A CALCULATION MODEL FOR STEAM PROPERTY VARIATION ALONG WELLBORE TRAJECTORY IN SAGD PROCESS

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By
Ning Ju
Regina, Saskatchewan
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Ning Ju, candidate for the degree of Master of Applied Science in Petroleum Systems Engineering, has presented a thesis titled, *A Calculation Model for Steam Property Variation Along Wellbore Trajectory in SAGD Process*, in an oral examination held on December 17, 2015. The following committee members have found the thesis acceptable in form and content, and that the candidate demonstrated satisfactory knowledge of the subject material.

External Examiner: Dr. Kelvin Ng, Environmental Systems Engineering

Supervisor: Dr. Gang Zhao, Petroleum Systems Engineering

Committee Member: Dr. Fanhua Zeng, Petroleum Systems Engineering

Committee Member: Dr. Yang Zhao, Department of Mathematics & Statistics

Chair of Defense: Dr. Guoxiang Chi, Department of Geology
ABSTRACT

Steam-Assisted Gravity Drainage is an effective approach for recovering heavy oil and bitumen and its essence idea is to introduce steam heat into cold reservoir and reduce oil viscosity. In a typical SAGD process, two horizontal wells were drilled inside the target formation, one is put on the top as steam injection well and another is usually put on the bottom of target formation as oil production well.

During SAGD process, a large amount of high pressure high temperature steam is injected into reservoir which occupies a big part of the whole SAGD project cost. The steam pressure and quality decreases during the steam flow inside vertical wellbore because the steam loses its heat through wellbore system to formation due to temperature gradient. And in horizontal wellbore, steam even flow into formation through slotted liner which takes away energy directly. For a SAGD project, the steam pressure and steam quality insides steam injection well are quite important parameters. A high enough quality can offer enough energy for steam chamber to develop and steam pressure will influence the oil production rate, steam trap control as well as ultimate oil recovery. In order to control the cost of SAGD production and offer evidence for steam injection, oil production strategy, the knowledge of steam properties (pressure, temperature, quality) along both vertical and horizontal wellbore are needed.
The steam flow inside wellbore is a two phase (dry steam and hot water) flow and the determination of phase void fraction is critical in predicting the pressure loss and heat transfer. The major difference between steam flow inside horizontal wellbore and vertical wellbore is the existence of wall outflow in horizontal wellbore part. This wellbore outflow has a significant effect on wellbore friction as well as flow pattern transition inside wellbore which make it difficult to describe flow pattern in horizontal wellbore by former technology.

A model describe steam flow inside wellbore during conventional Steam Assisted Gravity Drainage stage (after the steam chamber has achieved full height and lateral growth becomes the dominant mechanism for recovery) was built and solved in this thesis. A flow pattern independent drift flux model based void fraction correlation was introduced in this thesis in order to overcome the uncertainty problem in determining flow pattern and making is possible to describe steam flow in both vertical wellbore and horizontal wellbore in an unified way. A modified Reis’s drainage model was used in this thesis which combined steam flow inside wellbore and steam chamber development inside formation.

The steam properties (pressure, quality) distribution along wellbore trajectory were calculated, the effects of basic steam injection parameters (pressure, quality and mass flow rate) were analysed in this thesis. These steam property profiles along wellbore trajectory actually build correlations between wellbore flow and oil production, and will improve the understanding in steam injection strategy adjustment as well as oil production dynamic monitoring.
ACKNOWLEDGEMENTS

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DEDICATION

To

My best friend and companion, Ms. Shanshan Yao,

and my loving parents Mr. Yujun Ju and Ms. Xiumei Fan.
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NOMENCLATURE

\( A \) cross section area of wellbore, \( m^2 \)

\( A_{\text{slot}} \) area of each slot, \( m^2 \)

\( A_h \) cross section area of horizontal wellbore, \( m^2 \)

\( a \) temperature coefficient, dimensionless

\( C_o \) distribution coefficient, dimensionless

\( G_r \) Grashof number, dimensionless

\( C_{vr} \) initial reservoir volumetric heat capacity, \( J / m^3 \cdot K \)

\( C_{vo} \) overburden volumetric heat capacity, \( J / m^3 \cdot K \)

\( D \) hydraulic pipe/wellbore diameter, \( m \)

\( dQ \) heat loss to surround, \( J / m \)

\( dW \) work done by wall friction force, \( J \)

\( f(t) \) transient heat conduction function, dimensionless

\( f_o \) friction factor calculated by Colebrook equation, dimensionless

\( f_g \) in-situ gas phase void fraction, dimensionless

\( h_g \) enthalpy of dry steam, \( KJ / Kg \)

\( h_l \) enthalpy of liquid phase, \( KJ / Kg \)

\( h_c \) convective heat transfer coefficient of fluid inside annulus,

\( J / S \cdot m^2 \cdot K \)

\( h_r \) radiative heat transfer coefficient of fluid inside annulus,

\( J / S \cdot m^2 \cdot K \)
$h_f$  convective heat transfer coefficient of fluid, $J / S \cdot m^2 \cdot K$

$h$  height of reservoir above producer, $m$

$H_c$  heat stored inside steam chamber, $J / m$

$H_{out}$  heat accumulation ahead of steam chamber, $J / m$

$H_o$  heat loss to the overburden formation, $J / m$

$H_{inj}$  cumulative heat loss, $J / m$

$h_m$  enthalpy of steam, $KJ / Kg$

$k_e$  thermal conductivity of formation, $J / s \cdot m \cdot K$

$k_t$  thermal conductivity of tubing, $J / s \cdot m \cdot K$

$k_{cem}$  thermal conductivity of cement, $J / s \cdot m \cdot K$

$k_{cas}$  thermal conductivity of casing, $J / s \cdot m \cdot K$

$k_o$  effective oil permeability, $m^2$

$M_g$  gas phase mass flow rate, $Kg / s$

$M_l$  liquid phase mass flow rate, $Kg / s$

$Mm$  steam mass flow rate, $Kg / s$

$m$  viscosity coefficient, dimensionless

$N$  slot density, $1 / m$

$P$  steam saturatation pressure, $kPa$

$P_{an}$  annulus liquid pressure, $kPa$

$P_r$  Prandtl number, dimensionless

$Q_{inj}$  instantaneous heat loss to formation and steam chamber, $J / m \cdot s$
\( q_{\text{in}} \) mass flow rate into formation, \( \text{Kg/m} \cdot \text{s} \)
\( q_{\text{m}} \) steam (gas, liquid mixture) volume flow rate, \( \text{m}^3/\text{s} \)
\( q_{\text{g}} \) gas phase volume flow rate, \( \text{m}^3/\text{s} \)
\( q_{\text{l}} \) liquid phase volume flow rate, \( \text{m}^3/\text{s} \)
\( R_{\text{etp}} \) two-phase Reynolds number, dimensionless
\( R_{\text{rw}} \) Wall Reynolds number, dimensionless
\( R_{\text{z}} \) length of vertical segment, \( \text{m} \)
\( r_{\text{to}} \) radius of outside tubing, \( \text{m} \)
\( r_{\text{n}} \) radius of inside tubing, \( \text{m} \)
\( r_{\text{ins}} \) radius of insulation, \( \text{m} \)
\( r_{\text{co}} \) radius of outside casing, \( \text{m} \)
\( r_{\text{wb}} \) radius of wellbore, \( \text{m} \)
\( T \) steam saturation temperature, \( \text{K} \)
\( T_{\text{c}} \) water critical temperature under a specific pressure, \( \text{K} \)
\( T_{\text{e}} \) formation temperature, \( \text{K} \)
\( T_{\text{f}} \) fluid temperature inside tubing, \( \text{K} \)
\( T_{\text{h}} \) cement-formation interface temperature, \( \text{K} \)
\( v_{\text{qin}} \) steam flow rate into formation, \( \text{m/s} \)
\( v_{\text{g}} \) actual gas flow rate, \( \text{m/s} \)
\( v_{\text{l}} \) actual liquid flow rate, \( \text{m/s} \)
\( v_{\text{sg}} \) superficial velocity of gas phase, \( \text{m/s} \)
$v_{sl}$ superficial velocity of liquid phase, $m/s$

$\nu_d$ drift velocity of gas phase, $m/s$

$U_{to}$ over-all heat transfer coefficient, $J/S \cdot m^2 \cdot K$

$x$ dry steam quality, dimensionless

$\theta$ wellbore orientation measured from horizontal direction, $rad$

$\rho_g$ gas phase density, $Kg/m^3$

$\rho_l$ liquid phase density, $Kg/m^3$

$\rho_m$ steam density, $Kg/m^3$

$\varepsilon_{to}$ emissivity of outside tubing surface, dimensionless

$\varepsilon_{ti}$ emissivity of inside casing surface, dimensionless

$\mu_l$ liquid phase viscosity, $Pa \cdot s$

$\nu_{os}$ kinematic oil viscosity at steam temperature, $m^2/s$

$\alpha$ formation thermal diffusivity, $m^2/s$

$\Delta S_o$ initial oil saturation minus residual oil saturation, dimensionless

$\phi$ porosity, dimensionless

$\tau$ friction force between steam and liner, $N$
CHAPTER 1
INTRODUCTION

1.1 Steam-assisted Gravity Drainage

Heavy oil and oil sands bitumen account for 70% of the world proven oil reserves (World’s Oil and Natural Gas Scenario). Especially for Canada, 98% of oil reserves, 175 billion barrels, are reserved in the form of oil sands (Hein, Fran, 2008). Moreover, conventional oil (light oil) supply peaks eventually and enters into decline. Therefore the efficient development of heavy oil and oil sands becomes more important for Canada’s and even world’s energy security.

Heavy oil and oil sands bitumen are characterized by high viscosity compared with conventional oil. The gas-free viscosity of heavy oil under reservoir condition lies between 100cp and 10,000cp while the bitumen’s viscosity can reach 10,000,000cp. Such high viscosity makes it difficult for the oil to flow underground (Working Document of the NPC Global Oil & Gas Study). As a thermal based oil production method, the Steam-assisted Gravity Drainage (SAGD) technique appears and attracts enormous attention for its efficiency in reducing viscosity and producing oil.

The concept of Steam-Assisted Gravity Drainage (SAGD) becomes practical in the 1970s after the horizontal well was successfully introduced into petroleum industry. In 1978, a first modern horizontal producing well paired with a vertical steam-injection well was drilled at Cold Lake, Alberta, Canada. This is the initial attempt of the SAGD concept. In the 1980s, the first-ever twin SAGD
Figure 1.1-Schematic of SAGD process (Courtesy of IFPEN, 2015).
wells were drilled at Dover Underground Test Facility, Canada. The production success of this field test boosts commercial implementation of SAGD projects. In 2010 approximately 240,000 barrels of oil per day came from SAGD wells in Canada (M Medina, 2010).

**Figure 1.1** shows the schematic of SAGD process. With the SAGD technique, a pair of horizontal wells, situated 4 to 6 meters above the other, is drilled in a central well pad. Water is heated into steam in a plant nearby and then travels through above-ground pipelines to wells, enters the reservoir via the top well (injection well) and forms a steam chamber. The injected steam flows towards the edge of the chamber where the steam releases its latent heat to the cold oil and condenses. Oil viscosity reduces dramatically after receiving the energy. And the mixture of mobile oil and steam condensate flows down into the bottom well (production well) under gravity drive. As the oil flows away, the steam will fill the place originally occupied by cold oil and expands vertically and/or horizontally. In SAGD, steam injection and oil production happen continuously and simultaneously.

Commercial SAGD process usually involves five phases of operations: start-up, ramp-up, conventional SAGD, steam rampdown and blowdown operations (**Figure 1.2**). In start-up and ramp-up, SAGD well-pair are communicated and the steam chamber grows up vertically to reservoir top. Then the conventional
Figure 1.2-SAGD production stages (Lackey and Kane, 2013).
SAGD phase begins. The conventional SAGD operation can last for 6 years, which is much longer than other operations. In conventional SAGD phase, the steam chamber achieves full height and spreads laterally in the reservoir. Usually higher steam injection rates are necessary to guarantee steady oil production in this phase. Steam injection rate gradually reduces to zero during rampdown and blowdown. And SAGD wells are abandoned when oil production declines to an uneconomic rate (Lackey and Kane, 2013).

