity to settle out relatively massive particles from the surrounding water. Approximately one-third of investments in treatment facilities is used to finance settling tanks (Swamee and Tyagi 1996). The particle size distribution and suspended sediment concentration at the outlet of the tank are primary water quality parameters for wastewater treatment processes, and can be used as an indicator of overall water quality.

Normally, the greater the overflow rate, or discharge, the higher the sediment transport capacity (García 1999). The design of ideal sedimentation tanks is usually based on overflow rate, detention time, or both. If the settling velocity and the overflow rate are the same value, then the settling velocity is called the critical settling velocity. When settling velocity is greater than the critical settling velocity the suspended particles can be completely removed from water irrespective of particle size distribution. Conversely, when particle settling velocity is smaller than the critical settling velocity, the suspended particles have only partially settled to the bottom of the tank (Jin et al. 2000). However, due to the complexities of non-uniform sediment transport, this
designation principle has often failed to predict or explain the behavior of sediment transport under real operating situations. A design procedure based on the removal efficiency and the sludge scouring rate would be more reliable (Swamee and Tyagi 1996).

With proper design, the rectangular sedimentation tank more closely resembles an ideal sedimentation tank compared to a circular sedimentation tank. Rectangular sedimentation tanks can be divided into four different functional zones (1) the inlet zone: the region where the flow enters and is uniformly distributed over a vertical cross-section of the sedimentation chamber, (2) the settling zone: the region where pollutant settling mainly occurs under quiescent conditions, (3) the outlet zone: the region where clarified effluent is collected and discharge through a weir, and (4) the sludge zone: the region used for the collection of sludge below the settling zone. The length of the settling zone may vary from two to four times the width, and may be ten to twenty times the depth (Qasim et al. 2000).

1.2.2 Modeling Sedimentation Tanks

The conventional empirical models for sedimentation tanks are widely used today to predict the main characteristics of the effluent, such as suspended solids (SS) concentration and process design before real construction and operation (Olsson and Newell 1999). Computational fluid dynamics (CFD) models are used to predict some important features of the flow, and the suspended solids distribution throughout the tank (Matko et al. 1996). For the simulation of particle removal from water and sewage by sedimentation, some important modeling criteria must be known, such as the flow velocity distribution, the particles settling velocity distribution, re-suspension of settled sediments from the bottom of the tank, flow variation, sediment concentration

The general equation for the settling velocity is given by Stokes’ law. The particle settling velocities are determined by the size and density differences between particles and water. When the density difference is constant, particle size is the main determinant. Conventional models that select only one average settling velocity for all suspended particles are inaccurate for calculating particle compositions either in effluent or in sludge (Huang 2010). As suggested by Guo’s research, by subdividing the particles into groups based on particle sizes, the new model can give results with more realistic values.

When considering how to solve sediment concentrations, the concept of sediment carrying capacity is utilized. The sediment carrying capacity is defined as a sediment concentration under ideal conditions, which means deposits coming from upstream can be carried entirely downstream without any deposition or erosion occurring over the bed (Jin et al. 2000). If the suspended sediment concentration has a different value to the sediment carrying capacity, deposition or erosion will occur. Deposition occurs when the sediment concentration in the water exceeds the carrying capacity of a certain flow. Over time, the suspended sediment concentration will reach the same number as the carrying capacity. At this point, the corresponding flow is under saturated conditions. Operation beyond this will result in erosion, as the sediment concentration is less than the carrying capacity. The saturated condition, or the ideal condition, for a certain flow is also referred to as its equilibrium condition. In actuality, equilibrium conditions cannot be used directly in real situations, but they can be appropriately used in non-equilibrium conditions when additional assumptions and coefficients are incorporated (Guo and Jin 1999).
By using Stokes’ Law, the velocity of discrete particles can be known. Then, by introducing a flocculation factor, the velocities for corresponding floc can be determined. By applying the Lagrange interpolating polynomials method, floc size and floc size distribution can also be obtained. However, the floc sizes obtained by the Lagrange interpolating polynomials method does not reflect actual floc sizes, because with the same settling velocity the floc sizes are normally smaller than the discrete particles (Wu 2007). This is because the shape of a floc particle is not spherical. The flocculation factor and its components will be introduced in the following section.

1.2.3 Cohesive Sediment Transport

In a sedimentation tank many fine particles are present with cohesive properties. Fine particles are difficult to separate from water because they lack sufficient mass to settle by gravity, and they may be small enough to pass through the pores of filtration media. To remove the fine particles, the individual colloids may be encouraged to aggregate together to form a larger particle. The aggregation of fine particles can be considered in two distinct steps: (1) fine particles move toward each other and undergo inter-particle collision; and (2) the colliding particles coalesce if both particles are sufficiently destabilized. The first step is known as flocculation. The process of destabilization and transportation is known as coagulation.

After a brief review of the development and generally accepted theories of cohesive sediment transport, flocculation is found to be a significant factor affecting sediment transport when the particle size is below a certain threshold. This threshold is called critical particle size (Kranck 1980). In Dangwei Wang’s research, 0.009 mm to 0.032 mm was determined to be the best range for critical particle size when considering flocculation (Wang et al. 2007). In Kate Kranck’s research, when considering flocculation, the value of critical particle size was determined to be smaller
than 0.02 mm (Kranck 1980). In Wu’s research, it was suggested that flocculation could be neglected when particle sizes exceeded 0.03 mm.

The reason critical particle size exists is because all colloidal particles possess an electric charged, as shown in Figure 1.1. When the electrostatic force is greater than or equal to the force of gravity, fine-grained sediments may stick together to form bigger floc (Wu 2007). On the other hand, because larger particles undergo higher shear stress, the formation of large floc increases their tendency to disaggregation into finer particles and/or single particles. An electrostatic force is a type of dynamic non-contact force which arises between two separated particles at a distance. If particles with the same charges, either positive or negative, interact via the electronegative force, they repel each other. And the force is positive as the distance between them increases positively. If both charges are unlike each other, the particles will attract one another, and the force is negative as the distance between the particles decreases. The electrostatic force is a function of distance, with the force decreasing in magnitude as the distance between interacting particles increases.

As described by Stokes’ Law, larger particles have greater settling velocities. When fine grained particles form a larger floc particle, the sediment concentration is also affected. To calculate the floc settling velocity, the flocculation factor must be considered. Flocculation factor is affected by many factors, including sediment concentration, particle size, temperature, turbulence, salinity, etc. (Jianwei 1981; Wu 2007).
Figure 1.1  Schematic of General Potential Energy Curves
As summarized by Thorn (1981), Mehta (1986), and Wu (2007), flocculation is highly related to sediment concentration, which can be explained in two situations. First, when the sediment concentration is initially low but subject to an increase, the chances of an inter-particle collision increases. This means that particles have more opportunities to stick together and form larger floc, and the settling velocity increases. Second, when the sediment concentration is high, single-grain particles will easily form into giant-scale floc matrices which act like huge nets (Nicholson and O'Connor 1986). The settling velocity of these huge nets is very low. When the raising force equals the gravitational force the settling velocity can even be zero. In this scenario, the mixture of water and sediment becomes a non-Newtonian fluid.

Salinity influences flocculation significantly, as a result of the electrostatic force (Jianwei 1981; Wu 2007). In saline water, the repulsive force is reduced below a critical value and the attractive force becomes dominant. Under these conditions, particles stick together to form larger floc. When water salinity is low, the particle’s settling velocity increases rapidly with increasing salinity. When water salinity exceeds a certain value, it has limited influence on floc settling velocity.

Flocculation gets affected by turbulence intensity in two different ways (Haralampides et al. 2003; McConnachie 1991). For low turbulence intensity, increasing turbulence increases the chances of inter-particle collisions, which increases floc aggregation. For high turbulence intensity, increasing turbulence may disaggregate large floc into smaller floc and single particles, as a result flocculation decreases.

Temperature indirectly affects flocculation through its affect on flow velocity. Organic matter also affects flocculation, though the quantification of its influence currently requires further investigation (Wu 2007).
1.3 Research Objectives

This thesis entails the development of a relatively simple and practical numerical model for rectangular sedimentation tank with considering the flocculation influence by using depth-averaged method. The present study focuses not only on suspended sediment transport but also cohesive sediment transport in the sedimentation tank. The model can provide the results for the flocculation model and the suspended sediment transport model at the same time and perform the comparison analysis to achieve a better understanding of different types of sediment transport. A model was set up for rectangular sedimentation tanks with unsteady horizontal flow, non-uniform particle size distribution, and suspended particles divided into groups based on their size distribution.

The numerical model was designed to be comprehensive, accurate and stable. The objectives of the rectangular sedimentation tank simulation were: (1) to develop an full-scale, user-friendly model, capable of simulating and investigating behaviors of particle aggregation and settling transportation; (2) to make the simulation as general as possible, so that the theory for the sedimentation tank could be explored, and so the dimensionality of the simulation was not changed through simplified assumptions. For instance, assuming the particle size is uniform in each group and the numbers of groups depend on model users’ preference and the requirement of accuracy.

Compared to previous rectangular or circular sedimentation tank models, the new model is able to provide additional important information. Points of interest investigated by this model include water elevation, suspended sediment concentration, the size composition of bottom sludge and effluent suspension, the thickness of bottom sludge along the flow direction, and tank dimensions based on desired removal efficiency.
When flocculation is considered, the sediment removal efficiency predicted from the new model has good agreement with the El-Baroudi’s experimental data for larger particles. When flocculation is not considered, the removal efficiency predicted from the model has good agreement with El-Baroudi’s experimental results for smaller particles. Both of the results are better than the results generated from El-Baroudi’s numerical model and other applied models.

1.4 Thesis Organization

This thesis consists of six chapters.

Chapter 2 includes a literature review of numerical models which have been created for sedimentation tanks. Also reviewed is the ideal sedimentation tank design. This chapter details the flow and sediment transport information in sedimentation tanks.

Chapter 3 fully details the derivation of the flow pattern equations and sediment transport equations for the proposed model. The flocculation factor will be introduced to calculate the floc settling velocities. The governing equations for the new model are derived from the Reynolds two-dimensional mass conservation and momentum equations, and the depth-averaged method will be used to simplify the two-dimensional continuity and momentum equations into one-dimension. The suspended sediment transport equations for non-uniform particle size distributions in settling tanks and the equations which can be used to calculate the velocities for floc will be described.

Chapter 4 establishes the model step-by-step based on the numerical formulation of the governing equations. This includes a discretization of the differential equations, solving these equations to find the property values for all variables along the flow direction, and using MATLAB to implement the model computation. The coefficients used in the governing equations are also discussed in this chapter.
Chapter 5 discusses the numerical results from the ideal model, and the results from the flocculation model and the suspended sediment transport model. Comparison and analysis between models and experimental data are also provided.

Chapter 6 summarises the new flocculation model and provides the conclusions of the research. Recommendations for future works are also provided.
Chapter 2 Literature Review

A comprehensive literature review is presented here to gather the background and supporting information necessary for this sedimentation tank research. The original application of sedimentation occurred since ancient times all over the world. In China, the knowledge and technique of sediment transport has been in use since 4000 BC. The Great Yu achieved success by bringing the Huang He (the Yellow River) under control. The Yellow River is the sixth-longest river in the world but with the highest sediment concentration (Guo 2001). Mesopotamia is a fluvial area of the Tigris-Euphrates river system, which is formed by the accumulated sediments from rivers. The accumulated sediments from the rivers are rich in organic matters and minerals which helped to support a dense population and provided great agricultural development in this region.

In the 19th and early 20th centuries, a period known as the Gold Rush began. During this time a rush of migrate workers sought to find gold deposits. Workers would wash free gold particles loose from surface sediment. In recent times, sediment transport has been applied more effectively and widely in daily life, industry, and agriculture. The application of sedimentation tanks in water and wastewater treatment is a tremendous contribution which greatly improves the water quality and the quality of people’s lives. Because of this, numerous formulas for sedimentation have been developed since the beginning of the 20th century, and the exploration and research into sediment transport has continued ever since. The experimental works and numerical models which focus on particle settling in sedimentation tanks have become common with high requirements for accuracy.

2.1 Theoretical Background

Many great researchers have contributed to the study of hydraulic phenomena, including Bernoulli (1700-1782), Euler (1707-1783), Saint-Venant (1797-1886), Darcy
(1803-1858), Pitot (1695-1771), Lagrange (1736-1813), Chezy (1718-1798), Navier (1785-1836), Froude (1810-1879), Stokes (1819-1903), Manning (1816-1897), Reynolds (1842-1912), Boussinesq (1842-1929), and many others. From hundreds years ago to the present day, their contributions are remembered and their investigations are widely used in all aspects of hydraulic research. Saint-Venant Equation and the Stokes’ Law were mainly used in my research, there are detailed introduction in the following part.

2.1.1 Saint-Venant Equation

The one-dimensional Saint-Venant equation was derived by Saint-Venant (1797-1886). It is based on the shallow water hypothesis and has been widely used to model open-channel flow and surface runoff due to its efficiency and accuracy (Abbott and Minns 1998; Bakhmeteff and Scobey 1932). For rectangular sedimentation tanks, the one-dimensional Saint-Venant equation can be applied to solve for the elevation of the water. The one-dimensional Saint-Venant equation can also be used to describe the fluid motion when written in the following form (expressed in Cartesian Coordinates in the $x$ direction):

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2} \right) + f_x$$

where $x$ and $z$ are the horizontal (flow direction) and vertical coordinates, respectively; $t$ is the time scale; $u$ and $w$ are the longitudinal and vertical flow velocities, respectively; $p$ is the pressure of the flow; $\rho$ is the mass density of the fluid; $\nu$ is the kinematic viscosity; $f_x$ is the body force in the $x$ direction.

The Saint-Venant equation is a combination of the continuity equation and the momentum equation. As a result it is difficult to solve these two equations together. Many numerical technologies have been utilized in order to calculate the Saint-Venant
equation. Mahmood and Yevjevich (1975) provide some approaches to solve general open-channel problems. Molls and Molls (1998) developed a space-time conservation method in order to simulate the Saint-Venant equation with less simulation time. Hu and Sueyoshi (2010) use both numerical and experimental approaches to study the Saint-Venant equation for dam break cases. Steffler and Jin (1993) selected a depth-averaged method to simplify the equation by using a depth-averaged flow velocity instead of the velocity in the vertical direction.

### 2.1.2 Stokes’ Law

Stokes’ Law, named after George Gabriel Stokes’ (1819-1903), describes the drag force on spherical objects in continuous flow scenarios in which the Reynolds number is very small. The flow in rectangular sedimentation tanks should be slow and can be treated as laminar flow. When setting up a model, it is therefore valid to assume that the only forces acting on the particles are gravity and the drag force. Considering the gravitational force alone, particles accelerate as they settling out of suspension. However, as they accelerate, the drag force increases until it exactly balances the force of gravity. In other words, the change of the particles settling velocity will become constant. Particles will settle to the bottom of the sedimentation tank with a constant settling velocity. This settling velocity can be expressed by Stokes’ Law:

$$\omega = \frac{1}{18} \frac{g}{\mu} (\rho_1 - \rho) d^2$$  \hspace{1cm} (2.2)

in which $\omega$ = the settling velocity; $g$ = gravity acceleration; $\mu$ = the dynamic viscosity of the fluid; $\rho$ is the mass density of the fluid; $\rho_1$ = the mass density of the particle; $d$ = the diameter of the particle. The equation (2.2) has been widely used for many years because it has been examined experimentally by several researchers for many kinds of situations and shows good consistency.
From the physical quantities involved in equation (2.2), there are some issues that can be discussed relating to settling velocity: (1) When $\rho_1 > \rho$, the suspended particles fall with a settling velocity equal to $\omega$. When $\rho_1 = \rho$, the particles remain in suspension; in this situation, particles cannot be removed either by sedimentation or flotation. When $\rho_1 < \rho$, the particles float with a velocity equals to $\omega$. These particles can be removed by flotation. (2) The settling velocity $\omega$ is directly proportional to the square of the particle diameter $d$. By increasing the particle size, particle removal efficiency can be improved. (3) The settling velocity $\omega$ is inversely proportional to the dynamic viscosity of the fluid $\mu$. The viscosity decreases with increasing temperature. As a result settling velocity will also increase.

From Stokes’ law, it is observed that particle size and the density difference between particle and fluid are the main determinants for particle settling velocity. Sedimentation happens when there is a density difference between the particles and the fluid. Particles will move vertically due to the force of gravity. Specific gravities range from about 2.6 for fine grained particles to about 1.03 for floc containing 95% water (Camp 1946). In actuality, sediment particles vary in size, shape, specific gravity, and settling velocity. In order to achieve better performance, suspended particles and floc are separated into groups based on sieve analysis. Particles and floc sharing the same diameter, specific gravity, settling velocity, and shape are grouped together.

### 2.2 Sediment Transport Models

Presently, the methods used to evaluate suspended particle settling processes are generally based on two approaches. The first approach is to use the ideal settling tank concept which was introduced in the first chapter. The other approach is to use computational fluid dynamics (CFD) models. In the ideal settling tank concept, first developed by Hazen (1904) and Camp (1946), the preferred shape of the tank is set to
be rectangular (El-Baroudi 1969). Fluid flow into the settling zone is steady and uniformly distributed, the suspended particles in the flow direction are also uniformly distributed, and discrete particle settling is considered without flocculation. Once particles settle to the bottom of the tank these particles are considered to be completely removed (Krishnappan and Marsalek 2002).