Generally, in reservoirs the steam which contacts with in-situ oil comes directly from the injection well. The injection wellbore conditions are closed related to the chamber development and oil production. Hence monitoring wellbore conditions becomes necessary for optimizing steam injection and minimizing cost. Measuring devices can stay in the wellbore and collect real-time temperature and pressure data. The measuring devices include thermocouples, fiber optic, pressure gauges, etc. But such measuring devices are unstable in high temperature and also very expensive. Moreover, downhole steam quality is an important factor in SAGD production surveillance, which cannot be tested by measuring devices. Now wellbore modeling provides another option for monitoring wellbore conditions. Wellbore modeling by analytical and/or numerical methods can accurately predict temperature, pressure and steam quality profile along the wellbore.
1.2 Scope and Objectives of This Study

This study is focused on the steam properties (pressure, quality) distribution along wellbore (both vertical and horizontal) during conventional SAGD production phase since this period last the longest time and make a large contribution of the whole oil production in the SAGD well lifetime.

The concept of SAGD, initially proposed by Butler and his colleagues (Butler et al. 1981) has been used worldwide for the recovery of heavy oil and bitumen. Two horizontal wells are placed close to the bottom of a formation, with one above the other at a short vertical distance. Steam is injected continuously into the upper well, and rises in the formation forming a steam chamber. Cold oil surrounding the steam chamber is heated mainly by thermal conduction. As its temperature increases, oil becomes mobile and flows together with condensate along the chamber boundary toward the lower well that functions as a producer (Butler, 1997).

After steam is injected into steam injection, it begins to loss its heat to the wellbore system and formation around, and this energy loss decreases the ability of steam heating reservoir and also improves steam-oil ratio (which measures the volume of steam used to produce one unit volume of oil, the lower the ratio, the higher the efficiency of the steam use.) which leading to noneconomic oil production.

Many investigators have worked on the modelling and prediction of steam property insider vertical wellbore since the first introduction of thermal-based enhanced oil recovery method. A series of research work were done step by
step: Ramey gave out approximate solution to the transient heat-conduction problem in 1962. Satter (1965) improved Ramey’s analytical model by considering a depth-dependent overall heat transfer coefficient and phase and temperature-dependent fluid properties; Holst and Flock (1966) added the friction loss and kinetic energy effect to Ramey’s and Satter’s models; Willhite (1967) proposed the formation of overall heat transfer coefficient, Farouq-Ali (1981) put forward a comprehensive wellbore steam/water flow model for steam injection and geothermal application. After then, the basic idea and procedure of calculating the heat loss during vertical steam injection was established.

The main difference between horizontal steam injection wellbore and vertical injection wellbore are: (1) horizontal injection wellbore completion is more concise, usually with slotted liner and open hole condition, so the heat conduction through wellbore system is quite small; (2) horizontal wellbore completed with slotted liner leading to a directly mass transfer and heat transfer between wellbore and formation. Because of the two mainly characteristics of SAGD horizontal steam injection wellbore, the steam flow inside horizontal wellbore is dramatically changed when compared with that of vertical wellbore, and the technologies used to model vertical steam flow and technologies used to describe flow behavior in ordinary horizontal pipes are not working well here.

Ouyang (1998) conducted a comprehensive research work on pressure drop along wellbore, inflow effect on fluid behavior inside wellbore based on a 5 years’ experiment. However, this research work was mainly focused on the oil
production well which with a big difference in fluid flow behavior with steam injection well.

Considering this situation, this thesis was focused on describing fluid flow behavior inside horizontal steam injection well. The problem was studied from the basic mass balance, momentum balance, energy balance equation. A flow pattern independent drift flux model based void fraction correlation was introduced in this thesis in the aim to describe the two-phase flow in a one phase manner, which intended to avoid flow pattern related difficulties. A modified Reis’s drainage model was used within this model to calculate the mass transfer between wellbore and formation. A new two-phase friction factor was also used to consider the effect of steam outflow on axial wellbore fluid flow.
1.3 Organization of the Thesis

Five chapters comprise this thesis. Chapter 1 introduces the background knowledge of SAGD process and outlines the study objectives. Chapter 2 shows a comprehensive literature review on multiphase flow, heat transfer around wellbore and steam chamber as well as SAGD drainage model. Chapter 3 gives the detailed mathematical model of the injection wellbore in conventional SAGD operation. The mathematical model is developed by combing vertical wellbore model and horizontal wellbore model. Then Chapter 4 validates this model by using field data and other models. Chapter 5 analyzes modeling results under different wellbore and reservoir properties. Finally, Chapter 6 draws conclusions and provides recommendations.
CHAPTER 2

LITERATURE REVIEW

In order to predict steam properties variation along wellbore trajectory accurately, several works need to be done, this including the precise description of steam flow behavior inside wellbore, steam chamber development behavior inside formation, heat transfer behavior between steam and wellbore system as well as formation. In this chapter, the literature review was conducted towards these four topics.

2.1 Multiphase Flow in Wellbore

During the SAGD process, the steam was injected into the target formation through vertical wellbore and then horizontal wellbore, Steam flow along wellbores is gas-liquid two phase flow. And such multiphase flow effects in wellbores have a strong impact on the performance of reservoir and surface facilities. Incorrect consideration of pressure losses resulted by multiphase flow in wellbore may lead to a loss of production at the toe and/or overproduction at the heel. In order to optimize the performance of wells, multiphase flow models in wellbore must be considered.
Figure 2.1-Flow patterns in vertical co-current gas-liquid flow (Bar-Meir, 2013).
2.1.1 Multiphase Flow in Vertical Wellbore


**Figure 2.1** shows the flow patterns in vertical gas-liquid flow. In vertical bubbly flow, discrete bubbles with low velocity distribute uniformly throughout the continuous liquid phase. In slug flow the gas or vapor bubbles grow to Taylor bubbles which have almost same diameter as the pipe. Bubbles are separated from the pipe wall only by a thin film of liquid. The Taylor bubbles break down in churn flow. Unlike slug flow, the gas or vapor flows in a chaotic manner when churn flow occurs. For annular flow, the continuous gas phase flows through the core of the pipe while the liquid phase is dragged along the wall.

Since the determination of slow pattern is critical for modeling multiphase flow, many researchers have tried to find a way to describe different flow patterns. One technology is called the flow pattern map, is a method of represent the various flow patterns, and the individual patterns are areas on a two-dimensional (2D) graph, where coordinates are the actual superficial phase velocities or generalized parameters containing these velocities. One example of this kind is the flow pattern map put forward by Hewitt and Roberts (1969).

Considering that the flow pattern map are usually in the form of graph, and not easy to be used in mathematical model, Hasan and Kabir (1988) further
Table 2.1-Transition criteria for multiphase flow in vertical pipes (Hasan and Kabir, 1988).

<table>
<thead>
<tr>
<th>Flow Type</th>
<th>$v_{sg}$</th>
<th>$v_{m}^{1.12}$</th>
<th>$v_{sl}^{2} \rho_l$</th>
<th>$v_{sg}^{2} \rho_g$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bubbly Flow</td>
<td>$(0.429 v_{sl} + 0.357 v_{\infty}) &lt; 0.8 (0.429 v_{sl} + 0.357 v_{\infty})$</td>
<td>$&gt; 4.68d^{0.48} \left[ \frac{g \left( \rho_l - \rho_g \right)}{\sigma} \right]^{0.5}$</td>
<td>$&gt; 50$</td>
<td>$&lt; [17.1 \log_{10} (\rho_l v_{ls}^{2}) - 23.2]$</td>
</tr>
<tr>
<td>Slug Flow</td>
<td>$(0.429 v_{sl} + 0.357 v_{\infty}) &gt; 0$</td>
<td>$&gt; 50$</td>
<td>$&lt; 0.00673 (v_{ls}^{2} \rho_l)^{1.7}$</td>
<td></td>
</tr>
<tr>
<td>Churn Flow</td>
<td>$&lt; 3.1 \left[ \frac{\sigma_l (\rho_l - \rho_g)}{\rho_g^{2}} \right]^{0.25}$</td>
<td>$&gt; 50$</td>
<td>$&gt; [17.1 \log_{10} (\rho_l v_{ls}^{2}) - 23.2]$</td>
<td></td>
</tr>
<tr>
<td>Annular Flow</td>
<td>$&gt; 3.1 \left[ \frac{\sigma_l (\rho_l - \rho_g)}{\rho_g^{2}} \right]^{0.25}$</td>
<td>$&lt; 50$</td>
<td>$&gt; 0.00673 (v_{ls}^{2} \rho_l)^{1.7}$</td>
<td></td>
</tr>
</tbody>
</table>
provided transition criteria to determine different flow patterns. In Hasan’s method, each flow pattern corresponds to certain range of void fraction values, which are based on mathematical equations. Table 2.1 summarizes the criteria according to Hasan and Kabir (1988). This method became more popular since it allows the physical modeling of individual flow patterns to be expressed in the form of equations. Hasan also developed a mathematical model to calculate void fractions and pressure drop for multiphase flow in vertical pipes for different flow patterns.

2.1.2 Multiphase Flow in Horizontal Wellbore

Flow patterns during two-phase flow through horizontal pipes are similar to those observed during vertical flow. However, some important differences exist, arising mainly from the gravity effect. Unlike liquid phase, the gas phase usually concentrates toward the upper wall in horizontal multiphase flow. And phase distribution tends to be asymmetrical. Hence judgement in flow patterns for vertical flow causes difficulty in designate flow regimes of horizontal multiphase flow.

Taitel et al. (1978) classified flow regimes for gas-liquid two-phase flow in horizontal pipes. Figure 2.2 shows the flow patterns in horizontal flow. In stratified smooth flow, liquid phase flows at the bottom while gas move at the top. The interface between two phases is smooth. Stratified wavy flow is quite similar to stratified smooth flow except that the interface is wavy. For plug flow or elongated bubble flow, elongated gas bubbles at the upper part are separated by sections of continuous liquid. In slug flow, liquid slugs separated by gas
Figure 2.2-Flow patterns in horizontal flow (Taitel et al, 1978).
Figure 2.3-Flow pattern map for gas-liquid flow in horizontal pipes (Mandhane, et al., 1974).
pockets flow downstream violently. The liquid slugs may get aerated by distributed small bubbles at higher gas flow rates. Usually the degree of agitation and height of liquid film between slugs make slug flow different from plug flow. Annular flow takes place when gas flow rates are high. When the aeration in liquid phase is sufficiently high under large gas flow rates, gas forms a continuous phase and annular flow occurs. In annular flow, gas moves in the pipe center and liquid flows in the annular space. Furthermore liquid film at the bottom is thicker than that at the top. When liquid rates become high, small gas bubbles disperse in the continuous liquid phase and dispersed bubble flow forms. Normally the bubble density at the top is higher than that at the bottom.

Mandhane et al. (1974) Proposed transition criteria for multiphase horizontal flow based on 6,000 experiment data points. Flow patterns are determined according to superficial gas velocity \( v_{sg} \) and superficial liquid velocity \( v_{sl} \). Figure 2.3 shows the detailed flow pattern map in Mandhane et al.’s work.

Taitel et al. (1978) also gave the transition criteria for multiphase flow in horizontal wellbore. In stratified to intermittent or annular flow,

\[
v_g \geq \left(1 - \frac{h}{D}\right) \left[ \frac{(\rho_l - \rho_g)gA_g}{\rho_g A_i} \right]^{\frac{1}{2}}. \quad (2.1)
\]

In stratified smooth to stratified wavy flow,

\[
v_g \geq \frac{400u_i(\rho_l - \rho_g)g}{\rho_g v_i} \cdot \frac{1}{\varphi}. \quad (2.2)
\]

Here \( u_i \) is the kinematic viscosity. For intermittent to dispersed bubble flow,

\[
v_i \geq \frac{4A_g g}{S_i} \frac{1}{F_i} \left[ \frac{(\rho_l - \rho_g)}{\rho_l} \right]^{\frac{1}{2}}. \quad (2.3)
\]
Here

$h$ is liquid level,

$u$ is velocity in the $x$ direction,

$D$ is pipe diameter, also hydraulic diameter,

$A$ is flow cross-section area,

$A_h$ is differentiation with respect to $h$,

$F_i$ is the friction factor between well and liquid phase,

$S_i$ is the gas/liquid interface perimeter.
2.1.3 Multiphase Flow Modeling

So far three types of models can describe the multiphase flow in petroleum industry: the empirical correlations, homogeneous models and mechanistic models.

The empirical correlations are based on curve fitting of experimental data (Duns and Ros, 1963; Hagedorn and Brown, 1965). These empirical correlations are simple since no complex physical model is required. But the correlations can only apply to very limited set of flow conditions under which experiments are conducted.

Homogeneous models treat multiphase flow as pseudo-single phase flow. The multiple phases own same velocity, pressure, temperature and average fluid properties. Homogeneous models are simple, continuous and differentiable, which makes them applicable in numerical reservoir simulators. But Shi et al. (2005) pointed out some simple homogeneous models ignore slip between phases and fail to capture the complex relationship between in-situ volume fraction and the input volume fraction.

Mechanistic models model each phase separately and reveal the interplay of multiple phases. Such models apply local instantaneous conservation equations and develop average relations for variables of interest. The fundamental postulate in these models is that different flow patterns exist in multiphase flow. Mechanistic modeling approach is most rigorous of three modeling techniques. Correspondingly it provides reliable predictions of phase properties. However, several concerns of mechanistic models draw researchers'
attention. The conservation equations in these models come from experimental data matching and therefore may not be applicable to cases beyond the experiment conditions. In addition, for multiphase flow in a pipe with mass transfer across the pipe wall, no sufficient experimental data are available for mechanistic models. Furthermore, sometimes mechanistic models cannot converge during flow pattern transition boundaries. At last, Ouyang (2000) argued that mechanistic models implemented in a full field reservoir simulator could significantly slow down the entire process.

Drift flux model (Zuber and Findlay, 1965), a kind of homogeneous model, appears as an intermediate technique between rigorous mechanistic approach and simple homogeneous models. Drift flux models treat multiple phases as a mixture but also account for the velocity difference between gas phase and liquid phase. The drift flux model develops a relationship between the gas phase velocity and mixture velocity as:

### 2.1.4 Void Fraction Correlations

After understanding the differences between these three multiphase flow modeling technologies, it is a good choice to use drift-flux model to describe the steam flow inside wellbore, because it is simple to use and can also get the slip effect between different phases. By using drift flux-model, the steam flow inside wellbore is described as one homogeneous model. In this case, the phase void fractions become critical parameter to our calculation when the mixture parameters such as density and viscosity need to be determined.
Generally, there are four types of void fraction correlations: (1) slip ratio models; (2) $K_{eff}$ correlations; (3) drift flux correlations; (4) empirical correlations (Vijayan et al., 2000).

For annular flow, Zivi (1964) gave out a widely-used correlation of void fraction $f_g$ vs. quality $x$. His work was based on two hypothetical conditions: (1) the principle of minimum entropy production is applicable to a turbulent two-phase flow and (2) the wall shear stress is negligible which is far from reality in actual two-phase flows. The correlation is:

$$ f_g = \frac{1}{1 + \left(\frac{1-x}{x}\right)(\rho_g / \rho_l)^{2/3}} $$

Where $\rho_g$ is the vapor density and $\rho_l$ is the liquid density.

Hughmark (1965) developed a correlation to estimate the void fraction $f_g$ in horizontal slug flow according to the bubble velocity and the liquid slug Reynolds number:

$$ f_g = \frac{Q_g}{Q_g + Q_l + Re_l \left(\frac{Q_g + Q_l}{A}\right)A} $$

where $Q_g$ is the volumetric flow rate of gas phase, $Q_l$ is the volumetric flow rate of liquid phase, $A$ is the cross-section area and $Re_l$ is a function of the liquid Reynolds number.

Wallis (1970) proposed a simple theory for annular two-phase flow in terms of interfacial and wall shear stress. Wallis showed the correlation of void fraction $f_g$ and quality $x$ as
\[ x^2 = \frac{(1 - F_g)^2[1 + 75(1 - f_g)]}{f_g^{3/2}} \] .........................................................(2.6)

This equation was based on two simplified conditions: (1) there is no liquid entrainment and (2) the liquid film velocity is low compared with the velocity of the gas core.

Tandon et al. (1985) developed an analytical model to predict void fraction of two-phase annular flow. Tandon et al. described the annular flow as a vapour core in an axisymmetric liquid annulus. The 1D fluid flow is steady and no radial pressure gradient exists. In addition, both liquid and vapour flows are to be turbulent. Moreover, both phases have constant properties corresponding to the saturated state at any locations. The void fraction \( f_g \) in annular flow is as

\[ f_g = 1 - 1.928 \text{Re}_i^{-0.315} F(X_u)^{-1} + 0.9293 \text{Re}_i^{-0.63} F(X_u)^{-2}, 50 < \text{Re}_i < 1125 \] .... (2.7a)

\[ f_g = 1 - 0.38 \text{Re}_i^{-0.088} F(X_u)^{-1} + 0.0361 \text{Re}_i^{-0.176} F(X_u)^{-2}, 1125 < \text{Re}_i \] .............. (2.7b)

where

\[ F(X_u) = 0.15(X_u^{-1} + 2.85X_u^{-0.476}), \] ..............................................................(2.7c)

\[ X_u = \left( \mu_l / \mu_g \right)^{0.1} \left( \frac{1 - X}{X} \right)^{0.9} \left( \rho_g / \rho_l \right)^{0.5}. \] ..............................................................(2.7d)

\( \text{Re}_{el} \) is liquid Reynolds number,

\( \mu \) is dynamic viscosity.

Jepson and Taylor (1993) conducted an experimental study on slug flow behavior of air and water inside a 30cm steel pipe. They compared results with those in small diameter pipes (2.54cm and 5.08cm). Their results showed that the slug frequency and pressure gradient along pipe decrease as the pipe...
diameter increases. But the slug length increases if the pipe diameter becomes larger.

2.1.5 Summary

The drift flux model considers the slip effect between phases and it can also be solved easily. So the drift flux model becomes flexible and useful to model the gas-liquid two phase flow in the wellbore. However, there is a major shortcome for most of current drift flux models: they are developed for a specific flow pattern or need additional information related to the flow pattern which is sometimes difficult to identify clearly. Thus, judging the correct flow pattern of steam flow when using the drift flux model becomes a challenging issue and it affects its application. Furthermore, during the steam flow inside the wellbore, the flow patterns are changing along with time and locations. It’s difficult to notice the transition of one flow pattern to another. So it is hard to guarantee correct estimation of steam flow calculation. The flow-pattern-independent drift flux model based void fraction correlation is able to predict the void fraction of gas and liquid phases over a wide range of system pressure, pipe diameters and fluid properties. After introducing this correlation, this study tried to avoid flow pattern related non-continuity and non-convergence problem, and generated acceptable calculation results for the steam-wellbore system during SAGD process.
2.2 Heat Transfer around Wellbore

Steam pressure, temperature and density change once the steam is injected into the wellbore. Reasons of such changes mainly include the heat transfer between the steam and cold formation around wellbore, the friction between the steam and inner tubing surface and the change of the hydrostatic pressure with respect to depth.

In 1962, Ramey developed an analytical solution of the transient heat-conduction problem when hot fluids flow down the tubing. In Ramey’s work, the single-phase hot fluid is in steady state flow. Also the overall heat transfer coefficient in the model is independent of depth. The friction loss and kinetic energy were further neglected. Satter (1965) improved Ramey’s analytical model by considering a depth-dependent overall heat transfer coefficient and phase- and temperature-dependent fluid properties in heat conduction equations.

In 1966, Holst and Flock added the friction loss and kinetic energy effect to Ramey’s (1962) and Satter’s (1965) models. The friction losses lead to an extra pressure drop in the steam along wellbore.

In 1967, Wilhite proposed a widely-used method to estimate the overall heat transfer coefficient as.

\[
U_{to} = \left( \frac{r_{to}}{r_{to} h_f} + \frac{r_{to} \ln \frac{r_{to}}{r_{ai}}}{k_{tub}} + \frac{1}{r_{to} \ln \frac{r_{co}}{r_{ci}} + \frac{r_{to} \ln \frac{r_{h}}{r_{co}}}{k_{em}}} \right)^{-1}, \text{..................(2.8)}
\]

where

\( U_{to} \) is over-all heat transfer coefficient,
$r_{to}$ is outside radius of tubing,

$r_{ti}$ is inside radius of tubing,

$r_{ci}$ is inside radius of casing,

$r_{co}$ is outside radius of casing,

$k_{tab}$ is thermal conductivity of the tubing material at the average tubing temperature,

$k_{car}$ is thermal conductivity of the casing material at average casing temperature,

$k_{cem}$ is cement conductivity of the casing material at average cement temperature,

$h_{f}$ is film coefficient for heat transfer based pm the inside tubing or casing surface and the temperature difference between the flowing fluid and either of these surfaces,

$h_{c}$ is heat transfer coefficient for natural convection based on the outside tubing surface and the temperature difference between the outside tubing and inside casing surfaces.

The overall heat transfer coefficient represents the net resistance of the flowing fluid, tubing, casing annulus, casing wall and cement sheath to the flow of heat.

When heat is transferred from the flowing fluid inside wellbore to the formation outside wellbore, the fluid properties and heat transfer around wellbore are interrelated. Ali (1981) and Fontanilla and Aziz (1982) solved partial
differential equations (PDEs) numerically to estimate steam quality when steam flows along wellbore.

In 1994, Hasan and Kabir developed an analytical wellbore model to calculate the hot fluid temperature inside wellbore. They applied the Joule-Thompson coefficient to simplify PDEs into ordinary differential equations (ODEs) and solved the equations analytically under appropriate boundary conditions.

Livescu et al. (2008) proposed a comprehensive numerical non-isothermal multiphase wellbore model. They confirmed that decoupling the wellbore energy balance equations from mass balance equations is reasonable when densities of each phase fluid have smaller change with temperature than that with pressure.

In 2010, Bahonar et al. developed a semi-unsteady-state two-phase flow wellbore numerical model to calculate heat transfer between steam and formation around wellbore. This model coupled mass, momentum and energy balance equations and provides all necessary wellbore data with respect to depth and time for a predetermined surface condition.
2.3 Heat Transfer at Chamber Interface

The center idea of SAGD is to transfer heat from steam to cold bitumen. So it’s critical to understand the heat transfer amount and heat transfer mechanism at the steam chamber interface.

Based on simulation study, Ito and Suzuki (1996) concluded that heat conduction and convection coexist at the chamber interface. They showed that the heat conduction happens at the whole chamber in an even magnitude while heat convection happens at the upper part of chamber interface. Ali (1997) and Ito and Suzuki (1999) further indicated that main heat transfer mechanism in the SAGD process is by heat convection.

However, Edmunds (1999) argued that the heat transferred by convention only account for 5% of the heat transferred by conduction. Edmunds showed that in the chamber edge, water travels almost along the thermal isotherms and convention may reduce to zero.

Then Sharma and Gates (2011) re-examined heat transfer at the edge of steam chamber, and took account of convection of warm condensate into the oil sand at the edge of steam chamber. Their results showed that heat conduction dominates at the chamber edge approximately below 225°C. Even if the heat convection is important at above 225°C, it cannot enhance the oil recovery since oil mobility decreases. Although higher temperature reduces the oil viscosity, the convective flow of water into the oil sands also reduces the oil relative permeability. In terms of oil mobility, the effect of convective flow outweighs the temperature.
Irani and Ghannadi (2013) checked the relative roles of conductive and convective heat transfer at the edge of SAGD chambers based on their mathematical model and field data. They summarized that in oil-rich portion of the chamber edge, the contribution of heat conduction is less than 1% and can be neglected under common Athabasca reservoir conditions.

The calculation of total heat consumption in SAGD process is complex.

Reis (1992) divided the energy stored around steam chamber into three parts: (1) energy associated with expanding the steam zone; (2) energy required to preheat the tar; (3) energy lost to overburden.

Edmunds and Peterson (2007) presented an analytical model to estimate the cumulative steam/oil ratio of SAGD and other steam based recovery processes based on the research of Reis model. In this model the author made a simplification of Reis's model by introducing a constant account for heat stored below the steam chamber as a factor of the overburden transient losses which bring convenience in energy calculation. However, the author assume the value of effective sweep efficiency to be one-half and heat stored below moving front plus transient losses below vicinity of production well to be one-third of heat loss to overburden which is subjective and lack of evidence.

2.4 SAGD Drainage Model

Since the essence idea of SAGD is to transfer steam latent heat to cold reservoir and decrease oil viscosity, the steam consumption is directly correlated with steam chamber development inside formation and oil production. So a
concise and accurate SAGD drainage model is needed to calculate the amount of steam flow into the formation.