The typical models for understanding sediment transportation in sedimentation tanks come from Schamber and Larock (1981), Imam et al. (1983), El-Baroudi (1969), Ostandorf and Botkin (1987), Adams and Rodi (1990), Zhou and McCorquodale (1992), Swamee and Tyagi (1996), Jin et al. (2000), and Guo (2001). These models were set up for Type I sedimentation tanks which do not consider flocculation during the settling process.

The El-Baroudi’s model is a one-dimensional sedimentation tank model set up to discuss the influence from eddy diffusion. This model has also undergone experimental verification (El-Baroudi 1969). Imam’s model is a two-dimensional turbulent flow model of an incompressible-Newtonian fluid, which consists of a hydrodynamic sub-model and a transport sub-model. This model considers particle settling processes as a transport phenomenon and simulates it under neutral density conditions (Imam et al. 1983). Jin’s model is a one-dimensional model that considers steady gradually varied flow with non-uniform particle size distributions in a Type I settling tank. Jin’s model will be used in Chapter 5 in comparison with the author’s one-dimensional unsteady uniform flow model to discuss the performance of the new model.

2.2.1 Ideal Sedimentation Tank

To illustrate the working principles of the sedimentation tank, and to analyze the motion of the suspended particles, Hazen (1904) and Camp (1946) developed the
concept of an ideal settling tank. For an ideal sedimentation tank, the suspended particles undergo two kinds of movement once in the settling zone. On the one hand, the horizontal water flow direction moves the particles horizontally. The horizontal velocity of the particles is equal to the flow velocity:

\[ v' = \frac{Q}{A} = \frac{Q}{H'B} \]  

(2.3)

in which \( v' \) is the suspended particles' horizontal velocity. \( Q \) is the inflow rate. \( A \) is the cross-sectional area in the flow direction of the settling zone. \( H' \) is the water depth in the settling zone. \( B \) is the width of the settling zone.

On the other hand, particles may fall vertically due to the force of gravity. The vertical velocity of the particles is equal to its free falling velocity:

\[ \omega' = \sqrt{\frac{4g(\rho_1 - \rho) \cdot d}{3\lambda' \cdot \rho}} \]  

(2.4)

where \( \omega' \) is the particle’s free falling velocity. \( \lambda' \) is the drag coefficient. This value is dependent on the Reynolds number. Depending on the value of the drag coefficient, equation (2.4) can be used both in laminar and turbulent flow. Equation (2.4) is also called Newton’s Law, which is similar to Stokes’ Law. Stokes’ Law can be seen as a specific case of Newton’s Law, as it is limited exclusively to laminar flow systems (Huang 2010).

In reality, a particle’s motion trail is the vector addition of its horizontal velocity \( v' \) and its vertical velocity \( \omega' \). During the settling process, the particle’s motion trail will resemble a sloping line. The slope is given by:

\[ \text{slope} = \frac{\omega'}{v'} \]  

(2.5)

We assume the particles have a minimum settling velocity \( \omega_0 \). When a particle’s settling velocity \( \omega \) is greater than or equal to \( \omega_0 \), regardless of the where the particle entered the settling zone, it can be settled to the bottom of the tank and
removed completely. When \( \omega \) is less than \( \omega_0 \), two situations arise. In the first situation, consider a particle that entered into the settling zone at the water’s surface. Here, it will remain at the water’s surface and it will not settle to the bottom of the tank. Instead, it will remain suspended and flow out of the system. In the second situation, consider a particle that entered into the settling zone below the water surface. Here, it can settle to the tank bottom and be removed. So when \( \omega \) is less than \( \omega_0 \), particles can be partially removed.

As mentioned before, the overflow rate is a critical factor when considering an ideal sedimentation tank and the design of the ideal sedimentation tank is usually based on the simple rule of overflow rate. A supplementary parameter, the dimensionless velocity \( v'/\omega_0 \) is used here:

\[
\frac{v'}{\omega_0} = \frac{L}{H}
\]  

(2.6)

substituting equation (2.6) into the equation (2.3) yields:

\[
Q = \omega_0 \cdot \frac{L}{H} \cdot B = \omega_0 \cdot L \cdot B = \omega_0 \cdot A'
\]  

(2.7)

in which \( A' \) is the surface area of the rectangular sedimentation tank.

From equation(2.7), the overflow rate can be expressed as:

\[
\omega_0 = Q/A'
\]  

(2.8)

The \( Q/A' \) parameter can be used to estimate the settling efficiency of the sedimentation tanks. This parameter can also be assigned the variable \( q_v \) and is called the overflow rate or surface loading rate:

\[
q_v = Q/A'
\]  

(2.9)

From equation (2.8) and equation (2.9), it is observed that, in the ideal sedimentation tanks, \( \omega_0 \) and \( q_v \) share the same value but have different physical meanings. The unit for \( \omega_0 \) is meters per second (m/s), while \( q_v \), which represents the
flow rate per unit area per unit time, has the unit \( m^3/m^2 \cdot s \). Therefore, as long as the particle’s minimum settling velocity \( \omega_0 \) can be decided, the overflow rate \( q_o \) for the ideal sedimentation tank can be easily determined. Furthermore, equation (2.9) shows that the settling efficiency of an ideal sedimentation tank is related to the surface area of the tank \( A' \). The settling efficiency of an ideal sedimentation tank is not related to the elevation of the water \( H \), which also means that it is independent of the volume of the sedimentation tank \( V \).

Assuming the proportion of particles with a settling velocity equal to \( \omega_1 \) is \( dP\% \), among them \( ((h/H) \cdot dP\%) \) particles can be settled to the tank bottom and then removed. In the same settling time \( t_s \):

\[
\begin{align*}
h &= \omega_1 t_s \\
H &= \omega_0 t_s
\end{align*}
\]

Then

\[
\frac{h}{H} dP = \frac{\omega_1}{\omega_0} dP
\]

The removal efficiency for the particles with a settling velocity of \( \omega_1 \) (\( \omega_1 < \omega_0 \)) is:

\[
\int_0^{\omega_1/\omega_0} \omega_1 \cdot dP = \frac{1}{\omega_0} \int_0^{\omega_0} \omega_1 \cdot dP
\]

The total removal efficiency can be calculated as:

\[
\eta = (1 - P_0) + \frac{1}{\omega_0} \int_0^{\omega_0} \omega_1 \cdot dP
\]

where \( P_0 \) represents the proportion of particles with a settling velocity smaller than the minimum settling velocity \( \omega_0 \).
2.2.2 Swamee’s Sedimentation Tank Model

In 1996, Swamee published a paper about modifying the design method for rectangular sedimentation tanks. The method detailed in this paper is different from the conventional designing method. The traditional method focuses on the surface loading rate; Swamee’s method focuses on the preferred removal efficiency and the scouring rate of the accumulated sludge at the bottom of the tank.

Similar to the calculation for the removal efficiency of the ideal sedimentation tank, Swamee’s removal efficiency equation is based on particle size distribution, which means that the particles can be completely removed if their diameter exceeds a minimum particle size. In Swamee’s model, Stokes’ Law has been used to determine the particle’s settling velocity. Rouse’s equation (Rouse 1937) was then used to calculate the variation of suspended sediment concentration along the water depth in the channel.

In order to calculate removal efficiency, the particle size distribution has to be determined first. This can be done using Swamee and Ojha’s method (1991). After calculating particle size distribution, the particle-size-distribution curve can be plotted. In Swamee’s model, substituting Stokes’ Law and the slope of the particle-size-distribution curve into equation (2.14), allows removal efficiency to be determined (Swamee and Tyagi 1996). The removal efficiency can then be given by the designer at the sedimentation tank designing stage.

By introducing the shear velocity \( u_s \) and Darcy-Weisbach friction factor \( f \) with equation (2.2), the scour velocity for the rectangular sedimentation tank \( V_{sc} \) can be obtained (Swamee and Tyagi 1996):

\[
V_{sc} = \sqrt{\frac{8}{f} \cdot u_s} = \frac{6k_p Q}{BL} \left( \frac{Q}{VB} \right)^{0.125}
\]  

(2.15)
where $k_y$ is a dimensionless coefficient indicating the relationship between the shear velocity $u_*$ and the particle’s settling velocity $\omega$ from equation (2.2). The suggested value of $k_y$ is between 0.5 and 0.8 (Ingersoll et al. 1956).

Assuming the surface loading rate equals the value of the scouring velocity $V_{sc}$, the dimensions of the rectangular sedimentation tank can be written as:

$$B = \frac{3.51Q}{d_0 \sqrt{k_y (\rho_1 - \rho) gH}} \left[ \frac{\mu}{d_0 \sqrt{k_y (\rho_1 - \rho) gH}} \right]^{9/7}$$  \hspace{1cm} (2.16)

$$L = 5.31 k_y H \left[ \frac{d_0 \sqrt{k_y (\rho_1 - \rho) gH}}{\mu} \right]^{2/7}$$  \hspace{1cm} (2.17)

in which $d_0$ is the critical diameter of the suspended particles, which can be obtained based on the model user’s preferred removal efficiency.

Compare to the ideal model, Swamee’s model considers scouring accumulated sludge from the tank. This is a great contribution. Swamee’s model can be used to determine the dimensions of the rectangular sedimentation tank based on the user’s preferred removal efficiency. But without the removal efficiency along the length of the tank, it is hard to modify the dimensions of the tank to guarantee the design objectives (reduce cost, saving investment, etc.) without sacrificing the performance of the tank. Secondly, there is no consideration of the effluent quality in Swamee’s model. For wastewater treatment facilities, the effluent particle size distribution is an important parameter that needs to be estimated.

2.2.3 Lyn’s Model

In the research area of sediment transport, it is generally understood that low suspended sediment concentrations and low shear stress normally promote flocculation, whereas high suspended sediment concentrations and high shear stress appear to promote floc disaggregation. The effect of suspended sediment concentration appears to be greater than that of the shear stress (Dyer and Manning 1999). Test results from Lyn
et al. (1992) are consistent with conclusions from Dyer and Manning, which suggest that sediment density differences have more influence on changing flow pattern than that of the shear stress induced flocculation. The difference in suspended sediment concentration between the receiving water in the tanks and the influent has always existed, which may also affect the flow in the tank.

In 1992, Lyn et al. created a numerical model for the two-dimensional turbulent flows in rectangular sedimentation tanks. One of the most significant characteristics of Lyn’s model is its estimation of flocculation effects. Conventionally, the design of sedimentation tanks is mainly based on the empirical discovery and the ideal sedimentation tank model. This model provides another possibility for the design, performance prediction, and maintenance of the tank.

Settling characteristics such as size distributions, settling velocities, and the sediment concentrations of the suspended particles in the influent greatly influences the removal efficiency of the sedimentation tanks (Metcalf 2003). Using one uniform settling velocity for all kinds of suspended particles in the sedimentation tank may cause overestimation of the removal efficiency (E. H. H. Imam, 1981). From Camp (1946) researchers have a broader agreement about considering flocculation under the flow pattern of turbulent flow. The governing equations for the flow pattern use the \( \kappa - \varepsilon \) turbulence model. The governing flow equations are:

\[
\frac{\partial (u\kappa)}{\partial x} + \frac{\partial (v\kappa)}{\partial y} = \frac{\partial}{\partial x} \left( \nu_{\text{eff}} \frac{\partial \kappa}{\partial x} \right) + \frac{\partial}{\partial y} \left( \nu_{\text{eff}} \frac{\partial \kappa}{\partial y} \right) + P_{\kappa} - \varepsilon + R_{\text{Ht}} \frac{\nu_{\text{t}}}{\sigma_{\varepsilon}} \frac{\partial}{\partial y} \quad (2.18)
\]

\[
\frac{\partial (u\varepsilon)}{\partial x} + \frac{\partial (v\varepsilon)}{\partial y} = \frac{\partial}{\partial x} \left( \nu_{\text{eff}} \frac{\partial \varepsilon}{\partial x} \right) + \frac{\partial}{\partial y} \left( \nu_{\text{eff}} \frac{\partial \varepsilon}{\partial y} \right) + C_1 \frac{\varepsilon}{\kappa} P_{\kappa} - C_2 \frac{\varepsilon^2}{\kappa} \quad (2.19)
\]

where \( \kappa \) is the turbulent kinetic energy per unit mass. \( v \) is the vertical flow velocity in the \( y \) direction in a two-dimensional turbulent model. \( y \) is the distance from the tank bottom. \( \nu_{\text{eff}} \) is the effective viscosity; This factor considers both turbulent viscosity...
and molecular viscosity. $\sigma_\kappa$, $\sigma_c$, and $\sigma_\varepsilon$ are the turbulent Schmidt numbers for $\kappa$, $c$, and $\varepsilon$ (dimensionless number), which are defined as the ratio between the kinetic viscosity and the mass diffusivity. $P_\kappa$ is the rate of production of turbulent kinetic energy due to mean velocity gradients. $\varepsilon$ is the rate of dissipation of turbulent kinetic energy per unit mass. $Ri_H$ is the Richardson number based on inlet density difference, flow depth, and nominal basin velocity, $Ri_H = g\gamma \rho_0 H / U^2$. $\nu_t$ is the turbulent viscosity, $\nu_t = C_\mu \kappa^2 / \varepsilon$. $C_\mu$, $C_1$, and $C_2$ are the standard constants for the $\kappa - \varepsilon$ model. The formulas and values for all the variables and constants used in flow governing equation are given in Lyn’s paper.

For sedimentation, to preserve the accuracy of the numerical model results, the suspended sediments are divided into groups based on their sizes, each with the influent suspended sediment concentration marked as $c_i$ and each with the settling velocity marked as $\omega_{st}$. Therefore, the suspended sediment concentration equation for each particle group with flocculation considered is:

$$\frac{\partial(u c_i)}{\partial x} + \frac{\partial((u - \omega_{st}) c_i)}{\partial y} = \frac{\partial}{\partial x} \left( \frac{\nu_{eff} \partial c_i}{\sigma_c \partial x} \right) + \frac{\partial}{\partial y} \left( \frac{\nu_{eff} \partial c_i}{\sigma_c \partial y} \right) + F_{floc}^i \tag{2.20}$$

in which $F_{floc}^i$ is the flocculation model. For each particle group, the flocculation model $F_{floc}^i$ can be written as:

$$F_{floc}^i = \left( \frac{\varepsilon}{\nu} \right)^{1/2} \left( \alpha_i - \frac{\sigma_i^2}{\sigma_{i-1}} \right) \tag{2.21}$$

where $\alpha_i$ is the empirical constant for the flocculation model. This value ranges from 0.4 to 2.3. The exact value of $\alpha_i$ needs to be calibrated based on similar case studies.

Lyn’s model is capable of predicting the flow field, flocculation effects, and suspended sediment concentration distributions. But he used the $\kappa - \varepsilon$ model as the flow governing equation for the two-dimensional turbulent flow, which is a very complicated and arcane model to be used in numerical modeling. Secondly, for the
flocculation model, he assumed that collision only happened to particles of the same sizes. Once a particle collision produced a larger particle, the larger flocculated particle will advance into the next particle group to aggregate with the particles in this group. This assumption may introduce inaccuracy, since in reality, collisions may happen anytime, in any form, and between any type and size of particle. It is not limited by artificial grouping.
Chapter 3 Governing Equations

In this chapter the governing equations for the one-dimensional rectangular flocculated sedimentation tank with considering flocculation influence are derived, with unsteady flow and non-uniform sediment particles. Sediment carrying capacity is introduced to calculate the suspended sediment concentration. The settling velocity for discrete particles is given by Stokes’ law. The flocculation factor is an important parameter used to calculate the settling velocities for floc. It is composed of several correction factors, the details of which are outlined in this chapter. Particles are divided into groups according to sizes and within each group sediment transport is considered to be uniform. As usual, this model is capable of providing removal efficiency, particle size distribution in the sludge and effluent, the thickness of the accumulated sludge, and the dimensions of the tank based on desired removal efficiency or vice versa.

From the raw water flowing into the settling tank to the clarified water flowing out of the tank, the inlet zone, settling zone, outlet zone, and sludge zone are the four main parts of the fluid path. These zones are shown in Figure 3.1. The research for the sedimentation tank is only for the settling zone and sludge zone, because the sediment deposition in the inlet zone and outlet zone are negligible.

3.1 Derivation of Flow Equations

3.1.1 Reynolds Equations

The depth-averaged governing equations for incompressible flow pattern will be derived from the two-dimensional Reynolds mass conservation equation (3.1) and momentum equation (3.2) (Chaudhry 2008), with reference to the definition sketch shown in Figure 3.1:
where $\sigma_x$ is the Reynolds turbulent normal stress assigned along the $x$-direction; $\tau_{zx}$ is the Reynolds turbulent shear stress assigned at the bottom of the flow.

### 3.1.2 Depth-averaged Continuity Equation

The one-dimensional depth-averaged continuity equation and depth-averaged momentum equation for unsteady flow with a hydrostatic pressure distribution are derived by integrating the two-dimensional mass and momentum conservation equations over the depth of the tank in a Cartesian coordinate system. As shown in the definition sketch (Figure 3.1): $H$ represents the surface elevation of the water, $h$ represents the depth of the water in the settling zone, $z_b$ represents the total elevation from the tank bottom, and $z_b$ is comprised of two parts: $z_s$, which represents the thickness of the sediment deposition accumulated at the bottom of the tank, and $z_g$, which represents the elevation from the tank bottom. According to the definitions of these five items, the arithmetical relationship can be shown as:

$$ H = h + z_b \quad \text{(3.3)} $$

$$ z_b = z_s + z_g \quad \text{(3.4)} $$

Integrating the mass conservation equation over the depth of the water, which is $z_b$ to $H$ yields:

$$ \int_{z_b}^{H} \frac{\partial u}{\partial x} \, dz + \int_{z_b}^{H} \frac{\partial w}{\partial z} \, dz = 0 \quad \text{(3.5)} $$

By using Leibnitz’s integral rule, the two terms in equation (3.5) can be calculated as:
In which \( \bar{u} \) is the depth-averaged horizontal flow velocity; \( u_H \) and \( u_b \) are the horizontal flow velocities at the water surface and tank bottom, respectively. \( w_H \) and \( w_b \) are the vertical flow velocities at the water surface and tank bottom respectively. Now, equation (3.5) can be rewritten as:

\[
\frac{\partial h \bar{u}}{\partial x} - u_H \frac{\partial H}{\partial x} + u_b \frac{\partial z_b}{\partial x} + w_H - w_b = 0
\] (3.6)

with reference to the boundary conditions, equation (3.6) can be further simplified.

The kinematic and dynamic boundary conditions for the movable channel bed are (Steffler and Jin 1993):

\[
w_H = \frac{\partial H}{\partial t} + u_H \frac{\partial H}{\partial x} \tag{3.7}
\]

\[
P_H = \tau_{xzH} = \sigma_{xH} = 0 \tag{3.8}
\]

\[
w_b = \frac{\partial z_b}{\partial t} + u_b \frac{\partial z_b}{\partial x} \tag{3.9}
\]

where \( P_H \) is the water pressure at the flow surface; \( \sigma_{xH} \) and \( \tau_{xzH} \) are the Reynolds turbulence normal stress and shear stress in the \( x \)-direction at the flow surface, respectively. Substituting the water surface and bed kinematic boundary conditions from equation (3.7) and equation (3.9) into equation (3.6) gives the final form for the depth-averaged continuity equation.

\[
\frac{\partial h \bar{u}}{\partial x} + \frac{\partial h}{\partial t} = 0 \tag{3.10}
\]
Figure 3.1  Definition Sketch
3.1.3 Depth-averaged Momentum Equation

Taking the same procedure to the momentum equation, integrating equation (3.2) from \( z_b \) to \( H \) yields:

\[
\int_{z_b}^{H} \frac{\partial u}{\partial t} \, dz + \int_{z_b}^{H} \frac{\partial u^2}{\partial x} \, dz + \int_{z_b}^{H} \frac{\partial uw}{\partial z} \, dz = -\frac{1}{\rho} \int_{z_b}^{H} \frac{\partial p}{\partial x} \, dz + \frac{1}{\rho} \int_{z_b}^{H} \frac{\partial \sigma_x}{\partial x} \, dz + \frac{1}{\rho} \int_{z_b}^{H} \frac{\partial \tau_{zx}}{\partial z} \, dz
\]

(3.11)

Each term in the above equation will be calculated separately using Leibnitz’s integral rule.

(a) \[
\int_{z_b}^{H} \frac{\partial u}{\partial t} \, dz = \frac{\partial}{\partial t} \int_{z_b}^{H} u \, dz - u_H \frac{\partial H}{\partial t} + u_b \frac{\partial z_b}{\partial t} = \frac{\partial h \bar{u}}{\partial t} - u_H \frac{\partial H}{\partial t} + u_b \frac{\partial z_b}{\partial t}
\]

(b) \[
\int_{z_b}^{H} \frac{\partial u^2}{\partial x} \, dz = \frac{\partial}{\partial x} \int_{z_b}^{H} u^2 \, dz - u_H^2 \frac{\partial H}{\partial x} + u_b^2 \frac{\partial z_b}{\partial x} = \frac{\partial h \bar{u}^2}{\partial t} - u_H^2 \frac{\partial H}{\partial t} + u_b^2 \frac{\partial z_b}{\partial t}
\]

(c) \[
\int_{z_b}^{H} \frac{\partial uw}{\partial z} \, dz = w_H u_H - w_b u_b
\]

(d) \[
-\frac{1}{\rho} \int_{z_b}^{H} \frac{\partial p}{\partial x} \, dz = -\frac{1}{\rho} \left( \frac{\partial}{\partial x} \int_{z_b}^{H} p \, dz - p_H \frac{\partial H}{\partial x} + p_b \frac{\partial z_b}{\partial x} \right)
\]

\[
= -\frac{1}{\rho} \left( \frac{\partial h \bar{p}}{\partial x} - p_H \frac{\partial H}{\partial x} + p_b \frac{\partial z_b}{\partial x} \right)
\]

(e) \[
\frac{1}{\rho} \int_{z_b}^{H} \frac{\partial \sigma_x}{\partial x} \, dz = \frac{1}{\rho} \left( \frac{\partial}{\partial x} \int_{z_b}^{H} \sigma_x \, dz - \sigma_{xH} \frac{\partial H}{\partial x} + \sigma_{xb} \frac{\partial z_b}{\partial x} \right)
\]

\[
= \frac{1}{\rho} \left( \frac{\partial h \bar{\sigma}_x}{\partial x} - \sigma_{xH} \frac{\partial H}{\partial x} + \sigma_{xb} \frac{\partial z_b}{\partial x} \right)
\]

(f) \[
\frac{1}{\rho} \int_{z_b}^{H} \frac{\partial \tau_{zx}}{\partial z} \, dz = \frac{1}{\rho} (\tau_{zH} - \tau_{zxb})
\]

Where \( \bar{p} \) is the mean flow depth pressure; \( p_b \) is the water depth pressure at the flow bottom; \( \bar{\sigma}_x \) is the depth-averaged Reynolds turbulence normal stress; \( \sigma_{xb} \) and \( \tau_{zxb} \) are the Reynolds turbulence normal stress and shear stress in the \( x \)-direction at the bottom,
respectively. By using the depth-averaged continuity equation and substituting the
boundary conditions from equations (3.7), (3.8), and (3.9), the momentum equation can
be temporarily written as:
\[
\frac{h}{h} \frac{\partial \bar{u}}{\partial t} + h \bar{u} \frac{\partial \bar{u}}{\partial x} = -\frac{1}{\rho} \left[ \left( \frac{\partial h \bar{p}}{\partial x} - \frac{\partial h \bar{\sigma_x}}{\partial x} \right) + \left( p_b \frac{\partial z_b}{\partial x} - \sigma_{xb} \frac{\partial z_b}{\partial x} \right) + \tau_{zxb} \right]
\]  
(3.12)

with the assumption of hydrostatic pressure distribution for typical free surface flow,
the term \( \frac{1}{\rho} \frac{\partial h \bar{\sigma_x}}{\partial x} \) in equation (3.12) can be neglected (Guo 2001), and
\[
\bar{p} = 0.5p_b = 0.5\rho gh
\]  
(3.13)

Assuming the stress terms \( \sigma_{xb} \) and \( \tau_{zxb} \) have a relationship with the shear stress \( \tau_b \), it
can be expressed as (Steffler and Jin 1993):
\[
\tau_{zxb} = \tau_b (\cos^2 \alpha - \sin^2 \alpha)
\]  
(3.14)
\[
\sigma_{xb} = -2\tau_b \cos \alpha \sin \alpha
\]  
(3.15)

where \( \tan \alpha = \frac{\partial z_b}{\partial x} \) represents the bottom slope of the tank. By substituting equation
(3.14) and (3.15) into equation (3.12), two terms related to \( \sin \alpha \) arise:\( \frac{\sin 2\alpha}{c^2 \rho} \) and \( \frac{\sin 2\alpha}{\rho} \frac{\partial h}{\partial x} \)

The value of \( \sin 2\alpha \) may range from 0 to 1, the Chezy Coefficient, \( C \), ranges from 100
to 150 (Zloczower 2003), and \( \rho \) is 1000 \( kg/m^3 \) under normal conditions. Note that
\( \frac{\sin 2\alpha}{c^2 \rho} \) is at least smaller than \( 10^{-5} \), making it acceptable to be neglected from equation
(3.12). The same analysis method can be used to eliminate \( \frac{\sin 2\alpha}{\rho} \frac{\partial h}{\partial x} \) which is 4000 to
80000 times smaller than 1 (Huang 2010).

Another important assumption is that within each small enough time interval, \( \Delta t \),
the flow pattern is uniform. Therefore, the Chezy equation in the following form can be
used to express the relationship between the bed shear stress, \( \tau_b \), and the depth-
averaged horizontal flow velocity \( \bar{u} \) (Steffler and Jin 1993).
\[
\frac{\tau_b}{\rho} = \frac{\bar{u}^2}{C^2}
\]
(3.16)

where \(C\) = Chezy Coefficient, which can be determined by

\[
C = \frac{R^{1/6}}{n}
\]
(3.17)

where \(R\) = the hydraulic radius. \(n\) = the Manning Roughness Coefficient. The value of \(n\) will be discussed in a later chapter.

With the help of equations (3.13), (3.14), (3.15), (3.16), and (3.17), equation (3.12) can be written as the final form of the depth-averaged momentum equation in the \(x\)-direction:

\[
\frac{1}{2g} \frac{\partial \bar{u}^2}{\partial x} + \frac{\partial H}{\partial x} + \frac{\bar{u}^2}{ghC^2} = 0
\]
(3.18)

### 3.2 Derivation of Sediment Transport Equations

#### 3.2.1 Sediment Transport Equation

A one-dimensional sediment transport equation is derived by integrating the two-dimensional suspended sediment transport equation along the water depth. The general formula describing the two-dimensional suspended sediment transport in a diffusion model is written as (Rijn 1986):

\[
\frac{\partial s}{\partial t} + \frac{\partial us}{\partial x} + \frac{\partial ws}{\partial z} = \frac{\partial \omega s}{\partial z} + \frac{\partial}{\partial x} \left( \varepsilon_x \frac{\partial s}{\partial x} \right) + \frac{\partial}{\partial z} \left( \varepsilon_z \frac{\partial s}{\partial z} \right)
\]
(3.19)

where \(s\) is the time-averaged suspended sediment concentration in water, and \(\varepsilon_x\) and \(\varepsilon_z\) are the diffusion coefficients in the \(x\) and \(z\) directions, respectively.

Equation (3.19) is only applicable for water-sediment mixtures with fine sediments and low concentrations (Greimann and Holly Jr 2001). As described in Fick’s first law, the flux only moves from a high concentration region to a low concentration region, so diffusion in the \(x\) direction has a very limited influence on
sediment transport compared to the influence in the vertical direction. Therefore, the term \( \frac{\partial}{\partial x} (\varepsilon_x \frac{\partial s}{\partial x}) \) in equation (3.19) can be further omitted (Simons and Şentürk 1992).

Similar to the derivation of governing equations for flow pattern, by using the depth-averaged method to integrate equation (3.19), the integral form of the sediment transport equation from \( z_b \) to \( H \) can be written as:

\[
\int_{z_b}^{H} \frac{\partial s}{\partial t} \, dz + \int_{z_b}^{H} \frac{\partial u_s}{\partial x} \, dz + \int_{z_b}^{H} \frac{\partial w_s}{\partial z} \, dz = \int_{z_b}^{H} \frac{\partial \omega_s}{\partial z} \, dz + \int_{z_b}^{H} \frac{\partial}{\partial z} \left( \varepsilon_z \frac{\partial s}{\partial z} \right) \, dz
\]  

(3.20)

Based on Leibnitz’s rule, each term in equation (3.20) can be calculated separately.

(a) \[
\int_{z_b}^{H} \frac{\partial s}{\partial t} \, dz = \frac{\partial}{\partial t} \int_{z_b}^{H} s \, dz - s_H \frac{\partial H}{\partial t} + s_b \frac{\partial z_b}{\partial t} = \frac{\partial hl}{\partial t} - s_H \frac{\partial H}{\partial t} + s_b \frac{\partial z_b}{\partial t}
\]

(b) \[
\int_{z_b}^{H} \frac{\partial u_s}{\partial x} \, dz = \frac{\partial}{\partial x} (us)dz - u_H s_H \frac{\partial H}{\partial x} + u_b s_b \frac{\partial z_b}{\partial x}
\]

(c) \[
\int_{z_b}^{H} \frac{\partial w_s}{\partial z} \, dz = w_H s_H - w_b s_b
\]

(d) \[
\int_{z_b}^{H} \frac{\partial \omega_s}{\partial z} \, dz = \omega_H s_H - \omega_b s_b
\]

(e) \[
\int_{z_b}^{H} \frac{\partial}{\partial z} \left( \varepsilon_z \frac{\partial s}{\partial z} \right) \, dz = \left( \varepsilon_z \frac{\partial s}{\partial z} \right)_H - \left( \varepsilon_z \frac{\partial s}{\partial z} \right)_b
\]

where \( \bar{s} \) is the depth-averaged sediment concentration of the cross-section. \( s_H \) and \( s_b \) are the suspended sediment concentrations at the top of the flow and at the bottom of the tank, respectively. \( \omega_H \) and \( \omega_b \) are the particle falling velocities at the top of the flow and at the bottom of the tank, respectively.

Applying the flow surface and bed kinematic boundary conditions equations (3.7), (3.8), (3.9), and with the help of the depth-averaged continuity equation (3.10), the equation (3.20) can be rewritten as:
\[
\frac{h \bar{u}}{\partial x} = \omega_H S_H + \left( \varepsilon_x \frac{\partial s}{\partial z} \right)_H - \omega_b S_b - \left( \varepsilon_x \frac{\partial s}{\partial z} \right)_b
\]
(3.21)

where \(\omega_H S_H\) represents the quantity of sediments settled to the surface of the flow per unit time. According to the physical meaning of sediment transport, the sediment accumulation for free surface flow is small enough to be neglected compared to the sediment accumulation at the bottom of the tank.

The term, \(\left( \varepsilon_x \frac{\partial s}{\partial z} \right)_H\), which represents the quantity of particles re-suspended from the tank bottom to the surface of the flow, can be eliminated from equation (3.21). The existence of this term is due to the concentration gradient, which accounts for the phenomenon of bottom sediment erosion. The diffusion coefficient, \(\varepsilon_x\), in the \(z\)-direction depends on temperature, fluid viscosity, particle size, and depth of flow (Rijn 1984). When considering this term at the surface of the flow, where the water depth should be zero, the diffusion coefficient \(\varepsilon_x\) should be close to zero. Therefore, terms including the diffusion coefficient can be neglected from equation (3.21).

To solve for the remaining terms in equation (3.21), the definition of sediment carrying capacity may be used. The term \(\left( \varepsilon_x \frac{\partial s}{\partial z} \right)_b\) is similar to the term \(\left( \varepsilon_x \frac{\partial s}{\partial z} \right)_H\). This term represents the quantity of sediment re-suspended from the tank bottom to the flow very near the bed of the tank due to diffusion. The term \(\omega_b S_b\) represents the quantity of sediment settled to the bottom of the tank due to gravity. As mentioned in the first chapter, when equilibrium is reached, deposition or erosion do not occur and the sediment concentration in the water \(\left( S_b \right)\) is equal to the sediment carrying capacity \(\left( S_b^* \right)\). Thus, the quantity of sediment re-suspended from the tank bottom to the flow very near to the bed of the tank \(\left( \varepsilon_x \frac{\partial s}{\partial z} \right)_b\) should be equal to the quantity of sediment which accumulated at the tank bottom due to gravity \(\omega_b S_b\). This physical phenomenon can be expressed as:
Equation (3.22) is used under equilibrium conditions and it cannot be directly used in real non-equilibrium conditions. It can be assumed that the theories for equilibrium conditions are appropriate in non-equilibrium conditions through the mean sediment carrying capacity at the cross-section $s^*$, which can be used instead of the sediment carrying capacity near the bed at the same section $s_b^*$. The relationship between them can be expressed as (Guo 2001):

$$s_b^* = \beta_1 s^*$$  \hspace{1cm} (3.23)

To further simplify the sediment transport equation, two more coefficients can be used. First, the ratio, $\beta_2$, which represents the sediment concentration near the bottom of the tank ($s_b$) is proportional to the depth-averaged sediment concentration of the cross-section ($\bar{s}$).

$$s_b = \beta_2 \bar{s}$$  \hspace{1cm} (3.24)

Second, it can be assumed that the settling velocity near the bed ($\omega_b$) can be replaced by the mean settling velocity ($\omega$). Furthermore, to eliminate the influence of instead $\omega_b$ by $\omega$ and simplify the equation, the coefficient $\beta$ can be used to replace $\beta_1, \beta_2$. Thus, the equation (3.21) can be rewritten as:

$$h \bar{u} \frac{\partial \bar{s}}{\partial x} = -\beta \omega (\bar{s} - s^*)$$  \hspace{1cm} (3.25)

where $s^*$ represents the mean sediment carrying capacity. The mathematical expression for this parameter is (Jin et al. 2000):

$$s^* = k \left( \frac{\bar{u}^3}{h \omega} \right)^m$$  \hspace{1cm} (3.26)
where the \( k \) coefficient reflects the capacity of sediment transported by a certain flow. The value of this coefficient will be discussed in the next chapter. The \( m \) coefficient has a suggested value of 0.92 (Jin et al. 2000).