2.4.1 Analytical Drainage Models

In 1979, Butler et al. originally proposed an analytical model to describe the SAGD drainage process when the steam chamber has reached the reservoir top. The model is based on Darcy’s law, material balance equations and heat conduction principles. Two main assumptions were given out for this model: (1) The interface moved at a fixed velocity normal to the interface; (2) Heat transfer ahead of interface is by conduction only. The total drainage rate is calculated as

\[ Q = \frac{\sqrt{2Kg\alpha\phi S_o h}}{mv_s}. \]  \hspace{1cm} (2.9)

where

- \( Q \) is oil drainage rate in volume per unit length of well per unit time for each side of the chamber,
- \( K \) is effective permeability to oil flow,
- \( \alpha \) is the thermal diffusivity of the reservoir material,
- \( \phi \) is porosity,
- \( S_o \) is the difference between the initial oil saturation and the residual saturation,
- \( h \) is the distance from the bottom of the reservoir to the top,
- \( m \) is dimensionless constant,
- \( \nu_s \) is viscosity of the oil at steam temperature.
One main problem for this model is that the calculated chamber interface curves move away from the production well during production. When the lower part of the interface lies at the same height as the production well, it is unrealistic for the oil to move towards the production well without gravity force. Another concern is that the chamber tends to spread to infinity in horizontal direction.

In 1981, Butler and Stephens improved the earlier theory by introducing the “TANDRAIN” assumption. The chamber interface becomes straight in its lower part. This assumption enables the oil behind the interface to flow horizontally to production well. The drainage rate then becomes

\[ Q = \frac{\sqrt{1.5Kg\alpha\phi S_o h}}{mv_s} \]  

(2.10)

In addition, no-flow boundary was added in the modified model and the steam interface was only allowed to spread halfway between adjacent well pairs.

Reis (1992) developed a simplified analytical model for SAGD process. Reis assumed the steam chamber to be an inverted triangle in conventional SAGD process. The up-side chamber moves horizontally and the lower part are fixed at the production well. The oil production per unit length along the horizontal well in this model is

\[ Q_o = \sqrt{\frac{\phi\Delta S_o k_o gH\alpha}{2\alpha v_{so} m}} \]  

(2.11)

where \( \alpha \) is dimensionless temperature coefficient and equals 0.4. Reis also gave the steam oil ratio by considering the steam enthalpy and heat loss to overburden.
In 2005, Akin put forward a more comprehensive mathematical model based on Reis’s work. In previous study, oil viscosity was fully determined by temperature. This work considered the influence of asphaltene content on oil viscosity by using the theory of Werner et al. (1998) when fluid compositions are determined:

\[
\ln\left[\frac{\mu(P, T)}{\mu(P_0, T_0)}\right] = c\left(\frac{1}{T} - \frac{1}{T_0}\right) + E \ln\left(\frac{D + P}{D + P_0}\right) \tag{2.12}
\]

where

- \( c \) is parameter that determines the variation of viscosity as a function of temperature,
- \( P \) is pressure,
- \( T \) is temperature,
- \( D \) and \( E \) are parameters that determine the variation of viscosity as a function of pressure.

If fluid compositions are unknown, a reference viscosity under reference temperature and pressure is necessary for calculating arbitrary oil viscosity.

Azad and Chalaturnyk (2010) further considered the oil saturation variation and geo-mechanical effect in SAGD process. The volume of steam chamber can be divided into up to 90 slices. This is because though the number of slices is optional, choosing a high number increases computational time with no increase in accuracy. For simplicity in that study, the number of slices was assumed to be 90. The oil saturations change in every slice at the end of each time step. This allows for choosing different relative permeability in oil rate calculation according
to oil saturations. Moreover, limit equilibrium approach (Duncan, 1996) was applied to calculate porosity and permeability change with stress field.

2.4.2 Numerical Models

Numerical simulation has been used widely by many researchers to help understand the SAGD process. It can be a method to validate numerical model by history matching physical model results and it can also be used to investigate the effect of a specific reservoir parameter when there are inadequate measured physical properties. Chow and Butler (1996) conducted a numerical simulation of the SAGD process based on the Chung and Butler's (1988) experimental data. A two-dimensional, three-phase and two-component black oil numerical model for the SAGD process was developed and tested. The results showed that STARS numerical simulator can provide a good match of SAGD process during the steam chamber spreading sideways and downwards period while didn't model the rising steam chamber well for lacking of built-in physics to simulate water/oil emulsification and steam fingering. Ito and Suzuki (1999) conducted a simulation study on the Hangingstone SAGD project in order to improve the understanding of oil production mechanism and study subcooling temperature optimization for steam trap control. The geomechanical change of formation during SAGD process was discussed and the author also put forward the conclusion that convective energy carried by steam condensate dominates the heat transfer mechanism. Law and Nasr (2000) conducted a fiddle-scale numerical simulation of SAGD process in order to investigate the SAGD performance in the Athabasca oil sands in the presence of a top water zone. A
series of field-scale numerical modeling case were studied to analyse the applicability of SAGD process under different reservoir conditions as well as water zones and gas caps. This research extended the knowledge gained from lab-scale studies to predict field-scale SAGD performance. Chen (2009) conducted a numerical study of reservoir heterogeneity effects on SAGD process by using a stochastic model of shale distribution. The effect of reservoir heterogeneity on SAGD was studied separately in two regions called the near well region (NWR) and the above well region (AWR). The author found that the drainage and flow of hot fluids within the NWR are of short characteristic length and to be very sensitive to the presence and distribution of shale while the AWR affects the expansion of the steam chamber that is of characteristic flow length on the order of half of formation height. It is also shown that SAGD yields low or moderate oil production rate and recovery in the reservoir with poor vertical communication due to the presence of high percentage of shale.
In SAGD process, the steam quality in vertical wellbore drops with the depth because of the heat loss from steam to wellbore system towards formation. Inside the horizontal wellbore, the steam flows horizontally while some steam flows into the formation through the slotted liner. The steam into the formation offers the energy for heating the bitumen formation and keeping the steam chamber spread.

In this study, the steam flow process was divided into two sub-system. The first sub-system is the steam flow inside vertical wellbore, and it's a co-current gas-liquid two-phase flow with a constant mixture flow rate. And the second sub-system is the steam flow along the horizontal well with radial outflow into formation. It’s a co-current gas-liquid two-phase flow but the mixture flow rate decreases along horizontal wellbore.

3.1 Definitions

In this study, the steam is saturated wet steam, the mixture of gas (dry steam) and liquid (hot water). The steam mixture volumetric flow rate is the sum of gas phase rate and liquid phase rate:

\[ q_m = q_g + q_l \]

For this two-phase flow, neither phase occupies the entire wellbore cross area. The actual flow rates for each phase are
\[ v_g = \frac{q_g}{A_g}, \]  

(3.2a)

and

\[ v_l = \frac{q_l}{A_l}, \]  

(3.2b)

where \(A_g\) and \(A_l\) are actual cross-section area occupied by gas and liquid phase, respectively. The superficial velocities are defined as

\[ v_{sg} = \frac{q_g}{A}, \]  

(3.3a)

and

\[ v_{sl} = \frac{q_l}{A}, \]  

(3.3b)

where

\(v_{sg}\) and \(v_{sl}\) the superficial velocities of the gas and liquid phase,

\(q_g\) and \(q_l\) are the volumetric flow rate of the gas and liquid phase,

\(A\) is the cross-section area of the wellbore.

And the steam mixture velocity \(v_m\) is defined as

\[ v_m = v_{sg} + v_{sl}. \]  

(3.4)

The ratio of gas superficial velocity over steam mixture velocity \(\beta\) is defined as

\[ \beta = \frac{v_{sg}}{v_m} \]  

(3.5)

The relationship of steam saturation temperature (K) and saturation pressure (kPa) is given by (Tortike and Farouq Ali, 1989):

\[ T = 280.034 + 14.0856 \ln p + 1.38075 \ln p^2 - 0.101806 \ln p^3 + 0.019017 \ln p^4, 0.61 \text{kPa} \leq p \leq 22120 \text{kPa}. \]  

(3.6)
The relationship between the liquid phase viscosity \( \mu_l \) (Pa⋅s) and steam temperature (K) is by (Tortike and Farouq Ali, 1989):

\[
\mu_l = 27.1038T^{-1} - 23527.5T^{-2} + 1.01425 \times 10^7 T^{-3} - 2.17342 \times 10^9 T^{-4} + 1.86935 \times 10^{11} T^{-5} - 0.0123274 . \quad \text{.........(3.7)}
\]

\(273.15K \leq T \leq 645K\)

The two-phase Reynolds number is

\[
Re_{lp} = \frac{v_{lm} \rho D}{\mu_l} \quad \text{.........................................................(3.8)}
\]

And the two-phase Fanning friction factor between two-phase flow and pipe wall is represented in Colebrook equation (1939):

\[
\frac{1}{\sqrt{f_{lp}}} = -4.0 \log_{10}(\frac{\varepsilon/D}{3.7} + \frac{1.256}{Re_{lp} \sqrt{f_{lp}}}) \quad \text{......................................................... (3.9a)}
\]

In \textbf{Equation 3.9a}, \( \varepsilon \) is the wellbore tubing roughness.

When the outflow of steam exists at wellbore, the two-phase friction factor can be estimated by Ouyang equation (1998):

\[
f_{lp} = \frac{16.0}{R_{ep}} [1 - 0.0625 \frac{(-R_{ew})^{1.3056}}{(R_{ew} + 4.626)^{0.2724}}] \quad \text{laminar flow} \begin{array}{c} \text{......................................................... (3.9b)} \\ \text{......................................................... (3.9b)} \end{array}
\]

\[
f_{lp} = f_o \left(1 - 17.5 \frac{R_{ew}}{R_{ep}^{0.75}} \right) \quad \text{turbulent flow} \quad \text{.........................................................(3.9c)}
\]

here \( f_o \) is the friction factor calculated by Colebrook equation in Eq. (3.9a)

\[R_{ew} \quad \text{wall Reynolds number}\]

The in-situ steam void fraction is defined as the fraction of total cross-section area through which one phase flows. The in-situ steam void fraction \( f_g \) is

\[
f_g = \frac{A_g}{A} = \frac{V_{sg}}{V_g} \quad \text{......................................................... (3.10)}
\]
In the two-phase flow drift flux model of this study, the relationship between $v_g$ and $v_m$ can be written as

$$v_g = C_0 v_m + v_d, \quad \text{................................................................. (3.11)}$$

where $C_0$ is the distribution coefficient which describes the effect of the velocity and concentration profiles and $v_d$ is the drift velocity of gas which describe the buoyancy effect. Therefore the gas void fraction can be rewritten as

$$f_g = \frac{v_g}{C_0 v_m + v_d}, \quad \text{................................................................. (3.12a)}$$

where the parameters are calculated by (Bhagwat and Ghajar, 2014):

$$C_0 = \frac{2 - \left(\frac{\rho_g}{\rho_l}\right)^2}{1 + (\text{Re}_{tp}/1000)^2} + \left[\left((1 + \left(\frac{\rho_g}{\rho_l}\right)^2 \cos \theta \right)/\left((1 + \cos \theta)^{1-\alpha}\right)\right]^{2/5} + C_{0,1} \quad , \ldots (3.12b)$$

$$v_d = (0.35 \sin \theta + 0.45 \cos \theta) \times \sqrt{\frac{gD_h(\rho_i - \rho_g)}{\rho_l}} (1 - \alpha)^{0.5} C_2 C_3 C_4 \quad , \ldots \ldots \ldots (3.12c)$$

$$C_{0,1} = (0.2 - 0.2 \sqrt{\rho_g/\rho_l}) [(2.6 - \beta)^{0.15} - \sqrt{f_{tp}}] (1 - x)^{1.5}, \quad \ldots \ldots \ldots (3.12d)$$

$$C_2 = \begin{cases} \left(\frac{0.434}{\log_{10}(\mu_l/0.001)}\right)^{0.15}, & \mu_l > 10000 \\ 1, & \mu_l \leq 10000 \end{cases}, \ldots \ldots \ldots (3.12e)$$

$$C_3 = \left\{ \begin{array}{ll} \left(\frac{L_a}{0.025}\right)^{0.025}, & L_a < 0.025 \\ 1, & L_a \geq 0.025 \end{array} \right. \quad , \ldots \ldots (3.12f)$$

$T_c$ is critical temperature of water under a specific pressure;

$x$ is the steam quality

$\theta$ is wellbore orientation measured from horizontal direction.
In Equation 3.12, the steam void fraction can be determined without figuring out the steam flow patterns inside wellbore.

The steam quality $x$ is the fraction of gas mass flow rate in the total mass flow rate:

$$x = \frac{M_g}{M_g + M_l} = \frac{\nu_{sg} \rho_g}{\nu_{sg} \rho_g + \nu_{sl} \rho_l}.$$ .......................... (3.13)

According to void fraction, the steam mixture density can be defined as

$$\rho_m = \rho_g f_g + \rho_l (1 - f_g).$$ ................................................................. (3.14)

Enthalpy is defined as a thermodynamic potential. The enthalpy of liquid phase in steam is given by (Tortike and Farouq Ali, 1989):

$$h_l = 23665.2 - 366.232T + 2.26952T^2 - 0.00730365T^3 + 1.3024 \times 10^{-5}T^4$$
$$- 1.22103 \times 10^{-5}T^5 + 4.70878 \times 10^{-12}T^6, \quad 280K \leq T \leq 645K$$ ........................ (3.15)

The enthalpy of gas phase in steam is given by (Tortike and Farouq Ali, 1989):

$$h_g = -22026.9 + 365.317T - 2.25837T^2 + 0.0073742T^3 - 1.33437 \times 10^{-5}T^4$$
$$+ 1.26913 \times 10^{-5}T^5 - 4.9688 \times 10^{-12}T^6, \quad 273.15K \leq T \leq 640K$$ .......... (3.16)

The enthalpy of steam mixture is defined as

$$h_m = xh_g + (1 - x)h_l.$$ ................................................................. (3.17)
Figure 3.1-(a) Schematic of vertical wellbore system in SAGD. (b) Steam flow inside tubing.
### 3.2 Vertical Wellbore Model

In the vertical wellbore of SAGD injection well, heat is transferred from the steam inside wellbore to the formation mainly by heat conduction. Figure 3.1a represents the typical vertical part of a SAGD injection wellbore. In terms of symmetry, Figure 3.1a gives half of the complete vertical wellbore system. The vertical wellbore system includes two parts: The first part contains steam flow inside tubing; the second part including the whole wellbore system from tubing to the formation.

In this work, a mathematical model is developed by coupling the above two parts. By solving this wellbore model, the steam properties variations along vertical wellbore can be plotted.

In this mathematical model, five assumptions are made first:

1. The vertical part of SAGD injection wellbore is located in a homogeneous reservoir.
2. The properties of the whole wellbore system and formation keep constant with the change of temperature.
3. Steam injection conditions (pressure, quality, and mass flow rate) at vertical well-head keep constant.
4. Steam keeps steady state flow along the vertical wellbore.
5. Steam was under saturated state along the vertical wellbore.
3.2.1 Heat Loss from Tubing to Formation

As shown in Figure 3.1, the tubing, annulus, casing, and cement work in series as heat resistances. The heat from the steam goes through this series of heat resistances and finally is absorbed by the cold formation that surrounds the wellbore system. In order to model the steam properties inside tubing, \( Q \) or \( dQ_i \) should be solved first. Here \( Q \) is the linkage between the steam, wellbore system and formation.

In Figure 3.1a, the heat flow from tubing to the cement-formation interface is given by (Willhite, 1967)

\[
\frac{dQ_1}{dz} = 2\pi r_{to} U_{to} (T_f - T_h) . \quad \text{............................................... (3.18a)}
\]

\[
\frac{1}{U_{to}} = \frac{r_{to}}{r_{hi}h_f} + \frac{r_{to} \ln(r_{hi}/r_{ti})}{k_t} + \frac{r_{to} \ln(r_{ins}/r_{lo})}{k_{ins}} + \frac{r_{to} \ln(r_{co}/r_{ci})}{k_{cas}} + \frac{r_{to} \ln(r_{wb}/r_{co})}{k_{cem}} \quad \text{.............................................. (3.18b)}
\]

\[
U_{to} = \left(\frac{1}{h_c + h_r} + \frac{r_{to} \ln r_h}{k_{cem}}\right)^{-1} \quad \text{................................................ (3.18c)}
\]

The heat flow absorbed by formation is (Ramey, 1962)

\[
\frac{dQ_2}{dz} = \frac{2\pi k_e (T_h - T_e)}{f(t)} , \quad \text{................................................................. (3.19)}
\]

where time function, \( f(t) = -\ln \frac{r_{co}}{2\sqrt{\alpha t}} - 0.29 \)

By equalizing the heat flow \( Q \), \( Q_1 \) and \( Q_2 \), the heat loss from steam becomes

\[
\frac{dQ}{dz} = \frac{2\pi r_{to} U_{to} k_e}{k_e + r_{to} U_{to} f(t)} (T_f - T_e) . \quad \text{................................................ (3.20)}
\]
3.2.2 Steam Flow inside Tubing

Steam is injected into tubing from surface pipeline. During steam flow downstream into horizontal wellbore, the heat continues to escape out of tubing and enter into formation. Firstly, the vertical wellbore was divided into segments in \( Z \) direction, then for each segment, mass balance equation, momentum balance equation and energy balance equation are applied to describe this process.

3.2.2.1 Mass Conservation

In Figure 3.1b, steam mixture flows into tubing with density and \( \rho_m \) velocity \( v_m \). The tubing cross-section area is \( A \). The mass balance equation should be

\[
- \frac{d}{dz}(\rho_m v_m) = \frac{d\rho_m}{dt} \quad \cdots \quad (3.21)
\]

Under steady state flow, the mixture density is independent of time:

\[
- \frac{d}{dz}(\rho_m v_m) = 0 \quad \cdots \quad (3.22)
\]

3.2.2.2 Momentum Conservation

Based on momentum conservation equation and the Beggs-Brill method (Beggs and Brill, 1973), the pressure drop in \( i \)-th segment of tubing is calculated as

\[
- \frac{dp}{dz} = \frac{g_c \rho_m + f_y M_m v_m^2}{2 g_c D} \left[ \frac{1 - \rho_m v_m v_{tg}}{g_c P} \right] \quad \cdots \quad (3.23)
\]
3.2.2.3 Energy Conservation

The total energy that the steam mixture carries is the sum of enthalpy, kinetic energy and gravitational potential energy. The energy conservation equation for i-th segment of the tubing is

\[ \frac{dh_m}{dz} + \nu_m \frac{dv_m}{dz} + g = - \frac{1}{\rho_m \nu_m A} \frac{dQ}{dz}. \]  \hfill (3.24)

Where \( h_m \) is the enthalpy of the steam mixture and \( Q, J/s, \) is the heat loss out of tubing in unit time. In order to calculate the steam quality along wellbore, Equations 3.17, 3.20 and 3.22 are substituted into Equation 3.24 to get

\[ \frac{dx}{dz} = \frac{C}{B} x - \frac{E}{B}. \] \hfill (3.25a)

Where

\[ B = h_e - h_i, \] \hfill (3.25b)

\[ C = \left( \frac{dh_i}{dT} - \frac{dh_i}{dT} \right) \frac{dT}{dP} \frac{dP}{dz}, \] \hfill (3.25c)

\[ E = \frac{1}{\rho_m \nu_m A} \frac{2 \pi \nu_i U_{10} k_e}{k_e + r_{10} U_{10} f(t)} (T_f - T_e) + \frac{dh_i}{dT} \frac{dT}{dP} \frac{dP}{dz} \] \hfill (3.25d)

The detailed derivation is listed in Appendix A.

Now the mathematical model for vertical wellbore part of SAGD injection well is developed by coupling Equations 3.6, 3.23 and 3.25. It is quite difficult to solving these equations together with analytical methods. Therefore an iterative method is proposed to solve the vertical wellbore model in this study.
3.3 Horizontal Wellbore Model

For the multiphase flow in horizontal wellbore, the differences from fluid flow in vertical wellbore include:

(1) Horizontal flow and radial flow coexist: steam flow down from heel to toe while steam keeps flowing into formation radially. So the steam mass flow rate decreases along horizontal wellbore.

(2) The heat conduction from horizontal wellbore to the formation is ignored. In horizontal wellbore, radial steam flow into formation carries most of heat that supports the steam chamber inside reservoir. Compared to such energy, the heat conduction is quite small and ignored in this study.

(3) SAGD drainage model is coupled with this horizontal wellbore model. As a thermal oil recovery technology, the SAGD process uses the steam energy to decrease the viscosity of bitumen and produce oil. So the amount of production is closely related to the steam flow into formation. Therefore combination of drainage model and horizontal wellbore model make it possible to estimate wellbore conditions based on production data.

Thus the steam flow in horizontal wellbore is more complex than that in vertical wellbore. After dividing the horizontal wellbore into segments, mass balance equations, energy balance equations and momentum balance equations are applied to develop the horizontal wellbore model.

In this mathematical model, five assumptions are made first:

(1) The horizontal part of SAGD injection wellbore is located in a homogeneous horizontal reservoir with net pay thickness $h$ and depth $z$. 
(2) The petro-physical properties of the net pay zone keep constant during SAGD production.

(3) The steam pressure, temperature, steam quality and steam flow rate at the heel of horizontal wellbore is the same as those at the bottom of vertical wellbore and keep constant.

(4) The steam was injected at wellbore heel, steam keeps saturated state as well as steady state during the whole flow procedure inside horizontal wellbore.

(5) The heat needed to keep steam chamber development and heat oil is offered by steam latent heat only.
Figure 3.2- (a) Steam flow in injection horizontal wellbore. (b) The heated area around steam chamber.
3.3.1 Steam Flow into Formation

In Figure 3.2, most of steam flow into slotted liner will enter the formation, forms a steam chamber, transfer its latent to the cold reservoir and heating the formation around steam injection well. In order to get the steam properties variation in horizontal wellbore, this study needs to obtain the steam flow rate into formation first by using SAGD drainage model.

This SAGD drainage model is developed by Reis’s (1992), and the derivation of heat stored around steam chamber is also based on the work of Reis. Figure 3.2b also shows the heated area around injection in conventional SAGD production phase. Total energy to offset the heat loss of steam chamber in Figure 3.2b include the heat in the chamber $H_c$, the heat loss to the overburden $H_o$ and the heat accumulation ahead of steam chamber $H_{out}$. When the conventional SAGD phase lasts for time period $t$, the instantaneous heat loss rate $Q_{inj}$ that should be provided on unit length wellbore is

\[
Q_{inj} = \Delta T C_{vr} h \left[ \frac{2k_o g \alpha}{\phi \Delta S_o h a v_{os} m} + 4 \Delta T \sqrt{\frac{2k_o g \alpha k \tau}{\phi \Delta S_o h a v_{os} m \pi}} \right] + \frac{2C_{vr} \Delta T \alpha}{ah} \left[ \frac{2k_o g \alpha}{\phi \Delta S_o h a v_{os} m} \right] \tag{3.26}
\]

Derivation of Equation 3.26 is shown in Appendix B. The heat $Q_{inj}$ is equal to the latent heat that the steam carries when it flows out of wellbore. Accordingly the instantaneous steam mass flow rate $q_{in}$ and velocity $v_{qin}$ out of horizontal wellbore are written as

\[
q_{in} = \frac{Q_{inj}}{L_v}, \tag{3.27a}
\]
and

\[ v_{qin} = \frac{Q_{\text{inj}}}{\rho_m L N A_{\text{slot}}} \]  \hspace{1cm} (3.27b)
Figure 3. 3-Schematic representation of steam flow in the slotted liner.
3.3.2 Steam Flow in Horizontal Wellbore

3.3.2.1 Mass Conservation

Figure 3.3 shows that the simplified horizontal wellbore of SAGD injection well. The steam have different pressure $p$, steam mixture velocity $v_m$ and steam mixture density $\rho_m$ at different locations. The steam flow rate into formation $q_{in}$ and flow velocity $v_{qin}$ are equally distributed along the wellbore. The mass balance equation is

$$-\frac{d}{dt}(\rho_m v) - \frac{q_{in}}{A_h} = \frac{d\rho_m}{dt}, \quad \text{................................................................. (3.28)}$$

where $A_h$ is the cross-section area of horizontal wellbore.