### 3.2.2 Sediment Transport Continuity Equation

The sediment transport continuity equation is used to calculate the quantity of sludge accumulated at the bottom of the tank in a certain period of time. It also means the length at which the tank bottom increased due to sediment deposition, which is the reason this equation is also called the bed variation equation (Jin et al. 2000). This equation physically represents the difference in suspended sediment concentrations between two cross-sections, which should be equal to the quantity of sludge accumulated at the tank bottom in the same time period. The mathematical expression can be written as:

\[
-h\bar{u} \frac{\partial \bar{s}}{\partial x} = \beta \omega (\bar{s} - s^*) = \rho_d \frac{\partial z_s}{\partial t}
\]  

(3.27)

where \( \rho_d \) is the density of the sediment.

At this point, all of the derivations for the depth-averaged sediment transport equations are finished, and can be used in the next chapter to construct the rectangular sedimentation tank model.

### 3.3 Floc Settling Velocity

#### 3.3.1 Flocculation Factor

The gravitational force and drag force are the two main forces acting on particles in sedimentation tanks. For fine sediments another force exists: the electrostatic force. When the action of electrostatic forces is equal to or greater than the gravitational force, fine sediment particles may collide and stick together and form floc or larger particles. Flocculation affects sediment concentrations and settling characteristics by changing particle size. Flocculation is affected by sediment
concentration, sediment size, turbulence intensity, salinity, organic matters and temperature, etc. By introducing a flocculate factor, the settling velocity of floc can be determined as (Wu 2007):

\[ F = \frac{\omega_f}{\omega} = k_s k_d k_{sa} k_t \]  

(3.28)

where \( F \) is the flocculation factor, the ratio between floc settling velocity and the corresponding dispersed particle settling velocity. \( \omega_f \) is the settling velocity of floc. \( k_s \), \( k_d \), \( k_{sa} \), and \( k_t \) are the correction factors, which indicate the influence of sediment concentration, particle size, water salinity, and turbulence intensity, respectively. Using \( \omega_f \) instead of \( \omega \), the model for rectangular sedimentation tanks with flocculation considered, can be established.

3.3.2 Effect of Particle Size on Flocculation

The relation between correction factor \( k_d \) and particle size, based on the research from Dixit et al. (1982), Huang (1981), and Wu (2007) can be shown roughly in Figure 3.2. Based on this, the following approximation for the correction factor \( k_d \) (Wu 2007) can be given by:

\[ k_d = \left( \frac{d_r}{d} \right)^{n_d}, d \leq d_r \]  

(3.29)

where \( d_r \) represents a reference diameter between 0.011mm and 0.022mm. The value differs based on the experimental tests and similar case analysis. \( n_d \) is an empirical exponent between 1.8 and 2.0 (Lin and Wu 2013). Equation (3.29) is only applied to the situation when \( d \leq d_r \). When \( d > d_r \), \( k_d = 1 \). This means that flocculation will not happen during coarse particle collisions.
Figure 3.2  Flocculation Factor as a Function of Particle Size (Jianwei 1981)
Figure 3.3   Settling Velocity as a Function of Sediment Concentration (Jianwei 1981)
3.3.3 Effect of Sediment Concentration on Flocculation

According to the explorations of Huang (1981), Mehta (1986), and Lin and Wu (2013), the correction factor $k_s$ can be evaluated as:

$$
k_s = \begin{cases} 
1 + k_1s^{n_s}, & 0 < s < s_p \\
K(1 - k_2s)^r, & s < s_p 
\end{cases}
$$

where $k_1$ and $k_2$ are empirical coefficients, and $K$ is calculated by $(1 + k_1s_p^{n_s})/ (1 - k_2s_p)^r$ (Lin and Wu 2013). $n_s$ is an empirical exponent ranging between 1 and 2 with a mean value of 1.3. $r$ is also an empirical exponent ranging between 3 and 5. $s_p$ is the sediment concentration related to the properties of the water sediment mixture. Its value varies from 1.5 to 15 kg/m$^3$. The value of this item needs to be determined by experimental tests or similar case analysis. As shown in Figure 3.3, the value for $s_p$ decreases as the floc settling velocity decreases in the curve of $\omega_f$~$s$ (sediment concentration).

3.3.4 Effect of Raw Water Salinity on Flocculation

Based on Huang’s experimental conditions and results (Jianwei 1981), the correction factor $k_{sa}$ is determined by:

$$
k_{sa} = \begin{cases} 
(c_{sa}/c_{sap})^{n_{sa}} c_{sa,min} < c_{sa} \leq c_{sap} \\
1 & c_{sa} > c_{sap} 
\end{cases}
$$

where $c_{sa}$ is the raw water salinity. $c_{sap}$ is the salinity at which its influence on floc settling velocity is minimal. This value is determined by experimental tests and similar case analysis.

As shown in Figure 3.4, the floc settling velocity increases rapidly as salinity increases. However, when salinity continues to increase to a certain value, its influence on settling velocity becomes limited. $c_{sa,min}$ is the minimum threshold value for salinity for the raw water. $n_{sa}$ is an empirical exponent. According to Huang (1981),
the approximation curves in Figure 3.4 are obtained from equation (3.31), with $c_{sap} = 28 (ppt)$ and $n_{sat} = 0.53$. The legend $H$ shown in Figure 3.4 represents the settling distance measured in the vertical direction.

### 3.3.5 Effect of Turbulence Intensity on Flocculation

The influence of turbulence intensity on flocculation can be roughly calculated by the application of the correction factor $k_t$, expressed as:

$$k_t = \begin{cases} 
1 + k_{f1} \left( \frac{\tau_b}{\tau_p} \right)^{n_{t1}} & 0 < \tau_b \leq \tau_p \\
\left( 1 + k_{f1} \right) \left( \frac{\tau_b}{\tau_p} \right)^{-n_{t2}} & \tau_b > \tau_p 
\end{cases} \quad (3.32)$$

where $k_{t1}$ is an empirical coefficient. $n_{t1}$ and $n_{t2}$ are empirical exponents. $\tau_b$ is the bottom shear stress. $\tau_p$ is the bed shear stress when $k_t$ reaches its maximum value. Figure 3.5 shows the relationship between the residual turbidity and the stirrer speed, using three different types of stirrers. From McConnachie’s observation, with the stirrer speed increasing, the residual turbidity decreased first and then increased (McConnachie 1991). If there are no stirrers used in the sedimentation facilities, the first expression in Equation (3.32) is equal to 1. This means that the flow is in quiescent conditions and the influence on flocculation by turbulence can be neglected.
Figure 3.4  Settling Velocity as a Function of Salinity (Jianwei 1981)
Figure 3.5  Residual Turbidity vs. Stirrer Speed (McConnachie 1991)
3.3.6 Effect of Operation Temperature on Flocculation

Temperature indirectly affects flocculation through its direct influence on the viscosity of water. For numerical applications, equation (3.33) can be used to express the relationship between the settling velocity of flocculated sediments and the viscosity of water.

\[
\frac{\omega_{f1}}{\omega_{f2}} = \frac{\mu_{T1}}{\mu_{T2}}
\]  

(3.33)

where \(\mu_{T1}\) and \(\mu_{T2}\) are the dynamic viscosities of the water at temperature \(T_1\) and \(T_2\), respectively. Figure 3.6 shows Huang’s experimental results. Based on his observation, the floc settling velocity increases with the increasing of the water temperature. This is because the water viscosity tends to decrease as its temperature increases. The operational temperature can be controlled during laboratory experiments, and it will not change significantly during real operation time intervals each day.

At this point, all of the correction factors having influence on flocculation have been formulated and discussed. The floc settling velocity \(\omega_f\) can be used in the sediment transport equation, sediment transport continuity equation, and sediment carrying capacity equation instead of the discrete particle mean settling velocity \(\omega\). Calculations for floc settling velocity will be applied to each particle group, and the settling velocity for each floc group will be marked as \(\omega_{fp}\), and \(\omega_{fp}\) as:

\[
\omega_{fp} = \omega_p k_s k_d k_{sa} k_t
\]  

(3.34)
Figure 3.6  Settling Velocity as a Function of Temperature (Jianwei 1981)
Chapter 4 Model Construction

The governing equations for a rectangular sedimentation tank, with flocculation considered, are derived in the previous chapter. All of the equations which can be used to set up the model are summarized as follows:

Depth-averaged continuity flow equation:

$$\frac{\partial h \bar{u}}{\partial x} + \frac{\partial h}{\partial t} = 0$$

(4.1)

Depth-averaged momentum flow equation:

$$\frac{1}{2g} \frac{\partial \bar{u}^2}{\partial x} + \frac{\partial H}{\partial x} + \frac{\bar{u}^2}{ghc^2} = 0$$

(4.2)

Sediment transport equation:

$$h \bar{u} \frac{\partial \bar{s}}{\partial x} = -\beta \omega (\bar{s} - s^*)$$

(4.3)

Sediment continuity equation:

$$-h \bar{u} \frac{\partial \bar{s}}{\partial x} = \beta \omega (\bar{s} - s^*) = \rho_d \frac{\partial z_s}{\partial t}$$

(4.4)

Sediment carrying capacity:

$$s^* = k \left( \frac{\bar{u}^3}{h \omega} \right)^m$$

(4.5)

Flocculation factor equation:

$$F = \frac{\omega_f}{\omega} = k_s k_d k_{sa} k_t$$

(4.6)

After the derivation of the governing equations for flow pattern, sediment transport and the flocculation factor, the unknown items in those equations need to be solved by using the finite difference method. The solutions will be used to set up the numerical model. The model framework will be presented in section 4.1.2, to describe the sequence of solving for the unknown values. This chapter can be separated into several parts, including an introduction to finite difference approximation, solving the
hydraulic factors, solving for sediment concentration, solving floc settling velocities, determining the thickness of sediment depositions, modifying the water depth, and discussing the coefficients used in the governing equations.

4.1 Numerical Method and Model Framework

4.1.1 Finite Difference Method

The finite difference method (FDM) will be used to solve the governing equations. In essence, the finite difference is the discrete analog of the derivatives in the partial differential equation, which can be approximated by linear combinations of functional values at the node points. For a rectangular sedimentation tank, the grid points can be set by divided the length of the tank into numerous identical infinitesimal sections $\Delta x$. The dividing points are noted as $1, 2, \ldots, i, i + 1, \ldots$, from the beginning of the settling zone to the end of the settling zone along the flow direction. An expression sketch is shown in Figure 4.1.

The Taylor Series Expansion can be used to study the behavior of the numerical approximation to partial differential equations. The general purpose of the Taylor Series is to provide a prediction of a function value at one point in terms of the function value and its derivatives at another point (Chapra and Canale 2012). The algebraic expression of the Taylor Series Expansion for the function $f(x)$ is showing in equation (4.7):

$$f(x_{i+1}) = f(x_i) + f'(x_i)(x_{i+1} - x_i) + \frac{f''(x_i)}{2!}(x_{i+1} - x_i)^2$$

$$+ \frac{f'''(x_i)}{3!}(x_{i+1} - x_i)^3 + \cdots + \frac{f^{(n)}(x_i)}{n!}(x_{i+1} - x_i)^n + R_n$$

(4.7)

The equation (4.7) is an infinite series, and the remainder term is included to account for all terms from $n + 1$ to infinity:

$$R_n = \frac{f^{(n+1)}(x_i)}{(n + 1)!}(x_{i+1} - x_i)^{n+1}$$

(4.8)
where \( R_n \) is called a Truncation Error or Lagrange Remainder, which is the difference between the true value of a derivative and the finite difference approximation. Equation (4.7) shows the remainder for the \( n \)th-order approximation.

For rectangular sedimentation tank applications, both space and time scales need to be considered. The subscript \( i \) is used to denote space scale, while the superscript \( j \) is used to denote the time scale. Figure 4.2 shows \( i \) and \( j \) which construct the space and time scales for the model. By neglecting the truncation error influence (the high-order terms), the first derivative of the function \( f(x) \) for each nodal point based on each time scale can be approximated as:

\[
f'(x_i^j) = \frac{f(x_{i+1}^j) - f(x_i^j)}{\Delta x}
\]  

(4.9)

Equation (4.9) is called the forward difference scheme. Similar to equation (4.9), there is a backward difference scheme (Equation (4.10)) and a central difference scheme (Equation (4.11)) for the first derivative:

\[
f'(x_i^j) = \frac{f(x_{i}^j) - f(x_{i-1}^j)}{\Delta x}
\]  

(4.10)

\[
f''(x_i^j) = \frac{f(x_{i+1}^j) - f(x_{i-1}^j)}{2\Delta x}
\]  

(4.11)

The forward difference scheme (equation (4.9)) and the backward difference scheme (equation (4.10)) are first-order accurate. The central difference scheme (equation (4.11)) is second-order accurate.

By the finite difference approximation, the differential equations (4.2), (4.3), (4.4), (4.5), and (4.6) can be converted to algebraic equations using the appropriate difference scheme. All of the unknown variables in the governing equations can be properly solved for in a certain sequence.
Figure 4.1  Finite Difference Discretization of the Domain

Figure 4.2  Finite Difference Grids
4.1.2 Model Framework

The framework for constructing this one dimensional numerical model for a rectangular sedimentation tank with flocculation considered is shown in Figure 4.3. Note that there are two main calculation circles. The first one calculates all of the information for the sedimentation tank without considering flocculation. The other one calculates all of the information for the sedimentation tank with flocculation considered by using different settling velocities. Before calculating the floc settling velocity, the model needs to finish all of the calculations for the dispersed particles, like settling velocities, sediment concentrations, and sediment carrying capacity.

After calculating the settling velocities for dispersed particles, suspended sediment concentration, sediment carrying capacity, and the thickness of the sludge accumulated at the bottom of the tank can be determined. By comparing the particle sizes with the reference particle sizes, and by using the suspended sediment concentration from the last calculation circle, the flocculation correction factors can also be determined. The arithmetic product of the flocculation correction factors are defined as the ratio of the settling velocities for dispersed particles to the floc settling velocities. Substituting the floc settling velocities into the sediment transport equations allows the sediment concentration for the flocculated sedimentation tank to be calculated. Then, both results for the sedimentation tank, with or without flocculation, can be obtained, like removal efficiency, removal efficiency for each particle and floc groups, particle size distributions at outlet, sludge distribution in the effluent, and sludge thickness.
Figure 4.3 Model Framework

Water-sediment Mixture Properties

Settling Velocity $\omega$

Flocculation Correction Factors

Water Elevation $H$

Flocs Settling Velocity $\omega_f$

Water Depth $h$

Total Suspended Sediment Concentration along the Tank

Removal Efficiency along the Tank

Particle Size Distributions along the Tank

Sludge Size Distributions

Sediment Carrying Capacity $s^*$

Suspension Sediment Concentration $\bar{s}$

Sludge Thickness $z_b$

Operation time ends?

Yes

No

Next time step
4.2 Model Procedures and Descriptions

4.2.1 Solving Hydraulic Profiles

The only unknown variable in equation (4.2) is the water elevation $H$. The other variables such as water depth $h$ and the hydraulic radius $R$ can be determined through water elevation. The hydraulic radius can be mathematically expressed as:

$$R = \frac{A}{P} = \frac{Bh}{B + 2h}$$

(4.12)

where $A$ is the cross-sectional area of flow in a rectangular sedimentation tank. $P$ is the wetted perimeter. So the first step of model construction should be to decide the value of the water elevation $H$.

The mathematical expression for the depth-averaged horizontal flow velocity is:

$$\bar{u} = \frac{Q}{Bh}$$

(4.13)

where $Q$ represents the inflow rate, which is assumed to be constant in the model. $B$ represents the width of the rectangular tank.

Equation (4.2) can be solved by using the finite difference approximation. The equation can be rewritten as:

$$H_{i+1} - H_i + \frac{Q^2}{2gB^2} \left( \frac{1}{h_{i+1}^2} - \frac{1}{h_i^2} \right) + \frac{\Delta x Q^2 n^2}{2gB^2} \left( \frac{1}{h_{i+1}^3 R_{i+1}^{1/3}} + \frac{1}{h_i^3 R_i^{1/3}} \right) = 0$$

(4.14)

where $i$ indicates the point number, $i + 1$ and $i$ indicate the upstream and the downstream sections, respectively. $\Delta x$ is the distance between sections $i + 1$ and $i$. For the finite difference method, the boundary conditions are essential values and need to be decided to get a numerical solution. For sedimentation tanks, the water elevation at the outlet can be determined during the design phase before the tank is built, because it is only related to the dimensions of the tank. Therefore, the downstream water level $H_{i+1}$ is given, and the upstream water level $H_i$ can be calculated using equation (4.11).
After the value of $H_i$ is known, the procedure can be repeated to determine the value of $H_{i-1}$. The water elevation for each sectional point along the length of the rectangular sedimentation tank can be calculated from the outlet back to the inlet. The water elevation at the initial time step is shown as:

$$H^0_{\text{outlet}} \rightarrow \cdots \rightarrow H^0_{i+1} \rightarrow H^0_i \rightarrow H^0_{i-1} \rightarrow \cdots \rightarrow H^0_2 \rightarrow H^0_1$$

At the initial time step, particles are still suspended in the water and no sediment has accumulated at the bottom of the tank. As such, the profile of the water depth is the same as the profile of the water elevation, assuming the water elevation at a certain location does not change within the operation time. This assumption can be expressed as:

$$H^0_i = H^1_i = H^2_i = \cdots = H^j_i$$

where $j$ denotes the time step number. After calculating the first time step, the profile of the water depth will return to the start of the calculation circle. For the next time step, the profile of the water depth needs to be modified by using the thickness of the sediment depositions which accumulated at the last time step. This calculation circle needs to be repeated for every time step until the final time step is reached.