Under steady state flow, the mixture density is independent of time:

$$-\frac{d}{dt}(\rho_m v_m) = \frac{q_{in}}{A_h}, \quad \text{................................................................. (3.29)}$$

3.3.2.2 Momentum Conservation

Since mass flow rate decreases along wellbore, the momentum equation must be considered. For the horizontal wellbore, the momentum conservation equation is

$$\frac{dP}{dl} = -\frac{\tau}{A_h dl} = \frac{d}{dl}(\rho_m v_m^2) + \frac{1}{A_h}(q_{in} v_{qin}), \quad \text{................................................................. (3.30a)}$$

In Equation 3.30a, $\tau$ is the friction force between steam and the liner, given by (White, 2009) as $\frac{\pi D \rho_m v_m^2 \Delta l}{8}$. For the slotted liner, unit length liner is assumed to have $N$ slots. The surface area of each slot is $A_{slot}$. The flow velocity out of horizontal wellbore $v_{qin}$ is
\[ v_{qin} = \frac{q_{in}}{\rho_m N A_{slot}} \] .......................... (3.30b)

After considering ideal gas equation of state and Equation 3.29, Equation 3.30 can be rewritten as

\[ \frac{dP}{dl} = \frac{R}{S} \] .......................... (3.31a)

where

\[ R = 2v_m q_{in} - \frac{A_h}{A_h} (q_{in} V_{qin}) - \frac{\pi D \rho_m v_m^2}{8 A_h} \] .......................... (3.31b)

\[ S = 1 + \frac{M_m v_m}{A_h} \left( \frac{1}{T} \frac{dT}{dP} - \frac{1}{P} \right) \] .......................... (3.31c)

\[ M_m = \rho_m v_m A_h \] .......................... (3.31d)

The detailed derivation is listed in Appendix C.

3.3.2.3 Energy Conservation

The total energy in this study contains enthalpy, kinetic energy and gravitational potential energy. The energy conservation equation for the steam flow in horizontal wellbore is

\[ -\frac{d}{dl}[M_m (h_m + \frac{v_m^2}{2})] = \frac{dW}{dt \cdot dl} + q_{in} (h_m + \frac{v_{qin}^2}{2}) \] .......................... (3.32)

where \( dW = \tau \Delta = \frac{\pi D \rho_m v_m^2 \Delta l}{8} \).

Substituting Equation 3.31 and Equation 3.17 into Equation 3.32, the energy conservation equation becomes
\[
\frac{dx}{dl} = -\frac{N}{M} x + \frac{Q}{M}, \nonumber \tag{3.33a}
\]

where

\[
M = M_m(h_s - h_i), \nonumber \tag{3.33b}
\]

\[
N = M_m \left( \frac{dh_s}{dT} - \frac{dh_i}{dT} \right) \frac{dp}{dl} = M_m \left( \frac{dh_s}{dT} - \frac{dh_i}{dT} \right) \frac{dT}{dp} R \nonumber \tag{3.33c}
\]

\[
Q = -\frac{\pi D \rho_v v_m^3}{8} - q_m - \frac{v_q v_{qin}}{2} + 3v_m^2 - q_in \nonumber \tag{3.33d}
\]

The detailed derivation for Equation 3.33 can be found in Appendix C.

The steam inside horizontal wellbore are saturated steam. So steam saturation temperature \( T \) is correlated with steam saturation pressure \( p \). If steam pressure \( p \) distribution is calculated, the steam temperature \( T \) can be easily obtained. Also the steam flow rate \( q_{in} \) and steam flow velocity \( v_{qin} \) are critical to calculate the steam qualities along wellbore. Once \( q_{in} \) and \( v_{qin} \) are determined in Equation 3.27, the next focus of this study is to solve Equations 3.31 and 3.33 to get the pressure \( p \) and quality \( x \).
3.4 Solution Process

3.4.1 Vertical Wellbore Model Solution

An iterative technique is adopted to solve the vertical wellbore model. The main steps are given as follows:

(1) This work divides the horizontal wellbore into \( n \) segments and input steam parameters. The input constant steam parameters are from the steam injected from surface pipeline. These parameters are set at the upstream of the first vertical wellbore segment. These parameters include steam pressure \( p_0 \), steam temperature \( T_0 \), steam quality \( x_0 \), steam mixture density \( \rho_m0 \) and steam mixture velocity \( v_m0 \) for first horizontal segment.

(2) This solution procedure starts from the first segment. After the calculation for \( i \)-th segment finishes, this calculation results act as initial conditions for the \( i+1 \) segment. The calculation on \( i+1 \) segment then begins.

(3) Then a pressure difference \( \Delta p_i \) and a quality difference \( \Delta x_i \) are assumed for the vertical segment. Based on the assumed pressure and quality difference, calculate the void fraction \( f_g \) and overall heat transfer coefficient \( U_{io} \) with Equations 3.12 and 3.18 by the an iterative method.

(4) With \( \Delta p_i, \Delta x_i \) and steam properties at the upstream end of \( i \)-th segment, a new pressure difference \( \Delta p_{i,new} \) and steam quality difference \( \Delta x_{i,new} \) are calculated according to Equation 3.23 and Equation 3.25.

(5) Compare \( \Delta p_{i,new}, \Delta x_{i,new} \) with \( \Delta p_i, \Delta x_i \). If these two set of parameters are close enough, the \( \Delta p_{i,new} \) and \( \Delta x_{i,new} \) will be the output pressure and steam quality of \( i \)-th segment If not, \( \Delta p_{i,new} \) and \( \Delta x_{i,new} \) will be the new assumed pressure.
and quality difference in Step 3. Repeat Step 3 to Step 5 until the results converge.

(6) The steam temperature $T_i$ in $i$-th segment is calculated based on the average pressure $(p_i + p_{i-1})/2$ and Equation 3.6.

(7) The calculation for $i+1$-th segment go through again from Step 3 to Step 6.

The calculation flowchart for the horizontal wellbore model is shown in Figure 3.4.

3.4.2 Horizontal Wellbore Model Solution

An iterative technique is adopted to solve the horizontal wellbore model. The main steps are given as follows:

(1) This work divides the horizontal wellbore into $m$ segments and input steam parameters. The input steam parameters are from the steam out of the vertical wellbore. These parameters are set at the upstream of the first horizontal wellbore segment. These parameters include steam pressure $p_0$, steam temperature $T_0$, steam quality $x_0$, steam mixture density $\rho_{m0}$ and steam mixture velocity $v_{m0}$ for first horizontal segment.

(2) This solution procedure starts from the first segment. After the calculation for $i$-th segment finishes, this calculation results act as initial conditions for the $i+1$ segment. The calculation on $i+1$ segment then begins.

(3) For the targeted $i$-th segment, the steam rate $q_{in,i}$ and velocity $v_{qin,i}$ are calculated by Equation 3.27.

(4) Then a pressure difference $\Delta p_i$ and a quality difference $\Delta x_i$ are assumed for the horizontal segment. Based on the assumed pressure and quality
difference, calculate the void fraction $f_g$ and overall heat transfer coefficient with **Equation 3.12** by the iterative method.

(5) With $\Delta p_i$, $\Delta x_i$ and steam properties at the upstream end of $i$-th segment, a new pressure difference $\Delta p_{i,\text{new}}$ and steam quality difference $\Delta x_{i,\text{new}}$ are calculated according to **Equation 3.31** and **Equation 3.33**.

(6) Compare $\Delta p_{i,\text{new}}$, $\Delta x_{i,\text{new}}$ with $\Delta p_i$, $\Delta x_i$. If these two set of parameters are close enough, the $\Delta p_{i,\text{new}}$ and $\Delta x_{i,\text{new}}$ will be the output pressure and steam quality of $i$-th segment. If not, $\Delta p_{i,\text{new}}$ and $\Delta x_{i,\text{new}}$ will be the new assumed pressure and quality difference in Step 4. Repeat Step 3 to Step 5 until the results converge.

(7) The steam temperature $T_i$ in $i$-th segment is calculated based on the average pressure $(p_i+p_{i-1})/2$ and **Equation 3.6**.

(8) The calculation for $i+1$-th segment go through again from Step 3 to Step 6. The calculation flowchart for the horizontal wellbore model is shown in **Figure 3.5**.
Figure 3.4-Calculation flow chart for the vertical wellbore model.
Figure 3.5-Calculation flow chart for the horizontal wellbore model.
CHAPTER 4
MODEL VALIDATION

The complete wellbore model for SAGD injection well is the combination of vertical wellbore model and horizontal wellbore model. In this chapter, both the vertical wellbore model and horizontal wellbore model are validated. Vertical wellbore model is validated by comparing modeling results with field data. Horizontal wellbore model is validated based on comparison with other horizontal wellbore models.

4.1 Vertical Wellbore Model Validation

4.1.1 Field Case 1

Bleakley (1964) provided temperature profile data along vertical wellbore during a test on the 61-0 Martha Bigpond well. The pressure test was implemented at 309 hours after steam injection. Table 4.1 gives the basic wellbore parameters for this Bigpond well.

Figure 4.1 shows the comparison results between calculation results of this model, field test data and pressure drop model of Aziz as well as Beggs & Brill. In Figure 4.1, this model and Beggs & Brill pressure model both predict the right pressure changing trend along vertical wellbore while the model of Aziz cannot predict the right pressure changing trend. This over prediction of steam pressure drop was caused by the following reasons:

1) Aziz’s model was built to calculate the pressure drop in wells which producing oil and gas, this model was limited to single-phase fluid
and gas-liquid mixtures in the bubble and slug flow patterns. For steam injection case, it was a steam-liquid flow which has a large gas velocity and large steam quality. Under this kind of condition, the model of Aziz was not able to predict the steam pressure change inside wellbore.

2) Another reasons is that in Aziz’s model, the pressure drop related to friction force was over predicted by \( \frac{\partial p}{\partial z} = \frac{2 f_{ip} M_m v_m}{g_c d} \) compared with that of \( \frac{\partial p}{\partial z} = \frac{f_{ip} M_m v_m}{2 g_c d} \) in this study and Beggs & Brill’s model. And this over prediction of pressure drop related to friction force leading to the wrong trend of steam pressure change.

The calculated steam pressure profile in this study matches quite well with the field data in the range of test depth and the results fits better than results of B-B model (Beggs and Brill, 1973). In these three models, the model in this study generates the best prediction for steam pressure decrease inside vertical wellbore.

And if Hasan’s criteria was adjusted to judge flow pattern in Field test 2, in the well depth range of 0-350 m, steam flow pattern was annular flow; in the well depth range of 350-425 m, steam flow pattern was churn flow; and in the well depth range of 425-518 m, steam flow pattern was slug flow which was shown in the study of M. Bahonar et al. This indicates that this study can predict the steam pressure drop in this field case accurately without considering each flow pattern and flow pattern related equations to do the calculation.
<table>
<thead>
<tr>
<th>Parameter</th>
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</thead>
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<td>( r_h ) (m)</td>
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</tr>
<tr>
<td>( k_e ) (J/s·m·K)</td>
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</tr>
<tr>
<td>( \alpha ) (m²/s)</td>
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<tr>
<td>( T_m ) (°K)</td>
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<tr>
<td>( \lambda ) (°K/m)</td>
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<tr>
<td>( r_{so} ) (m)</td>
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<tr>
<td>( r_{co} ) (m)</td>
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<tr>
<td>( k_{cem} ) (J/s·m·K)</td>
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<td>( r_{ci} ) (m)</td>
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<tr>
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<td>( \varepsilon_{so} )</td>
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<tr>
<td>( \varepsilon_{ci} )</td>
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<tr>
<td>( M_m ) (Kg/s)</td>
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<tr>
<td>Depth (m)</td>
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<tr>
<td>( R_z ) (m)</td>
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<tr>
<td>Surface tension (Kg/m²)</td>
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</tr>
<tr>
<td>Casing depth (m)</td>
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</table>
Figure 4.1: Steam pressure comparison between model results and field data
Figure 4.2- Steam quality comparison between different models.
Figure 4.2 gives the corresponding prediction of steam quality based on the steam pressure profile in Figure 4.1 and all these three models predict the same trend of steam quality change. When steam flow downwards, part of heat continue to lose from steam to formation. And therefore the steam quality can reduce from 0.8 at surface to 0.53 at depth 518.16 m.

4.1.2 Field Case 2

This set of filed data comes from Herrera et al. (1978) for the 38-25 William Holding Well of the Cat Canyon field. This is a deep steam injection well (822.96 m) where a cyclic steam stimulation process was employed before a steam drive pilot was initiated. Table 4.2 summarizes the wellbore parameters and steam injection conditions. The well was equipped with a thermocouple to monitor its casing temperature at 822.96 m depth. The measured casing temperatures are shown in Figure 4.3. The temperature test was implemented at 168 hours after continuous steam injection.