### 4.2.2 Solving the Settling Velocities

For equation (4.3), the settling velocity and the sediment carrying capacity need to be fixed before the calculation of suspended sediment concentration can be performed. According to equation (3.26) and equation (4.10), the algebraic formula for the sediment carrying capacity at location $i$ and time step $j$ can be written as:

$$s^*_{ij} = k \left( \frac{Q^3}{\omega h_i^{j+1} B^3} \right)^{0.92}$$

(4.15)

Because the water depth at location $i$ and time step $j$ is available from the first calculation step, the settling velocity becomes the only unknown variable that needs to
be determined. Equation (2.1) is the mathematical expression from Stokes’ law for calculating the settling velocity. From this equation, the particle size and density difference between the water and the sediments is needed to determine the particle’s settling velocity.

Previous models often select only one settling velocity for all particles, but normally particles in the raw water have a wide range of sizes and diameters. Therefore, in this model particles need to be separated into groups based on their diameters (Guo and Jin 1999). In each group, the particle sizes can be considered uniform and the uniform particle size distribution leads to a single settling velocity $\omega_p$ to be used. This group division method can solve the non-uniform particle size problem. The more groups provided, the more accurate the results.

For the dispersed particles settling velocity calculation, an assumption needs to be made. The assumption is that the dispersed particles with different sizes and different falling velocities will not affect each other during the entire settling processes.

After solving the particle settling velocities for each group, the sediment carrying capacity at location $i$ and time step $j$ for group $p$ is:

$$s_i^j = k \left( \frac{Q^3}{\omega_p h_i^j B^3} \right)^{0.92}$$ (4.16)

Because the main feature of the new model is to take flocculation into consideration during the settling processes, after the calculation of settling velocity for dispersed particles, the next step is to decide the settling velocities for the floc. With the available settling velocity for each group $\omega_p$, equation (3.28) can be rewritten as:

$$F = \frac{\omega_{fp}}{\omega_p} = k_s k_d k_s a k_t$$ (4.17)

where $\omega_{fp}$ represents the settling velocity for each floc group. The correction factors $k_s$, $k_d$, $k_s a$, and $k_t$ need to be decided by using equations (3.29), (3.30), (3.31), and (3.32).
But there are some prerequisite conditions. For instance, by using equation (3.29), each group’s particle diameter $d_p$ needs to be compared with the reference diameter $d_r$. A similar discussion will apply to sediment concentration, water salinity, and turbulent force. By using equation (3.30), the sediment concentration needs to be determined.

### 4.2.3 Solving for Sediment Concentration

The next step of model construction is to decide the suspended sediment concentration along the tank at time step $j$ for each group. By integrating equation (4.3) along the flow direction from inlet to outlet, the finite difference formula will be:

$$\frac{Q \bar{s}_{i+1,p}^j - \bar{s}_{i,p}^j}{B \Delta x} = -\beta \omega_p (\bar{s}_{i,p}^j - \bar{s}_{i+1,p}^j)$$

(4.18)

where $\bar{s}_{i,p}^j$ and $\bar{s}_{i+1,p}^j$ are the suspended sediment concentration and the sediment carrying capacity at the upstream section $i$ at time step $j$ for each group, respectively. $\bar{s}_{i+1,p}^j$ is the suspended sediment concentration at the downstream section $i + 1$ at time step $j$ for each group. To make the calculation simpler, it should be noted that:

$$h \bar{u} = q = \frac{Q}{B}$$

(4.19)

where $q$ is the discharge per unit width, which is constant along the flow direction from inlet to outlet. Besides $q$, the initial suspended sediment concentration $\bar{s}_1$ and the particle size distribution for each group are constant. The particle size distribution $\zeta_p$ can be measured by raw water sample tests and sieve analysis (Huang 2010). For each group, the suspended sediment concentration at the initial section can be calculated as:

$$\bar{s}_{1,p} = \zeta_p \bar{s}_1$$

(4.20)

Hence, the suspended sediment concentration at the next section $i + 1$ at time step $j$ for each group $p$ can be calculated as:
\[
\tilde{s}_{i+1,p}^j = \left(1 - \frac{\Delta x \beta \omega_p}{q}\right) \tilde{s}_{i,p}^j + \frac{\Delta x \beta \omega_p}{q} \tilde{s}_{i,p}^{*j}
\]  \hspace{1cm} (4.21)

Like equation (4.20), \(\tilde{s}_{i,p} = \zeta_p \tilde{s}_i\) and \(s_{i,p}^{*} = \zeta_p s_i^*\), where \(\zeta_p\) is the percentage of the group of particles \(p\) in the sediment-carrying capacity. Noting \(\zeta_p^* = \zeta_p\) and \(\sum \zeta_p = 1\). It is different from the calculation procedure of water elevation, the calculation procedure for the suspended sediment concentration from inlet to outlet is:

\[
\tilde{s}_{1,p}^j \rightarrow \tilde{s}_{2,p}^j \rightarrow \cdots \rightarrow \tilde{s}_{i,p}^j \rightarrow \tilde{s}_{i+1,p}^j \rightarrow \cdots \rightarrow \tilde{s}_{\text{outlet},p}^j
\]

where \(\tilde{s}_{\text{inlet},p}^j = \tilde{s}_{1,p}^j\). After the same calculation procedure is applied to all of the particle groups at time step \(j\), the results will be stored and prepared to be used for the next calculation steps. The total suspended sediment concentration at section \(i\) can be decided by adding all group suspended sediment concentrations into the same section together:

\[
\tilde{s}_i^j = \tilde{s}_{i,1}^j + \tilde{s}_{i,2}^j + \cdots + \tilde{s}_{i,p}^j
\]  \hspace{1cm} (4.22)

The profile of the total suspended sediment concentration along the tank is a principal factor to decide other important parameters in the sedimentation tank model, such as the settling velocity for the floc, particle size distribution in the effluent, and removal efficiency.

4.2.4 Solving for Sludge Thickness

The thickness of the sludge along the tank can be calculated at the fourth step. This step is akin to calculating the changes in elevation from the tank bottom. By integrating equation (4.4) along the length of the rectangular tank, the thickness of the sludge which accumulated by group \(p\) at section \(i\) and time step \(j\) can be calculated as:

\[
z_{s,i+1,p}^j - z_{s,i,p}^j = \frac{\Delta t \beta \omega_p}{\rho_d} \left(\tilde{s}_{i,p}^j - s_{i,p}^{*j}\right)
\]  \hspace{1cm} (4.23)
where $\Delta t$ is the time interval. Equation (4.23) shows that bed elevation is modified with time. The bed elevation variation is equal to the initial bed level plus the thickness of the accumulated sludge in the time interval $\Delta t$.

$$z_{s,i+1,p}^j - z_{s,i,p}^j = \Delta z_{s,i,p}^j \tag{4.24}$$

Note that the initial sludge thickness $z_{s,1,p}^j$ is zero, meaning that initially there is no sludge accumulated at the bottom of tank. When the thickness of the sediment depositions accumulated at section $i$ and at time step $j$ for each group are known, the total sludge thickness at time step $j$ can be written as:

$$\Delta z_{s,i}^j = \sum_{p-1}^{N} \Delta z_{s,i,p}^j \tag{4.25}$$

The sludge size distribution (SSD) is another important parameter that can be used to evaluate the performance of sedimentation tanks. The SSD means the bottom sludge composition generated by different particle groups, which can be expressed as:

$$SSD_{i,p}^j = \frac{\Delta z_{s,i,p}^j}{\Delta z_{s,i}^j} \times 100\% \tag{4.26}$$

4.2.5 Modification of the Water Depth

After calculating sludge thickness, the water depth needs to be modified at the end of each time interval. For equation (4.23), assuming the sludge thickness accumulation will be measured at the end of each time interval, and the time intervals are small enough, the thickness of the sludge would not change across the time interval. The water depth should be modified between two sequential time steps. With the help of equations (3.3) and (3.4), the water depth at section $i$ for time step $j + 1$ can be calculated as:

$$h_{i}^{j+1} = H_i - z_{s,i}^j - z_{g,i} \tag{4.27}$$
where $z_{g,i}$ is the tank bottom elevation which can be fixed before construction. From this step, all of the variables for time step $j$ are available. The calculation circle may then be repeated for the next time step $j + 1$. This model is developed for unsteady flow, but the whole unsteady process was divided into several time intervals in which the flow pattern and the sediment transportation pattern are steady. Similar to the calculation at time step $j$, all of the information for the sedimentation tank is calculated for time step $j + 1$ and stored for the next calculation circle.

4.2.6 Particle Size Distributions

There are some important elements that need to be decided when calculating the settling velocities for dispersed particles and floc, suspended sediment concentration, and sludge thickness. The determination of these elements will be summarized in this section.

As mentioned in the previous section, the particle size distribution (PSD) profile can be calculated based on the suspended sediment concentration. The particle size distribution in the inflow can be determined by the raw water sample tests and sieve analysis. The particle size distribution along the flow direction for each particle group is useful in monitoring the treatment performance for different particle sizes. The PSD in the outflow is useful in evaluating the performance of the entire tank, and it is an important inflow parameter for further treatment operations. By comparing the particle size distribution in the effluent of a rectangular sedimentation tank with and without flocculation, the effect of floc trapping can be discussed in Chapter 5. The PSD along the length of the tank at location $i$ and time step $j$ for group $p$ can be mathematically expressed as:

$$PSD_{i,p}^{j} = \frac{s_{i,p}^{j}}{s_{i}^{j}} \times 100\% \quad (4.28)$$
Similarly, the PSD in the outflow at time step $j$ for group $p$ can be calculated by:

$$PSD_{\text{outlet, } p}^j = \frac{s_{\text{outlet, } p}^j}{s_{\text{outlet}}^j} \times 100\%$$ (4.29)

### 4.2.7 Removal Efficiency

Besides particle size distribution per group and effluent, removal efficiency (RE) is the most important factor which can be used to evaluate the performance of the sedimentation tanks. Removal efficiency can be identified as the ratio between the quantity of particles which settled to the tank bottom and the total quantity of particles present in the inflow. For sedimentation tanks, removal efficiency can be used to direct engineering design, estimate sedimentation tank performance, and modify tank dimensions. The comparison for removal efficiency between the sedimentation tank models, with and without flocculation, will be described in the next chapter. The removal efficiency can be applied as both removal efficiency per particle group and total removal efficiency along the length of the tank. Removal efficiency for particle groups at section $i$ and time step $j$ can be written as:

$$RE_{i, p}^j = \frac{s_{1, p}^j - s_{i, p}^j}{s_{i, p}^j} \times 100\%$$ (4.30)

From equation (4.30), the removal efficiency for each group will be available after the entire calculation. This information can be used to modify the dimensions of the rectangular sedimentation tank.

The same mathematical expression shown in equation (4.30) will be repeated for all particle groups and the results will be stored to calculate the total removal efficiency along the length of the tank. As such, the total removal efficiency along the length of the tank at a certain time step can be mathematically expressed as:
\[ RE_i^j = \frac{\bar{s}_i^j - \ddot{s}_i^j}{\bar{s}_1^j} \times 100\% \]  \hspace{1cm} (4.31)

In which,

\[ \ddot{s}_i^j = \sum_{p=1}^{N} \ddot{s}_{i,p}^j \]  \hspace{1cm} (4.32)

where \( N \) is the total number of particle groups. This number can be decided by the model user based on the sieve tests for the raw water.

### 4.3 Data Input and Output

After discretization and calculation for all the unknown variables in the governing equations at every sectional node along the length of the tank \( L \) and at all time steps, the results are saved and can be used by the model user. Such variables include water elevation, water depth, the thickness of the sludge accumulation, suspended sediment concentration along the length of the tank and in the effluent, sludge size distribution along the length of the tank and at the outlet, removal efficiency for each particle group, and total removal efficiency which can be used to decide the dimensions of the tank. For rectangular sedimentation tanks, the dimensions of tank, properties of the raw water, properties of the sediment particles and flocs, and operational situations are the available input data for the numerical model. Some coefficients also need to be included before setting up the numerical model. Table 4.1 lists all of the input data and coefficients used to establish the model.
Table 4.1  Input Data for Creating the Model

<table>
<thead>
<tr>
<th>Dimensions of the Tank</th>
<th>$L, B, H_{outlet}, slope$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Raw Water</td>
<td>$Q, \bar{s}_1, \rho$</td>
</tr>
<tr>
<td>Sediment Particles and Flocs</td>
<td>$d, N, \omega, PSD$</td>
</tr>
<tr>
<td>Precision</td>
<td>$\Delta x, \Delta t, Operation time$</td>
</tr>
<tr>
<td>Coefficients</td>
<td>$n, k, \beta, m,$</td>
</tr>
<tr>
<td></td>
<td>Coefficients for floc settling velocity calculation</td>
</tr>
</tbody>
</table>

MATLAB can save the output data in many large matrices which model users can then view in MATLAB, or save to an Excel file or TXT file. The model can be used to produce figures showing two related variables. The figures can provide detailed information about the removal efficiency along the length of the tank, particle size distribution along the length of the tank, sludge size distribution along the length of the tank, sludge thickness along the length of the tank, and detailed information for each particle group. The resulting output and detailed analysis will be given in the next chapter.

4.4 Discussion

The inflow rate $Q$, the suspended sediment concentration in the inflow $\bar{s}_1$, and the water elevation at outlet $H_{outlet}$ are constant during the calculation processes, and principal information needs to be known when setting up the numerical model. The inflow rate $Q$, can be determined by using a flow meter, with data collected hourly every day. To solve the unsteady inflow rate, model users can separate the whole operation time into sections. The flow rate in each section is relatively constant so that inflow rate per section can be assumed steady. The suspended sediment concentration in the inflow $\bar{s}_1$, which can be determined by the raw water sample tests.
The dimensions of the rectangular sedimentation tank, such as the water elevation at the outlet $H_{\text{outlet}}$, the bottom slope of the tank, the length of the tank, and the width of the tank can be fixed by the model user during the design stage. The particle diameters and the particle size distributions in the inflow can be decided by sieve analysis and by particle counters. The density of the dry sediments can be obtained by the raw water sample tests. The particle settling velocity can be calculated using Stokes’ law after the particle diameters and density of dry sediments are decided. The group numbers can be decided by the model user based on the particle sizes.

For the coefficients, the Manning’s roughness is determined by the material which is used in the tank’s construction. Table 4.2 shows some of the values of Manning’s roughness for common materials used to build sedimentation tanks (Chow 1959). In the numerical model performance comparison in the next chapter, the Manning’s roughness $n$ is suggested as 0.011.

The coefficient $k$ in equation (4.15) is a reflection of the sediment carrying capacity of a certain flow. It increases as the particle size and water depth decrease, and it is proportional to the flow velocity. In applications, data from similar case analysis or experimental tests can be used to calibrate this value. In the numerical experiments shown in the next section, by comparing with other numerical models, a value of 0.01 is suggested to yield good results (Jin et al. 2000).

The coefficient $\beta$ in equation (4.18) represents the ratio of the sediment concentration at the bottom of the tank $s_b$ to the depth-averaged suspended sediment concentration of the cross-section $\bar{s}_i$. From the collection of experimental data and analysis of real operational data, $s_b$ is greater than $\bar{s}_i$, so the coefficient $\beta$ should be greater than unity (Jin et al. 2000). Similar to the coefficient $k$, the coefficient $\beta$ also needs to be calibrated by lab experiments and real operational data collected from
similar cases. The suggested value for $\beta$ is 1.2 in the numerical application in the next chapter. Although constant values for coefficients $k$ and $\beta$ are given when creating the numerical model, calibration is still encouraged.

$\Delta x$ and $\Delta t$ are the space interval and time interval, respectively. The values of these two factors are mainly based on the model user’s preference. Smaller $\Delta x$ and $\Delta t$ values lead to more accurate results. These values are also dependent on the operation time and the length of the rectangular sedimentation tank. The longer the operational hours and length of the sedimentation tank, and the smaller the time interval $\Delta t$ and sectional interval $\Delta x$, the longer the modeling time.

The coefficients $d_r$ and $n_d$, which are used in equation (3.29), can be determined using Table 4.3 (Dixit et al. 1982; Jianwei 1981; Migniot 1968; Qian et al. 1980; Wu 2007). As shown in Table 4.3, the value for the coefficient $n_d$ shows minor variation between all of the experimental studies listed. The value for the coefficient $d_r$ changes significantly under different experimental conditions. This may be due to experiments performed at different flow rates, salinity, and inflow sediment concentrations (Wu 2007). The numerical experiments in the next chapter will refer to the coefficients from Huang’s experiment, because the range of sediment concentration is fitted.

The coefficients which are used in equation (3.30) are $k_1$, $k_2$, $n_d$, $r$, and $s_p$. In Mehta’s study, the sediment concentration is from 0.1 $kg/m^3$ to 100 $kg/m^3$ (1986). So the values of coefficients will be followed by their suggestions: $s_p$ is equal to 3.5 $kg/m^3$, $k_1=0.513$, $k_2=0.008$, $n_d = 1$ to 2 (mean value ~ 1.3), $r = 4.65$ with a range from 3 to 5 (Wu 2007).