The field data are compared with calculated steam temperature at 822.96 m depth. The model results match well with measured temperature. Also Figure 4.3 shows the predicted temperature profile at 168 hours after steam injection. It confirms that steam temperature may be slightly higher at the bottomhole than at surface during steam injection.

The matching results with two sets of field data verify that the vertical wellbore model in this study is accurate and reliable in calculating the steam pressure and temperature profile in vertical part of SAGD injection wellbore.
Table 4.2-Well parameters and injection conditions in Field Test 2 (Herrera et al. 1978)

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<td>$\lambda$ (°K/m)</td>
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<tr>
<td>Depth (m)</td>
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</tr>
<tr>
<td>$R_z$ (m)</td>
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</tr>
<tr>
<td>Surface tension (Kg/m$^2$)</td>
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</tr>
<tr>
<td>Casing depth (m)</td>
<td>822.96</td>
</tr>
</tbody>
</table>
Figure 4.3- Steam temperature comparison between model results and field data.
4.2 Horizontal Wellbore Model Validation

In 2013, Chen et al. studied the steam injection performance of horizontal wells completed with slotted liner and developed a horizontal wellbore model to evaluate steam performance. For horizontal wellbore model validation, results of this model are compared with Chen’s model.

Table 4.3 lists the parameters used for model validation. The parameters include steam injection conditions, reservoir properties and horizontal wellbore configurations. Table 4.4 shows the comparison between two results. It shows that the predicted pressure distribution and steam quality of this study are very close to Chen et al.’s results. The relative error for steam pressure is smaller than 1%. And the relative error for steam quality is smaller than 10%.

The reason this model get a larger pressure drop and smaller steam quality decrease than the model of Chen et al. can be explained as follows:

1) Though the model of Chen et al. calculate the steam flow into the formation by using the method of Willams et al.(1980), Chen et al. only put this as a sink/source term in the mass balance equation. The influence of this steam out flow was not considered in the momentum change equation for steam inside wellbore which is not correct.

2) For Chen et al.’s model, the heat loss in the energy equation is calculated by the method of Ramey (1962). This calculation method ignored the truth that in horizontal wellbore, the steam and formation has a directly mass and heat transfer so most of the energy loss is taken by steam flow into formation through heat convection instead of heat conduction.
Table 4.3 Basis parameters for wellbore and steam injection.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wellbore Length (m)</td>
<td>300</td>
</tr>
<tr>
<td>Steam Pressure (kPa)</td>
<td>14000</td>
</tr>
<tr>
<td>Steam Quality (%)</td>
<td>50</td>
</tr>
<tr>
<td>Steam Injection Rate (Kg/s)</td>
<td>2.89</td>
</tr>
<tr>
<td>Reservoir Pressure (kPa)</td>
<td>10</td>
</tr>
<tr>
<td>Oil Viscosity (mPa·s)</td>
<td>440</td>
</tr>
<tr>
<td>(d_{is} (m))</td>
<td>0.15708</td>
</tr>
<tr>
<td>(d_{os} (m))</td>
<td>0.1778</td>
</tr>
<tr>
<td>(d_{w} (m))</td>
<td>0.19446</td>
</tr>
<tr>
<td>Slot Length (m)</td>
<td>0.1</td>
</tr>
<tr>
<td>Slot Width (m)</td>
<td>2e-4</td>
</tr>
<tr>
<td>Slot Density (m(^{-1}))</td>
<td>200</td>
</tr>
<tr>
<td>Permeability (m(^2))</td>
<td>5e-12</td>
</tr>
</tbody>
</table>
Table 4.4 Comparison between this model and Chen et al.'s model.

<table>
<thead>
<tr>
<th>Horizontal length(m)</th>
<th>0</th>
<th>100</th>
<th>200</th>
<th>290</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P \ (kPa)$</td>
<td>14000.0</td>
<td>13969.6</td>
<td>13943.0</td>
<td>13919.9</td>
</tr>
<tr>
<td>in this model</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P \ (kPa)$</td>
<td>14000.0</td>
<td>13996.5</td>
<td>13994.2</td>
<td>13993.5</td>
</tr>
<tr>
<td>in Chen et al.'s model</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Relative Error (%)</td>
<td>0</td>
<td>0.192</td>
<td>0.366</td>
<td>0.526</td>
</tr>
<tr>
<td>Steam quality</td>
<td>0.5</td>
<td>0.4875</td>
<td>0.4750</td>
<td>0.4625</td>
</tr>
<tr>
<td>in this model</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Steam quality</td>
<td>0.5</td>
<td>0.4758</td>
<td>0.4573</td>
<td>0.4162</td>
</tr>
<tr>
<td>in Chen et al.'s model</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Relative Error (%)</td>
<td>0</td>
<td>2.40</td>
<td>3.73</td>
<td>10.00</td>
</tr>
</tbody>
</table>
Because of these two concerns, the outflow of steam actually causing an additional pressure drop and heat loss inside horizontal wellbore and results in a smaller pressure value at a specific horizontal location. Since the pressure along wellbore is smaller, the latent heat of steam is actually increasing, and the amount of steam needed to offer latent heat to keep steam chamber develop and oil production is smaller, which can explain a smaller quality drop along wellbore.

It can be concluded that the horizontal wellbore model in this study is reliable in describing the steam behavior along horizontal wellbore in SAGD production.
CHAPTER 5

RESULTS AND DISCUSSION

In this chapter, the performance of SAGD process is analyzed based on the above validated model. In order to reveal the steam behavior in injection wellbore clearly, the analysis of SAGD performance are separated for horizontal and vertical wellbores. Then this chapter displays the steam properties variation along the whole wellbore by combining the two wellbore models.

For the vertical part of wellbore, the controlling conditions include steam injection pressure, steam injection rate and steam injection quality at the well head. For the horizontal part of wellbore, the critical parameters are steam injection pressure, steam injection rate and steam injection quality at the heel as well as pay zone properties.

5.1 Performance Analysis on Vertical Wellbore

5.1.1 Effect of Steam Injection Pressure

Figure 5.1 shows the effects of steam injection pressure at the well head on the profiles of steam thermophysical properties along the vertical wellbore. The steam injection pressure increases from 6000kPa to 12000kPa. Since the injected steam is saturated, steam injection temperature also increases correspondingly (Equation 3.6). The mass injection rate at well head is 4 kg/s. In addition, steam injection quality at wellhead is 0.6. Under constant steam injection quality and mass flow rate, steam quality reduces faster when steam
Figure 5.1-Effect of steam injection pressure on steam quality distribution in vertical wellbore.
Figure 5.2-Effect of steam injection quality on steam pressure distribution in vertical wellbore.
injection pressure becomes larger. Because the saturated steam at high pressure has a higher temperature and this results in a higher temperature gradient between steam and formation which results in a larger heat loss to the formation along the vertical wellbore. Thus steam quality decreases faster.

Although the injection pressure is inversely correlated with steam quality variations, it is inappropriate to decrease the injection pressure as much as possible. Low injection pressure at well head cannot support enough steam flow into the formation in the horizontal part of injection well.

5.1.2 Effect of Steam Injection Quality

Figure 5.2 displays the effect of steam injection quality on the pressure profile in the vertical wellbore. The injection pressure is constant at 8000kPa and injection mass flow is 6kg/s. The steam quality at well head increases from 0.6 to 0.9. Under certain injection pressure and rate, high steam quality at well head leads to higher pressure drop inside the wellbore. This is because with higher steam quality, the steam mixture density decreases and accordingly steam flow velocity increases. The friction between wet steam and pipe wall is proportional to the production of mass flow rate and flow velocity. The larger flow velocity becomes, the higher the pressure loss caused by friction force. So the steam pressure drop faster.

Steam with high quality carries more heat because vapor enthalpy is much higher than that of hot water. However, it does not mean that steam injection with high steam quality is always better than injection with lower quality. If the steam quality is too high at well head, the steam pressure at the heel of
horizontal wellbore would become very low and steam cannot flow further through the rest part of wellbore. Also a higher steam quality means more energy consumption which will increase oil production cost.
Figure 5.3 - Effect of steam injection rate on (a) steam quality distribution and (b) pressure distribution in vertical wellbore.
5.1.3 Effect of Steam Injection Rate

Figure 5.3 gives the effect of steam mass injection rate on the profiles of thermophysical properties in the vertical wellbore. Steam injection quality keeps constant at 0.6. And steam injection pressure is 10000 kPa. The steam mass flow rate at well head increases form 1 kg/s to 6 kg/s.

Figure 5.3a shows that steam quality decreases along the vertical wellbore. With a higher steam injection rate, steam quality inside wellbore decrease slower.

Figure 5.3b shows that steam pressure along vertical wellbore may increase or decrease. Under given conditions, when steam mass injection rate is smaller than 4 kg/s, steam pressure increases along the wellbore. But when steam injection rate is higher than 4 kg/s, steam pressure decreases with increasing vertical depth.

The pressure gradient of fluid flow can be written as (Beggs and Brill, 1973):

\[- \frac{dp}{dz} = (\frac{\partial p}{\partial z})_d + (\frac{\partial p}{\partial z})_{ac} + (\frac{\partial p}{\partial z})_f, \] ..............................................................(5.1)

Here \((\frac{\partial p}{\partial z})_f = \frac{f \nu M_m v_m}{2 g_c d}\) is the frictional pressure gradient caused by friction between fluid and pipe, and it tends to decrease the pressure. And

\[(\frac{\partial p}{\partial z})_{ac} = - \frac{\rho_m v_m v_{sg}}{g_c p} \frac{dp}{dz}\] is the acceleration pressure gradient and this term is usually negligible for most practical case. In addition, \((\frac{\partial p}{\partial z})_{el} = \frac{g}{g_c} \rho_m \sin \theta\) is the pressure gradient caused by elevation change. Under the vertical down flow
situation, it tends to make the pressure increase. The pressure gradient in vertical wellbore is large under a larger steam injection rate. The friction pressure gradient tends to be larger than the elevation pressure gradient, which leading to the decrease of wellbore pressure. If the steam injection rate is small, the elevation pressure gradient is larger than the friction pressure gradient, which leading to the increase of wellbore pressure, the smaller the steam injection rate, the faster wellbore pressure will increase.

With the increase of steam injection rate, at a specific vertical depth, the steam pressure tends to be smaller, as well as the steam temperature, which leading to a smaller temperature gradient from steam to formation as well as a smaller heat loss. This is why with the steam injection rate increase, the steam quality decreases slower.
5.2 Performance Analysis on Horizontal Wellbore

5.2.1 Effect of Steam Pressure at Heel

Figure 5.4 shows the effects of steam pressure at the heel on the profiles of steam thermophysical properties along the horizontal wellbore. The steam pressure at the heel increases from 6000kPa to 12000kPa. Since the injected steam is saturated, steam temperature increases correspondingly. The mass flow rate at heel is 2.5 kg/s. In addition, steam quality at the heel is 0.7. With constant steam quality and mass flow rate at the heel, steam quality reduces faster when steam pressure at the heel becomes larger.

Since mass flow rate inside horizontal wellbore can be written as \( M_m = \rho_m v_m A \) and the work done by friction force can be written as \( dw = \frac{\pi D M_m v_m }{8A} \Delta l \). Under a certain steam injection rate, if the pressure increase, the density of wet steam decrease, and the velocity of wet steam increases which leads to a larger energy loss due to friction. At the same time, with an increase of steam pressure, the latent heat of wet steam decreases, then a larger amount of wet steam flow into formation to offer the latent heat needed to keep the constant oil production. Thus with steam injection pressure increases the steam quality decreases faster.
Figure 5. Effect of steam pressure at the heel on steam quality distribution along horizontal wellbore.
Figure 5.5-Effect of steam quality at the heel on steam pressure distribution along horizontal wellbore.
5.2.2 Effect of Steam Quality at Heel

Figure 5.5 displays the effect of steam quality at the heel on the pressure profile along the horizontal wellbore. At the heel, the steam pressure is constant at 6000 kPa and steam mass flow is 1.5 kg/s. The steam quality increases for 0.6 to 0.9 at the wellbore heel. Under such fixed steam pressure and rate, high steam quality at wellbore heel leads to higher pressure drop inside the wellbore.