The other coefficients which are used to calculate the flocculation correction factor will not be used in the numerical experiments in the next chapter. The discussion
for those coefficients are neglected here. Research regarding very fine particle sediment transportation and the properties of cohesive sedimentation can be used to determine the values for all of the coefficients which are used to decide the flocculation factor.

Table 4.2 Hydraulic Roughness (Manning’s) Values

<table>
<thead>
<tr>
<th>Surface Material</th>
<th>Manning’s Roughness Coefficient n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asbestos Cement</td>
<td>0.011</td>
</tr>
<tr>
<td>Asphalt</td>
<td>0.016</td>
</tr>
<tr>
<td>Brickwork</td>
<td>0.015</td>
</tr>
<tr>
<td>Cast – Iron, New</td>
<td>0.012</td>
</tr>
<tr>
<td>Clay Tile</td>
<td>0.014</td>
</tr>
<tr>
<td>Concrete – Steel Forms</td>
<td>0.011</td>
</tr>
<tr>
<td>Concrete (Cement) - Finished</td>
<td>0.012</td>
</tr>
<tr>
<td>Concrete – Wooden Forms</td>
<td>0.015</td>
</tr>
<tr>
<td>Concrete – Centrifugally Spun</td>
<td>0.013</td>
</tr>
<tr>
<td>Galvanized Iron</td>
<td>0.016</td>
</tr>
<tr>
<td>Gravel</td>
<td>0.029</td>
</tr>
<tr>
<td>Masonry</td>
<td>0.025</td>
</tr>
<tr>
<td>Polyethylene PE</td>
<td>0.009 – 0.025</td>
</tr>
<tr>
<td>Polyvinyl Chloride PVC – With Smooth Inner Walls</td>
<td>0.009 – 0.011</td>
</tr>
<tr>
<td>Steel – Coal – Tar Enamel</td>
<td>0.010</td>
</tr>
<tr>
<td>Steel - Smooth</td>
<td>0.012</td>
</tr>
<tr>
<td>Steel – New Unlined</td>
<td>0.011</td>
</tr>
</tbody>
</table>
Table 4.3  Coefficients $d_r$ and $n_d$ of the flocculation factor

<table>
<thead>
<tr>
<th>Experimental study</th>
<th>Experimental conditions</th>
<th>$n_d$</th>
<th>$d_r$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Migniot (1968)</td>
<td>Muddy sediments, salinity = 30 ppt, sediment concentration = 10 kg/m$^3$</td>
<td>1.8</td>
<td>0.0215</td>
</tr>
<tr>
<td>Qian (1980)</td>
<td>River sediments, sediment concentration = 30 kg/m$^3$</td>
<td>2.0</td>
<td>0.011</td>
</tr>
<tr>
<td>Huang (1981)</td>
<td>Lianyun Harbor mud, salinity = 30 ppt, sediment concentration = 0.08 – 1.8 kg/m$^3$</td>
<td>1.9</td>
<td>0.022</td>
</tr>
<tr>
<td>Dixit et al. (1982)</td>
<td>No salinity, sediment concentration = 1.2 – 11 kg/m$^3$</td>
<td>1.8</td>
<td>0.012</td>
</tr>
<tr>
<td>Lin &amp; Wu (2013)</td>
<td>Fox River, no salinity, sediment concentration = 50 - 150 kg/m$^3$</td>
<td>1.8</td>
<td>0.012</td>
</tr>
</tbody>
</table>
Chapter 5 Model Results and Comparison

5.1 Model Application

Based on the calculation steps listed in the previous chapter, a numerical model for rectangular sedimentation tanks can be coded in MATLAB. In this chapter the model will be used to simulate the performance of a rectangular sedimentation tank under two conditions: with and without the influence of flocculation considered. The results from both conditions can be stored and summarized to evaluate the performance of the numerical model for the rectangular sedimentation tank. Firstly, the model without considering flocculation was evaluated by comparing it to other numerical models in similar simulation conditions. Secondly, by comparing the results from both conditions, the advantages and disadvantages for both models can be discussed. Sensitivity analysis is necessary to improve the model application.

The proposed model can be used to simulate the real settling processes in rectangular sedimentation tanks. In practical applications, this numerical model can be used to help engineers make rational predictions of the removal efficiency and the quantities of sludge collection based on the dimensions of the tank, properties of the raw water, and properties of the discrete particles and floc. As a research tool, this numerical model can be used to help researchers achieve a better understanding of complex theories in flow pattern and sediment transport in rectangular sedimentation tanks.

5.2 Initial Data Input

Initial data input includes the water flow into the rectangular sedimentation tank from one side, horizontal flow along the entire settling zone, and flow out from the other side of the sedimentation tank. Normally, the sludge zone is designed under the settling zone, and the bottom of the rectangular sedimentation tank has a slope and tilt.
to the outlet zone. The initial input data for this new model comes from Guo’s rectangular sedimentation tank model (Guo 2001), so the results from both models should be identical.

The dimensions of the rectangular sedimentation tank used in this model are as follows: the length of the tank $L = 30.0 \, m$, the depth of the water $h = 2.0 \, m$, the width of the settling zone $B = 3.0 \, m$, the elevation of the tank from the bottom $z_g = 2.0 \, m$, the elevation of the water surface at the outlet $H = 4.0 \, m$, with no slope. There are two more items: the detention time $T = 34 \, min = 2040 \, s$, and the overflow rate (surface loading rate) $q_v = 0.000977 \, m/s$. These values are needed to process the sensitivity analysis.

The properties of the raw water which flow into the settling zone include: the inflow rate $Q = 7600 m^3/day = 0.088 \, m^3/s$, the suspended sediment concentration at the inlet $\bar{s}_i = 0.5 \, kg/m^3$, the mass density of the raw water $\rho = 1000 \, kg/m^3$, and the mass density of the dry sediment $\rho_d = 1200 \, kg/m^3$.

The properties for the suspended sediment particles in each group include: particle size $d$, particle size distribution (PSD), and the settling velocity $\omega$ in the influent. These properties are shown in Table 5.1.

The precision degree includes the time interval $\Delta t$, the space interval $\Delta x$, and the operation time for the sedimentation tank model. The operation time is decided by the model user. In Guo’s model, the sedimentation tank model was operated for one day to collect the settling information. As suggested by Huang (2010), the time interval $\Delta t$ should take at least 1/1000 of the operation time for a reliable and accurate prediction. For constructing this model, the time interval $\Delta t = 30 \, s$ will be chosen. For the space interval $\Delta x$, the smaller the space interval $\Delta x$ the smaller the Lagrange Remainder will be and the better numerical results given. However, if the value of $\Delta x$ is infinitesimal,
the model will take a lot of time to give the results. From author’s experience, based on the time interval is chosen to be 30s, the space interval has to smaller than 0.5m. For constructing this numerical model, $\Delta x = 0.01m$ was selected as an example.

<table>
<thead>
<tr>
<th>Group Number</th>
<th>Particle Size (mm)</th>
<th>Settling Velocity (m/s)</th>
<th>PSD (%) (influ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>&lt; 0.005</td>
<td>0.0000095</td>
<td>2.0</td>
</tr>
<tr>
<td>2</td>
<td>0.005 – 0.01</td>
<td>0.0000536</td>
<td>8.0</td>
</tr>
<tr>
<td>3</td>
<td>0.01 – 0.025</td>
<td>0.000299</td>
<td>17.0</td>
</tr>
<tr>
<td>4</td>
<td>0.025 – 0.05</td>
<td>0.00134</td>
<td>22.0</td>
</tr>
<tr>
<td>5</td>
<td>0.05 – 0.1</td>
<td>0.00536</td>
<td>20.0</td>
</tr>
<tr>
<td>6</td>
<td>0.1 – 0.25</td>
<td>0.0172</td>
<td>14.0</td>
</tr>
<tr>
<td>7</td>
<td>0.25 – 0.5</td>
<td>0.0404</td>
<td>11.0</td>
</tr>
<tr>
<td>8</td>
<td>0.5 – 1</td>
<td>0.0828</td>
<td>6.0</td>
</tr>
</tbody>
</table>

The values of the coefficients which can be used to establish the suspended sediment transport model and the flocculation model were listed and discussed in Chapter 4. The values of the coefficients for the suspended sediment transport model mainly came from Guo’s one-dimensional rectangular sedimentation tank model (Guo 2001) and Huang’s one-dimensional circular sedimentation tank model (Huang 2010). The values of the coefficients for the flocculation model mainly come from references from Wu’s published book (Wu 2007) and Lin & Wu’s recently published paper (Lin and Wu 2013). The discussions about these coefficients are written in the previous chapter. Further modification of these coefficients will be considered after the results are compared between the experimental test and the corresponding numerical models.

5.3 Numerical Results

The results discussion is mainly separated into two parts. The first part is about testing the suspended sediment transport model. The second part is about evaluating the
flocculation influence on sediment transport by using floc settling velocity instead of suspended particle settling velocity. The numerical results analysis for both the suspended sediment transport model and the flocculation model are including in several parts: the total removal efficiency along the length of the tank, the removal efficiency for each particle group, particle size distribution along the length of the tank, sludge size distribution along the flow direction, total thickness of the accumulated sediment at the bottom of the sedimentation tank, and the sludge thickness accumulated by each particle group.

### 5.3.1 Floc Settling Velocity

By defining the flocculation factor $F$, the settling velocity for the flocculation particles can be shown in the following table.

<table>
<thead>
<tr>
<th>Group No</th>
<th>Particle Size (mm)</th>
<th>Settling velocity (m/s)</th>
<th>PSD (%) (influent)</th>
<th>Flocs Settling velocity (m/s)</th>
<th>Flocculation factor $F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>&lt; 0.005</td>
<td>0.0000095</td>
<td>2.0</td>
<td>0.000159</td>
<td>16.72</td>
</tr>
<tr>
<td>2</td>
<td>0.005 – 0.01</td>
<td>0.0000536</td>
<td>8.0</td>
<td>0.000242</td>
<td>4.51</td>
</tr>
<tr>
<td>3</td>
<td>0.01 – 0.025</td>
<td>0.000299</td>
<td>17.0</td>
<td>0.000305</td>
<td>1.02</td>
</tr>
<tr>
<td>4</td>
<td>0.025 – 0.05</td>
<td>0.00134</td>
<td>22.0</td>
<td>0.0014</td>
<td>1.04</td>
</tr>
<tr>
<td>5</td>
<td>0.05 – 0.1</td>
<td>0.00536</td>
<td>20.0</td>
<td>0.0055</td>
<td>1.03</td>
</tr>
<tr>
<td>6</td>
<td>0.1 – 0.25</td>
<td>0.0172</td>
<td>14.0</td>
<td>0.0175</td>
<td>1.02</td>
</tr>
<tr>
<td>7</td>
<td>0.25 – 0.5</td>
<td>0.0404</td>
<td>11.0</td>
<td>0.0409</td>
<td>1.01</td>
</tr>
<tr>
<td>8</td>
<td>0.5 – 1</td>
<td>0.0828</td>
<td>6.0</td>
<td>0.0832</td>
<td>1.00</td>
</tr>
</tbody>
</table>

The flocculation factor for particles smaller than 0.005mm is 16.72. When particles are between 0.01mm and 0.005mm the flocculation factor is 4.51. When particles are bigger than 0.01mm but smaller than 0.025mm the flocculation factor is approximately 1. The changing flocculation factor shown in Table 5.2 reveals that
flocculation only impacts fine particles, with smaller particles more significantly affected. Table 5.2 can also be used to decide the critical diameter of the flocculation particles is happened between 0.01 to 0.025 mm. For the particles in Table 5.2, when $F=1$, the critical particle size for flocculation is 0.022 mm.

5.3.2 Removal Efficiency

The performance of the sedimentation tank and the choice of the following treatment facilities have great influence on the quality of effluent in water and wastewater treatment. Removal efficiency is the most important item that can be monitored during water and wastewater treatment processes. Not only it is the most reliable parameter to evaluate the performance of the dimensioned sedimentation tank, but it is also the primary consideration for the engineers who design the treatment facilities after the sedimentation operation. Table 5.3 shows the total removal efficiency and removal efficiency for each group obtained from four rectangular sedimentation tank models. The models include the ideal model, Guo’s model, the new suspended sediment transport model, and the flocculation model.

Total Removal Efficiency

Figure 5.1 shows the total removal efficiency along the flow direction of the tank (length of the tank) after one day operation. As mentioned before, when setting up the model, it can be assumed that the settling process only happens in the settling zone, so the effective length of the tank is equal to the length of the settling zone. The $x$-axis represents the length of the settling zone which is 30 meters. The $y$-axis represents the removal efficiency.
Figure 5.1  Total Removal Efficiency Along the Length of the Tank
Table 5.3 Removal Efficiency

<table>
<thead>
<tr>
<th>Group No</th>
<th>Ideal Mode *</th>
<th>Guo’s Mode *</th>
<th>Without Flocculation</th>
<th>Flocculation Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.97%</td>
<td>1.03%</td>
<td>0.71%</td>
<td>17.69%</td>
</tr>
<tr>
<td>2</td>
<td>5.5%</td>
<td>6.4%</td>
<td>6.24%</td>
<td>25.67%</td>
</tr>
<tr>
<td>3</td>
<td>30.6%</td>
<td>30.76%</td>
<td>30.67%</td>
<td>31.26%</td>
</tr>
<tr>
<td>4</td>
<td>100%</td>
<td>80.69%</td>
<td>80.75%</td>
<td>82.15%</td>
</tr>
<tr>
<td>5</td>
<td>100%</td>
<td>99.86%</td>
<td>99.86%</td>
<td>99.89%</td>
</tr>
<tr>
<td>6</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>7</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>8</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>Total</td>
<td>78.66%</td>
<td>73.76%</td>
<td>74.55%</td>
<td>76.78%</td>
</tr>
</tbody>
</table>

Note: “*” data came from Jin and Guo’s paper (Jin et al. 2000)

From Table 5.3, the total removal efficiency from Guo’s model and the suspended sediment transport model (without flocculation) are very close to each other. The difference is less than 1%. This observation means that the suspended sediment transport model can generate good results consistent with present numerical models. The total removal efficiency in the flocculation model is 76.78%, compared to Guo’s 73.76%; an increase of 3% can be noticed. This is because the total removal efficiency is largely dependent on the contribution of the finer particles removal efficiency (Imam 1981). The difference of total removal efficiency between the flocculation model and the ideal model is less than 2%, but the flocculation model is more reliable. This is because, compared to the assumptions made in the ideal model, the assumptions made in the flocculation model are more practical.

Figure 5.1 shows that more than 50% of the suspended sediment has been removed from the water in the first 5 meters of the settling zone. This means that the bulk of the sedimentation happens in the area close to the inlet zone. A large increase in
the length of the settling zone contributes only a little improvement to the generation of deposits. Thus, increasing the length of the sedimentation tank to improve total removal efficiency is not economical. In the first 8 meters, the total removal efficiency in the flocculation model, Guo’s model, and the suspended sediment transport model are almost the same; the differences between them are less than 1%. All in all, the total removal efficiency which is predicted by the Ideal model, Guo’s model, and the proposed numerical models (the suspended sediment transport model and the flocculation model) are similar. The results from Figure 5.1 can be used to modify the length of the settling zone based on the model user’s preferred total removal efficiency.

**Removal Efficiency for Each Particle Group**

From Table 5.3, it is obvious that removal efficiency increases with increasing particle size, and that the removal efficiency in the flocculation model is comparable to the other models. This observation supports the belief that flocculation only happens to fine particles, with smaller particles being more significantly affected. In the flocculation model, the removal efficiency for Group1 and Group2 are 17.69% and 25.67%, which is many time larger than the same groups’ removal efficiency in other models. This is because these two groups of particles in the flocculation model are not spherical. Instead, the dispersed particles stick to each other to form a net shape. These nets are many times larger than 0.005mm and 0.01mm and therefore have the ability to capture other particles.

When particle sizes are smaller than 0.025mm, the removal efficiency in the Ideal model, Guo’s model, and the suspended sediment transport model are nearly the same. When the particle sizes are in the range of 0.025mm to 0.05mm, the removal efficiency in Guo’s model, the suspended sediment transport model, and flocculation model are very close. In the Ideal model, the removal efficiency reached 100% for all
particle groups if their particle sizes were larger than 0.025\(mm\). From Guo’s model, the new suspended sediment transport model, and the flocculation model, particles can be completely removed only when their sizes exceed 0.05\(mm\). The prediction for the removal efficiency of larger particles from the numerical models and the ideal model are similar to each other.

Figure 5.2 shows the removal efficiency for each particle group in the flocculation model. In this figure, particles in Group 8, 7, 6, and 5 can be removed completely, but the full settling distance required for each group is quite different. The particles in Group 8 only need 2\(m\) to be totally removed from the water. The particles in Group 7 need 5\(m\) for full settling, and Group 6 needs 9\(m\). The particles in Group 5 need 25\(m\) for full removal; this distance is nearly the entire length of the sedimentation tank. From Table 5.2, the settling velocity for particles in Group 8, 7, 6, and 5 are much greater than the overflow rate \(q_v\). Combining the analysis from before, these particles can be completely settled, which supports the Ideal sedimentation tank theory.

From the analysis of the total removal efficiency and the removal efficiency for each particle group, it is obvious that the existence of flocculation can slightly improve the total removal efficiency, and that flocculation has a more significant influence on small suspended dispersed particles than larger ones. Because flocculation happens in real sedimentation processes, it is recommended to use the flocculation model to simulate the settling process during tank design.
Figure 5.2  Removal Efficiency for Each Particle Group Along the Length of the Tank
5.3.3 Particle Size Distributions (PSD)

Table 5.4 shows the suspended particle size distribution at the outlet in the Ideal model, Guo’s model, the suspended sediment transport model, and the flocculation model with the same inflow particle size distribution. The PSD at the outlet in Guo’s model and the suspended sediment transport model have identical values for all of the eight particle groups. However, in the flocculation model, the PSD in the effluent for Group1 and Group2 have the smallest values in comparison with the other three numerical models. This demonstrates again that flocculation has a more significant effect on finer particles. The PSD for Group3 in the flocculation model has more differences than any other group. This is because of the value of the critical particle size is 0.022mm which belongs to this group. The PSD for the rest of the five groups in the flocculation model have similar values to Guo’s model and the suspended sediment transport model.

Table 5.4  Particle size distributions (PSD)

<table>
<thead>
<tr>
<th>Group No</th>
<th>PSD (%) (Influent)</th>
<th>PSD (%) (Effluent)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ideal Model</td>
<td>Guo’s Model</td>
</tr>
<tr>
<td>1</td>
<td>2.0</td>
<td>9.28</td>
</tr>
<tr>
<td>2</td>
<td>8.0</td>
<td>35.43</td>
</tr>
<tr>
<td>3</td>
<td>17.0</td>
<td>55.29</td>
</tr>
<tr>
<td>4</td>
<td>22.0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>20.0</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>14.0</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>11.0</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>6.0</td>
<td>0</td>
</tr>
</tbody>
</table>
Figure 5.3   Particle Size Distributions at the Inlet and Outlet
Figure 5.3 shows the suspended particle size distributions in the effluent of the suspended sediment transport model and the flocculation model with the same inflow particle size distributions. In the influent only 50% of the particles are larger than 0.05mm. In the effluent, all particles are smaller than 0.05mm in both models. This means that all particles with a size larger than 0.05mm have been removed from the raw water completely, which also shows that the particles in the effluent are much finer than those in the influent. In Group3, the difference between SST model and flocculation model is bigger than other groups, this is because the critical particle size is 0.022mm. The particle size in the effluent has important influence in the design of further treatment facilities, so the particle size distribution in the outlet is one of the important achievements of both models.

5.3.4 Sludge Particle Size Distribution

Sludge particle size distributions at the different dimensions of the outlet in Guo’s model, the suspended sediment transport model, and the flocculation model with the same influent sludge particle size distributions are shown in Table 5.5. Comparison between Guo’s model and the suspended sediment transport model shows that increasing the settling zone length decreases the difference between the two models. This may be because the particle settling conditions are more stable beyond the inlet and outlet areas. Compared to the suspended sediment transport model, the flocculation model shows an increase in the proportion of small particles in the effluent, and relatively less large particles. However, real sludge particle sizes in the effluent from the flocculation model cannot be known without a sieve test.
Table 5.5  Particle size distributions (PSD)

<table>
<thead>
<tr>
<th>Group No</th>
<th>Sludge (%) (Effluent)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Guo’s Model</td>
</tr>
<tr>
<td></td>
<td>5m</td>
</tr>
<tr>
<td>1</td>
<td>0.01</td>
</tr>
<tr>
<td>2</td>
<td>0.41</td>
</tr>
<tr>
<td>3</td>
<td>4.77</td>
</tr>
<tr>
<td>4</td>
<td>23.86</td>
</tr>
<tr>
<td>5</td>
<td>48.9</td>
</tr>
<tr>
<td>6</td>
<td>20.54</td>
</tr>
<tr>
<td>7</td>
<td>1.5</td>
</tr>
<tr>
<td>8</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Figure 5.4 show the sludge particle size distributions at the inlet and at the 30m outlet in the three models. At the inlet, 90% of the sludge particles have a size larger than 0.1mm. The larger particles make up a significant portion of the sludge. At the 30m outlet, 99% of the sludge particles are smaller than 0.05mm in all of the three models. This means that finer sludge particles remain near the outlet while coarser sludge particles accumulate at the bottom of the settling zone. These results also show that coarser sludge particles are deposited earlier, while finer sludge particles need a very long distance to settle out. From the figure, because of the existence of critical particle there is a minor different between two models.
Figure 5.4 Sludge Particle Size Distributions at the Inlet and Outlet (30 m)
5.3.5 Sludge Thickness Along the Length of the Tank

The simulation results in Figure 5.5 show how the accumulated sludge thickness, with flocculation considered, changes with the length of the sedimentation tank. The thickness of the accumulated sludge collected after one day of operation possessed a floc density of 1200 kg/m³. The peak of the sludge accumulation happened near the beginning of the settling zone. After the peak, the thickness decreased very sharply along the length of the tank. This observation can be used to aid in the construction of the sludge scouring functions. From Figure 5.5, the value of the peak is about 0.47 m in the flocculation model. In the proposed suspended sediment transport model the peak of the sludge accumulation is 0.46 m. Data from this model was not shown in Figure 5.5 because the trend lines from the two models greatly overlap. The figure also shows that more than 90% of the sludge accumulated in the first five meters of the settling zone.

Like removal efficiency, this model also provides the sludge thicknesses along the length of the settling zone for each particle group. Figure 5.6 shows the sludge thickness along the length of the settling zone for each particle group. The peaks of accumulated sludge thicknesses occur near the very beginning of the settling zone, especially for particles in groups 6, 7, and 8, which are completely settled to the bottom of the tank within the first five meters. This observation shows that the coarser particles are falling to the bottom much faster than the finer particles.

The thickness of the sludge is affected by the inflow sediment concentration, the operation hour, and the density of the dry particles. The sludge scouring operation should be designed based on the thickness of the accumulated sludge.
Figure 5.5   Sludge Thickness Along the Length of the Tank
Figure 5.6  Sludge Thickness Along the Length of the Settling Zone for Each Group
5.4 Sensitivity Analysis

The sensitivity analyses are based on the relationship between some key parameters which should be considered in the rectangular sedimentation tank model and their influence on the removal efficiency, such as hydraulic detention time (HDT), surface loading rate (SLR), and tank dimensions. With constant inflow rate, operation time, and particle settling information, the surface loading rate and the tank dimensions will be changed to explore the differences.

Table 5.6 shows a set of numerical experimental results with different detention times, overflow rates, and tank dimensions. The following observations can be made from these results:

1. For cases 1, 5, 8, and 12, with constant detention time and tank volume, the total removal efficiencies decrease with increasing surface loading rate. This is identifying the ideal settling theory, particles with smaller settling velocity than SLR can be removed partially.

2. For the cases which are sharing the same surface loading rate, the removal efficiencies are almost the same no matter how large the tanks are and how long the detention time is. Increasing the size of the rectangular sedimentation tank is not an economic or effective method.

3. To study the influence of tank dimensions on total removal efficiency, cases with fixed water depth and tank width show that removal efficiency is greater with increasing tank length. For the fixed tank surface area, the removal efficiency is inversely proportional to the water depth. So reducing the depth of the sedimentation tank does not affect the performance of the tank; however it does reduce cost and space requirements. The influence of the water depth on the tank’s performance seems greater than that of the length of the settling zone.
From the above analysis, once the detention time is constant, the surface loading rate becomes the most important influencing factor for the performance of the rectangular sedimentation tanks. Additional benefits can be gained by reducing the depth of the sedimentation tank, increasing the cross-sectional area in the flow direction of the settling zone, and reducing the surface loading rate.

Table 5.6   Removal efficiency for varying key parameters

<table>
<thead>
<tr>
<th>Case No.</th>
<th>HDT (min)</th>
<th>SLR (m/min)</th>
<th>Depth (m)</th>
<th>Width (m)</th>
<th>Length (m)</th>
<th>Total RE (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>34.10</td>
<td>0.044</td>
<td>1.5</td>
<td>4.0</td>
<td>30.0</td>
<td>80.55</td>
</tr>
<tr>
<td>2</td>
<td>45.45</td>
<td>0.044</td>
<td>2.0</td>
<td>3.0</td>
<td>40.0</td>
<td>80.54</td>
</tr>
<tr>
<td>3</td>
<td>68.18</td>
<td>0.044</td>
<td>3.0</td>
<td>2.0</td>
<td>60.0</td>
<td>80.53</td>
</tr>
<tr>
<td>4</td>
<td>25.57</td>
<td>0.059</td>
<td>1.5</td>
<td>4.0</td>
<td>22.5</td>
<td>76.78</td>
</tr>
<tr>
<td>5</td>
<td>34.10</td>
<td>0.059</td>
<td>2.0</td>
<td>3.0</td>
<td>30.0</td>
<td>76.77</td>
</tr>
<tr>
<td>6</td>
<td>51.14</td>
<td>0.059</td>
<td>3.0</td>
<td>2.0</td>
<td>45.0</td>
<td>76.76</td>
</tr>
<tr>
<td>7</td>
<td>20.45</td>
<td>0.073</td>
<td>1.5</td>
<td>4.0</td>
<td>18.0</td>
<td>73.76</td>
</tr>
<tr>
<td>8</td>
<td>34.10</td>
<td>0.073</td>
<td>2.5</td>
<td>3.0</td>
<td>24.0</td>
<td>73.75</td>
</tr>
<tr>
<td>9</td>
<td>40.90</td>
<td>0.073</td>
<td>3.0</td>
<td>2.0</td>
<td>36.0</td>
<td>73.74</td>
</tr>
<tr>
<td>10</td>
<td>17.05</td>
<td>0.088</td>
<td>1.5</td>
<td>4.0</td>
<td>15.0</td>
<td>71.26</td>
</tr>
<tr>
<td>11</td>
<td>22.73</td>
<td>0.088</td>
<td>2.0</td>
<td>3.0</td>
<td>20.0</td>
<td>71.25</td>
</tr>
<tr>
<td>12</td>
<td>34.10</td>
<td>0.088</td>
<td>3.0</td>
<td>2.0</td>
<td>30.0</td>
<td>71.24</td>
</tr>
</tbody>
</table>

5.5 Model Performance Analysis

By comparing Guo’s model, the suspended sediment transport model, and the flocculation model, the flocculation model does improve the settling performance for the fine particles. The following section is used to assess the accuracy of the flocculation model by comparing it with experimental data and other models.
5.5.1 El-Baroudi’s Laboratory Work

First of all, the experimental data reported by El-Baroudi (1969) are used for model comparison. The experiments were conducted in a rectangular sedimentation tank with length of 1.2446m and the width of the settling zone was 0.4572m. Since the sediment concentrations for El-Baroudi’s experiments are 0.1-0.35kg/m³ which is within the range of experimental conditions in Huang (1981) and Mehta (1986). The values of coefficients are: \( n_d = 1.9 \), \( d_r = 0.022 \) mm, and \( s_p = 3.5 \) kg/m³, \( k_1 = 0.513 \), \( k_2 = 0.008 \), \( n_s = 1.3 \), \( r = 4.65 \). Five of the experiments were modelled and the inflow rate, overflow rate, detention time, and the depth of the water are summarized in Table 5.7. Figure 5.7 shows the comparison between flocculated model and experimental data by plotting the removal efficiency versus \( \omega L/q \).

Table 5.7 Experimental parameters (El-Baroudi 1969)

<table>
<thead>
<tr>
<th>Run No.</th>
<th>Discharge ( Q \times 10^{-4} )m³/s</th>
<th>SLR ( \times 10^{-4} )m/s</th>
<th>HDT (min)</th>
<th>Depth (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-7</td>
<td>2.21</td>
<td>3.88</td>
<td>8.8</td>
<td>0.205</td>
</tr>
<tr>
<td>2-3</td>
<td>1.26</td>
<td>2.22</td>
<td>25.0</td>
<td>0.333</td>
</tr>
<tr>
<td>2-4</td>
<td>1.58</td>
<td>2.77</td>
<td>22.0</td>
<td>0.366</td>
</tr>
<tr>
<td>2-5</td>
<td>1.89</td>
<td>3.32</td>
<td>18.3</td>
<td>0.365</td>
</tr>
<tr>
<td>2-8</td>
<td>3.00</td>
<td>5.26</td>
<td>11.6</td>
<td>0.366</td>
</tr>
</tbody>
</table>

The critical particle size is an important parameter to learn the flocculation, for all these experiments: the critical particle size for run1-7 is 0.0226mm, run 2-3 is 0.0221, run 2-4 is 0.023, run 2-5 is 0.0223, run 2-8 is 0.0226. In Figure 5.7, the flocculation model produced good predictions in comparison to El-Baroudi’s experimental results. More specifically, in the following figure, when the value of \( \omega L/q \) between 0.3 to 1, the removal efficiency predicted by the newly developed models are almost identical to El-Baroudi’s experimental observations, especially when
it equals approximately 1. This means for large particles, the flocculated sedimentation tank model gave a better prediction for removal efficiency compared to the prediction for removal efficiency for finer particles. This is because flocculation made fine particles stick together to form larger flocs.
Figure 5.7 Comparison of removal efficiencies between flocculation model and experimental data
5.5.2 Swamee’s Model

The models are further assessed by comparing with Swamee and Tyagi’s model (Swamee and Tyagi 1996). In their model, the removal efficiency for the rectangular sedimentation tank is fixed at 85%. To achieve this removal efficiency the tank’s dimensions are: length=38m, width=3.0m, and water depth=1.5m. The numerical experimental conditions are: inflow rate=0.1m³/s, average particle size=3.173 × 10⁻⁵ m, and particle falling velocity=0.00134m/s. By using the suspended sediment transport model to achieve a removal efficiency of 85%, the length of the tank is 39.1m, which is quite close to 38m. In the flocculation model, with the settling zone length of 38m, the calculated removal efficiency is 88.8%. The differences of results between these models are small but noticeable.

5.5.3 Two Different Models from Wu (Lin and Wu 2013; Wu 2007)

In the flocculated sedimentation tank model, there are some correction factors which are used to calculate the flocculation factor. In Governing Equations Chapter 3, equation (3.30) is used to calculate the correction factor $k_s$, which is caused by the inflow sediment concentration. The form of equation (3.30) is from Lin and Wu’s published paper in 2013:

$$k_s = \begin{cases} 1 + k_1s^{n_s}, & 0 < s \leq s_p \\ K(1 - k_2s)^\gamma, & s > s_p \end{cases}$$ (5.1)

In Wu’s book, published in 2007, there is a different equation which can also be used to calculate the flocculation influence caused by the sediment concentration:

$$k_s = \begin{cases} k_1s^{n_s}, & 0 < s \leq s_p \\ K(1 - k_2s)^\gamma, & s > s_p \end{cases}$$ (5.2)

where $K$ is equal to: $k_1s_p^{n_s}/(1 - k_2s_p)^\gamma$. 

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Figure 5.8 Removal Efficiencies along the Length of the Tank by using Different Equations
As shown in Figure 5.8, by substituting equation (5.2) into the flocculated sedimentation tank model, the total removal efficiency is calculated to be 90% with the same numerical model set up as Guo’s model. To assess the model’s performance, equation (5.2) needs to be compared with El-Baroudi’s experimental observation. Compared with all of the five experimental runs, the removal efficiencies calculated using equation (5.2) showed quite a big difference to the experimental observations. However, the results are quite close to the Ideal model’s results. Because of these, equation (5.1) was chosen and applied to the flocculated sedimentation tank model.
Chapter 6 Conclusions and Recommendations

6.1 Conclusions

A one-dimensional numerical model has been developed in this study to simulate the suspended sediment transportation and flocculation in a rectangular sedimentation tank. The depth-averaged method, the finite difference approximation, and the MATLAB program were used to solve the flow governing equations, the sediment transport equations, and the flocculation correction factors to construct these models. The suspended sediment transport model is a modification of Jin’s Class I sedimentation tank model (Jin et al. 2000) in order to be used for both steady flow and unsteady flow. The flocculation sedimentation tank model is a modification of the suspended sediment transport model in order to produce reliable results considering the flocculation of fine particles.

The models divided multiple-particles into groups based on the particle size, in each particle groups the depth-averaged settling velocity is constant. The flocculation model takes flocculation factor into consideration through the floc settling velocity. The flocculation model can be used for practical applications such as predicting RE, particle and sludge size distribution from inlet to outlet, and sludge thickness. The model also shows that flocculation only impacts on fine particles and the critical particle size exists.

Sensitivity analyses have verified that the effect of surface loading rate on the total removal efficiency is quite significant. With fixed water depth, removal efficiency varies slightly with changing cross-sectional areas in the flow direction. With increasing water depth, the total removal efficiency decreases. As such, an increase in the dimensions of the rectangular sedimentation tank is not the best way to improve the tank’s performance. With a constant surface loading rate, significant changes in hydraulic detention time and tank volume do not significantly affect the total removal
efficiency. The flocculation model produced a better removal efficiency prediction for large particles by comparing with El-Baroudi’s experimental data.

6.2 Recommendations

The equations for the flocculation correction factors seem to be complex, but because they are separated into different conditions, they actually make the model more reliable when used. The flocculation sedimentation tank model is set up, so the direct application of this model is simple.

The newly developed models were set up to calculate all the useful information for rectangular sedimentation tanks based on one day of operation at a tank length of 60 meters. If the model user wanted to calculate a larger domain, the calculation time will be increased.

The coefficients which were used to create the models should be further calibrated by experimental data and similar case studies to improve the accuracy of the model’s results. These coefficients will not be the same with different inflow situations, different suspended particles size conditions, and different sedimentation tank properties.

The model was programmed in MATLAB and model users will require sufficient skill in MATLAB to run this model. Of course, with all the applied equations listed in the previous chapters, the model user can translate the MATLAB code into any other programming language they prefer.
References


Appendix

Program for calculating the flocs settling velocity:

clc;
clear all;
%calculate the settling velocities for flocs
%Input Initial Data
SS=0.5;
Q=0.088;
L=60;
B=2;
H=3.5;
q=Q/B;
dx=0.1;
Density=1200;
t=6;
dt=60;
slope=0.02;
n=0.011;
K=0.01;
m=0.92;
alpha=1.2;
Groups=8;

Group_U=[0.0000095; 0.0000536; 0.000299; 0.00134; 0.00536; 0.0172; 0.0404; 0.0828];

Group_PSD=[0.02; 0.08; 0.17; 0.22; 0.20; 0.14; 0.11; 0.06];
Group_Size=[0.005; 0.01; 0.025; 0.05; 0.1; 0.25; 0.5; 1.0];

dr=0.022;

nd=1.9;

cp=1.5;

n1=1.3;

K1=0.513;

K2=0.008;

r=4.65;

%Solving H

g=9.8;

StoreH(1,:)=H;

for i=1:L/dx

\[ R(i,:) = B \times \text{StoreH}(i,:) / (B + 2 \times \text{StoreH}(i,:)) \]

\[ p(i,:) = \text{StoreH}(i,:) + q^2 / (2 \times g \times \text{StoreH}(i,:)^2) + dx \times (n^2 \times (q^2) / (2 \times g \times \text{StoreH}(i,:)^3 \times R(i,:))^{0.33}) \]

\[ v(i,:) = q^2 / (2 \times g) \]

\[ w(i,:) = dx \times (n^2 \times (q^2) / (g \times 2 \times R(i,:)^{0.33})) \]

\[ \text{Equa}(i,:) = [1 - p(i,:) \ 0 \ v(i,:) - w(i,:)] \]

\[ \text{S_Equa}(i,:) = \text{roots}(\text{Equa}(i,:)) \]

\[ \text{Sa}(i,:) = \text{S_Equa}(1,i) \]

\[ \text{StoreH}(i+1,:) = \text{Sa}(i,:) \]

\[ i = i + 1 \]
end

\[ \text{Store}_h = \text{zeros}(L/dx+1, 60*60*t/dt+1) \]

for i=1:L/dx+1
if i==L/dx+1
    Store_h(i,:)=StoreH(L/dx+2-i,:);
else
    Store_h(i,:)=StoreH(L/dx+2-i,:) - slope*(L-dx*(i-1));
end
i=i+1;
end

% Calculation of Suspended Sediment concentration SS
% Calculation of Sediment Carrying Capacity SCP
Store_SSCP=zeros(L/dx+1,60*60*t/dt+1,Groups);
Store_SS=zeros(L/dx+1,60*60*t/dt+1,Groups);
Store_dz=zeros(L/dx+1,60*60*t/dt+1,Groups);
Total_dz=zeros(L/dx+1,60*60*t/dt+1);
for j=1:60*60*t/dt+1
    for k=1:Groups
        Store_SSCP(1,j,k)=K*((q^3/(Store_h(1,j)^4*Group_U(k,:)))^m);
        Store_SS(1,j,k)=SS*Group_PSD(k,:);
        for i=2:L/dx+1
            Store_SS(i,j,k)=(1-dx*alpha*Group_U(k,:)/q)*Store_SS(i-1,j,k)+(dx*alpha*Group_U(k,:)/q)*Store_SSCP(i-1,j,k);
            Store_SSCP(i,j,k)=K*((q^3/(Store_h(i,j)^4*Group_U(k,:)))^m);
            if Store_SSCP(i,j,k)>Store_SS(i,j,k)
                Store_SSCP(i,j,k)=Store_SS(i,j,k);
            end
        end
    end
end
\[
\text{Store}_{dz}(i,j,k) = \alpha \cdot \text{Group}_U(k,:) \cdot \text{dt} \cdot (\text{Store}_{SS}(i,j,k) - \text{Store}_{SCP}(i,j,k)) / \text{Density};
\]

\[
i = i + 1;
\]

\[
k = k + 1;
\]

\[
\text{end}
\]

\[
\text{for } i = 1: \text{L/dx+1}
\]

\[
\text{for } k = 1: \text{Groups}
\]

\[
\text{Total}_{dz}(i,j) = \text{Total}_{dz}(i,j) + \text{Store}_{dz}(i,j,k);
\]

\[
k = k + 1;
\]

\[
\text{end}
\]

\[
i = i + 1;
\]

\[
\text{end}
\]

\[
\text{j} = j + 1;
\]

\[
\text{end}
\]

\[
\%	ext{Calculation of flocs}
\]

\[
\text{Store}_{FSCP} = \text{zeros}(\text{L/dx+1,60*60*t/dt+1,Groups});
\]

\[
\text{Store}_{FSS} = \text{zeros}(\text{L/dx+1,60*60*t/dt+1,Groups});
\]

\[
\text{Store}_{Fdz} = \text{zeros}(\text{L/dx+1,60*60*t/dt+1,Groups});
\]

\[
\text{Total}_{Fdz} = \text{zeros}(\text{L/dx+1,60*60*t/dt+1});
\]

\[
\text{Group}_u = \text{zeros}(\text{Groups,1});
\]

\[
\text{for } j = 1:60*60*t/dt+1
\]

\[
\text{for } k = 1: \text{Groups}
\]

\[
\text{Store}_{FSS}(1,j,k) = \text{SS} \cdot \text{Group}_{PSD}(k,:);
\]

\[
\text{if } \text{Group}_{Size}(k,:) \geq \text{dr}
\]

\[
\text{end}
\]

\[
\text{end}
\]

\[
\text{end}
\]
kd=1;
else Group_Size(k,:)<dr
    kd=(dr/Group_Size(k,:))^nd;
end
if 0<Store_FSS(1,j,k)<=cp
    Ks=1+K1*Store_FSS(1,j,k)^n1;
else Store_FSS(1,j,k)>cp
    Ks=((1+K1*cp^n1)/(1-K2*cp^r))*(1-K2*Store_FSS(1,j,k))^r;
end
Group_u(k,:)=Group_U(k,:)*kd*Ks;
Store_FSCP(1,j,k)=K*((q^3/(Store_h(1,j)^4*Group_u(k,:)))^m);
for i=2:L/dx+1
    Store_FSS(i,j,k)=(1-dx*alpha*Group_u(k,:)/q)*Store_FSS(i-1,j,k)+(dx*alpha*Group_u(k,:)/q)*Store_FSCP(i-1,j,k);
    Store_FSCP(i,j,k)=K*((q^3/(Store_h(i,j)^4*Group_u(k,:)))^m);
    if Store_FSCP(i,j,k) >= Store_FSS(i,j,k)
        Store_FSCP(i,j,k)=Store_FSS(i,j,k);
    end
    Store_Fdz(i,j,k)=alpha*Group_u(k,:)*dt*(Store_FSS(i,j,k)-Store_FSCP(i,j,k))/Density;
    i=i+1;
end
k=k+1;
end
for i=1:L/dx+1
for k=1:Groups
    Total_Fdz(i,j)=Total_Fdz(i,j)+Store_Fdz(i,j,k);
    k=k+1;
end
i=i+1;
end
for i=1:L/dx+1
    Store_h(i,j+1)=Store_h(i,j)-Total_Fdz(i,j)-Total_dz(i,j);
    i=i+1;
end
j=j+1;
end
Group_u

Program for results calculation and figures plot:
clc;
clear all;

% Input Initial Data
SS=0.5;
Q=0.088;
L=30;
B=3;
H=4;
q=Q/B;
dx=0.1;
Density=1200;
t=24;
dt=60;
slope=0.02;
n=0.011;
K=0.01;
m=0.92;
alpha=1.2;
Groups=8;
Group_u=[0.0001588; 0.00024163; 0.00030522; 0.0014; 0.0055; 0.0175; 0.0409; 0.0832];
Group_PSD=[0.02; 0.08; 0.17; 0.22; 0.20; 0.14; 0.11; 0.06];
Group_Size=[0.005; 0.01; 0.025; 0.05; 0.1; 0.25; 0.5; 1.0];

%CALCULATION PART

%Solving H

g=9.8;
StoreH(1,:)=H;
for i=1:L/dx
    R(i,:)=B*StoreH(i,:)/(B+2*StoreH(i,:));
p(i,:)=StoreH(i,:)+q^2/(2*g*StoreH(i,:)^2)+dx*(n^2)*(q^2)/(2*g*StoreH(i,:)^3*R(i,:)*0.33);
v(i,:)=q^2/(2*g);
w(i,:)=dx*(n^2)*(q^2)/(g*2*R(i,:)^0.33);
Equa(i,:)=[1-p(i,:) 0 v(i,:) -w(i,:)];
S_Equa(:,i)=roots(Equa(i,:));
Sa(i,:)=S_Equa(1,i);
StoreH(i+1,:) = Sa(i,:);

i = i+1;

end

Store_h = zeros(L/dx+1,60*60*t/dt+1);

for i=1:L/dx+1
    if i==L/dx+1
        Store_h(i,:) = StoreH(L/dx+2-i,:);
    else
        Store_h(i,:) = StoreH(L/dx+2-i,:)-slope*(L-dx*(i-1));
    end
    i = i+1;
end

% Major Calculation
% Calculation of Suspended Sediment concentration SS
% Calculation of Sediment Carrying Capacity SCP

Store_SCP=zeros(L/dx+1,60*60*t/dt+1,Groups);

Store_SS=zeros(L/dx+1,60*60*t/dt+1,Groups);

Store_dz=zeros(L/dx+1,60*60*t/dt+1,Groups);

Total_dz=zeros(L/dx+1,60*60*t/dt+1);

for j=1:60*60*t/dt+1
    for k=1:Groups
        Store_SCP(1,j,k)=K*((q^3/(Store_h(1,j)^4*Group_u(k,:)))^m);
        Store_SS(1,j,k)=SS*Group_PSD(k,:);
    end
    for i=2:L/dx+1
Store_{SS}(i,j,k) = (1 - dx*alpha*Group_u(k,:)/q)*Store_{SS}(i-1,j,k) + (dx*alpha*Group_u(k,:)/q)*Store_{SCP}(i-1,j,k);

Store_{SCP}(i,j,k) = K*((q^3/(Store_h(i,j)^4*Group_u(k,:)))^m);

if Store_{SCP}(i,j,k) >= Store_{SS}(i,j,k)
    Store_{SCP}(i,j,k) = Store_{SS}(i,j,k);
end

Store_{dz}(i,j,k) = alpha*Group_u(k,:)*dt*(Store_{SS}(i,j,k) - Store_{SCP}(i,j,k))/Density;

i = i+1;
end

k = k+1;
end

for i = 1:L/dx+1
    for k = 1:Groups
        Total_dz(i,j) = Total_dz(i,j) + Store_{dz}(i,j,k);
        k = k+1;
    end
    i = i+1;
end

for i = 1:L/dx+1
    Store_h(i,j+1) = Store_h(i,j) - Total_dz(i,j);
    i = i+1;
end

j = j+1;
end
%Removal Efficiency for each group

Store_RE=zeros(L/dx+1,60*60*t/dt+1,Groups);

for k=1:Groups
    for j=1:60*60*t/dt+1
        for i=1:L/dx+1
            Store_RE(i,j,k)=(Store_SS(1,j,k)-Store_SS(i,j,k))/Store_SS(1,j,k)*100;
        end
        i=i+1;
    end
    j=j+1;
end

%Total Removal Efficiency

Total_RE=zeros(L/dx+1,60*60*t/dt+1);
Total_SS=zeros(L/dx+1,60*60*t/dt+1);

for j=1:60*60*t/dt+1
    for i=1:L/dx+1
        for k=1:Groups
            Total_SS(i,j)=Total_SS(i,j)+Store_SS(i,j,k);
        end
        Total_RE(i,j)=(SS-Total_SS(i,j))/SS*100;
        i=i+1;
    end
    j=j+1;
end
end

%Suspended Particle size at inlet and outlet

Particle_In=zeros(Groups,1);
Group_PSD_Out=zeros(Groups,1);
Particle_Out=zeros(Groups,1);
for k=1:Groups
    if k==1
        Particle_In(k,1)=Group_PSD(k,1);
    else
        Particle_In(k,1)=Group_PSD(k,1)+Particle_In(k-1,1);
    end
    Group_PSD_Out(k,1)=Store_SS(L/dx+1,60*60*t/dt+1,k)/Total_SS(L/dx+1,60*60*t/dt+1);
    if k==1
        Particle_Out(k,1)=Group_PSD_Out(k,1);
    else
        Particle_Out(k,1)=Particle_Out(k-1,1)+Group_PSD_Out(k,1);
    end
    k=k+1;
end

%Sludge Particle Size Distribution at inlet and outlet

Group_Sludge_In=zeros(Groups,1);
Group_Sludge_Out=zeros(Groups,1);
Sludge_In=zeros(Groups,1);
Sludge_Out=zeros(Groups,1);
for k=1:Groups
    Group_Sludge_In(k,1)=(Store_SS(1,60*60*t/dt+1,k)-
    Store_SS(2,60*60*t/dt+1,k))/(Total_SS(1,60*60*t/dt+1)-Total_SS(2,60*60*t/dt+1));
    Group_Sludge_Out(k,1)=(Store_SS(L/dx,60*60*t/dt+1,k)-
    Store_SS(L/dx+1,60*60*t/dt+1,k))/(Total_SS(L/dx,60*60*t/dt+1)-
    Total_SS(L/dx+1,60*60*t/dt+1));
    if k==1
        Sludge_In(k,1)=Group_Sludge_In(k,1);
        Sludge_Out(k,1)=Group_Sludge_Out(k,1);
    else
        Sludge_In(k,1)=Sludge_In(k-1,1)+Group_Sludge_In(k,1);
        Sludge_Out(k,1)=Sludge_Out(k-1,1)+Group_Sludge_Out(k,1);
    end
    k=k+1;
end

%FIGURE DRAWING PART
%Figure2 Removal Efficiency for Each Group
for k=1:Groups
    plot(0:dx:L,Store_RE(:,60*60*t/dt+1,k));
    hold on
    k=k+1;
end
axis([0 L 0 100])
xlabel('Length of the tank (m)')
ylabel('Removal Efficiency (%)')
title('Removal Percentage for Each Group size')
hold off
grid on

% Figure 1: Total Removal Efficiency
Accumulation_RE=ones(L/dx+1,1);
for i=1:L/dx+1
    for j=1:60*60*t/dt+1
        Accumulation_RE(i,1)=Total_RE(i,j)-Accumulation_RE(i,1);
        j=j+1;
    end
    i=i+1;
end
for i=1:L/dx+1
    if i==L/dx+1
        RE(i,1)=Accumulation_RE(i,1)-Accumulation_RE(i-1,1);
    else
        RE(i,1)=Accumulation_RE(i+1,1)-Accumulation_RE(i,1);
    end
    i=i+1;
end
figure
plot(0:dx:L,Total_RE(:,60*60*t/dt+1),0:dx:L,RE(:,1),'blue--');
grid on
axis([0 L 0 100])
xlabel('Length of the tank (m)')
ylabel('Removal Efficiency (%)')

title('Accumulated and Longitudinal Removal Efficiency Distribution')

%Figure3 Suspended Particle size at inlet and outlet

Particle_In_Percent=Particle_In*100;
Particle_Out_Percent=Particle_Out*100;

figure
plot(Group_Size,Particle_In_Percent,Group_Size,Particle_Out_Percent,'blue--');
axis([0 Group_Size(Groups,1) 0 100])
xlabel('Particle Size (mm)')
ylabel('Percentage of finer (%)')
title('Suspended Particle Size Distributions at Inlet and Outlet')
grid minor

%Figure4 Sludge Particle Size Distribution at inlet and outlet

Sludge_In_Percent=Sludge_In*100;
Sludge_Out_Percent=Sludge_Out*100;

figure
plot(Group_Size,Sludge_In_Percent,Group_Size,Sludge_Out_Percent,'blue--');
grid minor
axis([0 Group_Size(Groups,1) 0 100])
xlabel('Particle Size (mm)')
ylabel('Percentage of finer (%)')
title('Sludge Particle Size Distributions at Inlet and Outlet')
grid minor

%Figure5 Total Sludge Thickness

Accumulation_dz=zeros(L/dx+1,1);
for i=1:L/dx+1
for j=1:60*60*t/dt+1
    Accumulation_dz(i,1)=Accumulation_dz(i,1)+Total_dz(i,j);
    j=j+1;
end
i=i+1;
end
figure
plot(0:dx:L,Accumulation_dz(:,1),'black-');
grid on
axis([0 L 0 0.3])
xlabel('Length of the tank (m)')
ylabel('Sludge thickness (m)')
title('Sludge Thickness along the Length of The Tank')

Figure 6: Sludge Thickness for Each Group

figure
for k=1:Groups
    plot(0:dx:L,Store_dz(:,60*60*t/dt+1,k));
    hold on
    k=k+1;
end
hold off
axis([0 L 0 0.0002])
xlabel('Length of the tank (m)')
ylabel('Sludge thickness (m)')
title('Sludge Thickness for Each Group')
grid on