According to $M_m = \rho_m v_m A$ and $dW = \frac{\pi D M_m v_m \Delta l}{8 A}$ with the steam quality increases, the wet steam density decreases then the steam velocity inside wellbore increases which results in a larger energy loss due to friction, and this leading to the pressure inside wellbore decreases faster.
Figure 5.6-Effect of steam flow rate at the heel on (a) steam quality distribution and (b) on steam pressure distribution along the horizontal wellbore.
5.2.3 Effect of Steam Rate at Heel

Figure 5.6 gives the effect of steam mass flow rate at the heel on the profiles of thermophysical properties in the horizontal wellbore. Steam quality keeps constant at 0.7 at the heel. And steam pressure at the wellbore heel is 8000 kPa. The steam mass flow rate at well head increases form 1.5 kg/s to 3 kg/s.

Figure 5.6a shows that steam quality decreases along the horizontal wellbore. With a higher steam flow rate at the heel, steam quality inside wellbore decrease slower. And the rate of steam quality decrease is much smaller than that in the vertical wellbore.

Figure 5.6b shows that steam pressure along horizontal wellbore decrease as steam flow towards the toe. Under given conditions, steam pressure decrease slower when steam mass flow rate at the heel becomes smaller.

According to \( M_m = \rho_m v_m A \) and \( d_w = \frac{\pi f D M_m v_m}{8A} \Delta l \), under a certain steam injection pressure and steam quality, when the steam injection rate increases, the steam velocity inside horizontal tends to increase and this increases the energy loss due to friction, and the steam pressure inside wellbore tends to decrease faster. At the same time, with the decrease of steam pressure, the latent heat of wet steam increases and a smaller amount wet steam is need to offer the energy to keep a constant oil prediction, this is why with a larger injection rate the steam quality decreases slower.
5.2.4 Effect of Reservoir Permeability

Figure 5.7 gives the effect of reservoir permeability on the profiles of thermophysical properties in the horizontal wellbore. Steam quality keeps constant at 0.6 at the heel. And steam pressure at the wellbore heel is 6000 kPa. The steam mass flow rate at well head keeps 3 kg/s. The reservoir permeability increases from 2D to 50D.

The oil production rate tends to increase with higher reservoir permeability. The reservoir with high permeability needs more steam flow into the formation heating cold oil. For this reason, more steam flow inside formation at higher reservoir permeability, results in the mass flow rate inside wellbore decrease faster, and a small flow rate decrease the energy lost caused by friction, and makes the pressure inside wellbore decrease slower. Since the pressure at a specific wellbore location become higher, the latent heat of steam becomes small. This is another reason more steam slow inside formation, so the steam quality will decrease faster with higher reservoir permeability.
Figure 5.7 - Effect of reservoir permeability on (a) steam pressure distribution and (b) on steam quality distribution along the horizontal wellbore.
5.2.5 Effect of Oil Viscosity

Figure 5.8 gives the effect of oil viscosity on the profiles of thermophysical properties in the horizontal wellbore. Steam quality keeps constant at 0.6 at the heel. And steam pressure at the wellbore heel is 8000 kPa. The steam mass flow rate at well head becomes 2 kg/s. The oil kinematic viscosity increases from $2.43 \text{ m}^2/\text{d}$ to $121.5 \text{ m}^2/\text{d}$.

With higher oil viscosity, oil need more energy to become mobile and this tends to slow the steam chamber develop inside formation and decrease oil production rate accordingly. Then the amount of steam flow into formation is small, and the axial mass flow rate inside wellbore tend to increase which increase the friction and leading to a larger pressure drop. Then at a horizontal location, a relative lower pressure means a higher steam latent heat, this also decrease the steam quantity flow inside formation and a higher steam quality is predicted.
Figure 5. Effect of oil viscosity on (a) steam pressure distribution and (b) steam quality distribution along the horizontal wellbore.
CHAPTER 6

CONCLUSIONS AND RECOMMENDATIONS

6.1 Conclusions

After analysing the mechanism of heat and mass transfer between steam inside wellbore and formation around, a calculation model was put forward in this thesis which is able to calculate the steam properties (pressure, quality) distribution along wellbore trajectory during conventional Steam Assisted Gravity Drainage process. The main conclusions of this thesis are summarized next.

- During SAGD process, large amount steam is injected into reservoir through injection well. During steam flow inside horizontal wellbore, steam pressure, temperature as well as steam quality decrease because of the wellbore friction, heat and mass transfer between inside wellbore and formation.

- Based on wellbore completion technology, the steam inside vertical wellbore loses its energy mainly by heat conduction through the whole wellbore-formation system while steam inside horizontal wellbore loses its heat mainly through heat convection caused by steam flow into formation. Thus, within this thesis, the steam flow inside vertical wellbore and steam flow inside horizontal wellbore behavior are modeled and solved separately.

- The steam (hot water and dry steam mixture) flow inside wellbore will go through different flow patterns as the pressure, velocity, flow rate are changing with time and location. Though the former researchers have
worked out many flow maps or transition criteria to describe flow behavior, they are limited to some specific flow directions or specific flow patterns.

- The concept of phase flow pattern is one of the critical concept that describe flow behavior of steam flow inside well. Within this thesis, a flow pattern independent drift flux model based void fraction correlation (Bhagwat and Ghajar, 2014) is introduced and it make it possible to transfer this two phase flow problem into a homogeneous one phase flow while still considering the slip effect between phases.

- According to the sensitivity analysis, within vertical wellbore increase steam injection pressure tends to accelerate steam quality decrease; increase steam injection quality tends to slow pressure decrease; increase steam injection rate tends to accelerate pressure drop and slow quality decrease. While in horizontal wellbore, increase steam injection pressure tends to accelerate the steam quality decrease inside wellbore; increase steam quality tends to increase pressure drop; increase steam injection rate tends to accelerate pressure drop and slow steam quality decrease.

6.2 Recommendations

- The flow pattern independent drift flux model based void fraction correlation was introduced in this study intended to avoid the flow pattern determination related non-continuity and non-convergence problem and bring great convenience in steam property calculation. Like any other correlations, this correlation also has its limitations towards liquid viscosity and fluid Reynolds
number range. More research work is needed to correct some parameters inside this correlation further in order to extend its application range.

- Since the steam outflow into formation has an important effect on steam flow inside wellbore, more research, especially strict experimental study need to be done to test the single-phase and multi-phase friction factor on wellbore when outflow exists.

- Within this study, the wellbore system and reservoir parameters are assumed to be constant, which is not the situation during the actual SAGD process in field. Reservoir parameters such as reservoir temperature, formation thermal conductivity and oil relative permeability are changing as the reservoir was heated and heated oil and condensate water keep on flowing into production wellbore. Thus in the future research, these parameters should be kept updating with steam injection process.

- The steam flow inside wellbore is influenced by the steam injection conditions (pressure, temperature, quality, flow rate) at well head, the wellbore systems (tubing size, cement process, completion method) and also the reservoir conditions (pressure, temperature, oil viscosity, permeability), so more cases needed to be run to investigate the steam flow distribution law inside wellbore so that the optimized combination of steam injection pressure, quality and flow rate can be found under a specific SAGD project.
LIST OF REFERENCES


World’s Oil and Natural Gas Scenario, DOI 10.1007/978-3-642-14234-5_2.


APPENDIX A

DERIVATION OF \( \frac{dx}{dz} \)

The derivation of \( \frac{dx}{dz} \) for the vertical wellbore in Section 3.2 is listed as:

1. According to Equation 3.17, the partial derivative of steam mixture enthalpy over depth is rewritten as

\[
\frac{dh_m}{dz} = \frac{d}{dz} \left( x h_g + (1-x) h_l \right) \quad \text{..............................................(A.1)}
\]

\[
= (h_g - h_l) \frac{dx}{dz} + x \frac{d h_g}{dz} + (1-x) \frac{d h_l}{dz}
\]

The enthalpy of gas phase \( h_g \) and enthalpy of liquid phase \( h_l \) in steam mixture are functions of temperature \( T \) and pressure \( P \). At the same time, steam temperature are correlated to pressure for saturated steam (Equation 3.6). Therefore,

\[
\frac{d h_g}{dz} = \frac{d h_g}{dT} \frac{d T}{dP} \frac{d P}{dz} \quad \text{..............................................(A.2)}
\]

and

\[
\frac{d h_l}{dz} = \frac{d h_l}{dT} \frac{d T}{dP} \frac{d P}{dz} \quad \text{..............................................(A.3)}
\]

Substituting Equations A.2 and A.3 into Equation A.1 leads to

\[
\frac{dh_m}{dz} = (h_g - h_l) \frac{dx}{dz} + \left( \frac{d h_g}{dT} - \frac{d h_l}{dT} \right) \frac{d T}{dP} \frac{d P}{dz} x + \frac{d h_l}{dT} \frac{d T}{dP} \frac{d P}{dz} \quad \text{..............................................(A.4)}
\]
(2) Because of the mist state of steam injection process and large gas flow rate, the gas equation of state is applied. The density of steam mixture based on the equation of state is

\[
\frac{1}{\rho_m} = \frac{V}{nM} = \frac{Z_s RT}{MP}.
\] .................................(A.5)

Here \( M \) is the mole mass of this mixture.

\[
d(\frac{1}{\rho_m}) = \frac{1}{\rho_m} (\frac{1}{Z_s} \frac{dZ_s}{dP} + \frac{1}{T} \frac{dT}{dP} - \frac{1}{P}) dP .................................(A.6)
\]

\( dZ_s/dP \) is quite small and can be ignored. So

\[
\frac{d}{dz} \left( \frac{1}{\rho_m} \right) = \frac{1}{\rho_m} \left( \frac{1}{T} \frac{dT}{dP} - \frac{1}{P} \right) \frac{dP}{dz} .................................(A.7)
\]

Substituting **Equation A.7** into **Equation 3.22** leads to

\[
\frac{dv_m}{dz} = v_m \left( \frac{1}{T} \frac{dT}{dP} - \frac{1}{P} \right) \frac{dP}{dz} .................................(A.8)
\]

Finally, substituting **Equations A.4, A.8 and 3.20** can get **Equation 3.25**.
APPENDIX B

DERIVATION OF HEAT STORED AROUND SAGD STEAM CHAMBER

The total energy stored around steam chamber can be divided into three parts:

(1) Heat accumulation inside steam chamber (Reis, 1992):

\[ H_c = \frac{1}{2} * 2W_s * h \Delta T_{C_{vr}} = W_s h \Delta T_{C_{vr}} \]

.................................................................(B.1)

Since

\[ W_s = \frac{2k_{s \alpha} g \alpha}{\sqrt{\phi \Delta S_{s \alpha} \gamma V_{o \alpha} m t}} \] .................................................................(B.2)

\[ H_c = h \Delta T_{C_{vr}} \frac{2k_{s \alpha} g \alpha}{\sqrt{\phi \Delta S_{s \alpha} \gamma V_{o \alpha} m t}} \] .................................................................(B.3)

(2) Heat stored in front of steam chamber (Reis, 1992):

\[ H_{out} = \frac{C_{vr} \Delta T_h}{h^2} \left( h^2 + W_s^2 \right) \sqrt{\frac{\phi \Delta S_{e \alpha} V_{o \alpha} m h \alpha}{2ak_{s \alpha} g}} \] .................................................................(B.4)

Put Equation B.2 into Equation B.4
\[
H_{out} = \frac{C_v \Delta T_h}{h^2} (h^2 + \frac{2k_o g \alpha}{\phi \Delta S_o h a v_{os, m}} t^2) \sqrt{\frac{\phi \Delta S_o V_{os, m} m h \alpha}{2a k_o g}} \quad \ldots \quad (B.5)
\]

(3) Heat loss to the overburden formation (Edmunds and Peterson, 2007):
\[
H_o = \frac{4}{3} A \Delta T \left[ \frac{k_v C_{vo} t}{\pi} \right] \quad \ldots \quad (B.6)
\]

Put Equation B.2 into Equation B.6:
\[
H_o = \frac{4}{3} * 2 * \left[ \frac{2k_o g \alpha}{\phi \Delta S_o h a v_{os, m}} t \Delta T \right] \left[ \frac{k_v C_{vo} t}{\pi} \right] = \frac{8}{3} \Delta T \left[ \frac{2k_o g \alpha k_v C_{vo} t^3}{\phi \Delta S_o h a v_{os, m} \pi} \right] \quad \ldots \quad (B.7)
\]

The cumulative heat loss to the formation then can be calculated by:
\[
H_{inj} = H_c + H_o + H_{out}
\]
\[
= h \Delta T C_v \sqrt{\frac{2k_o g \alpha}{\phi \Delta S_o h a v_{os, m}}} t + \frac{8}{3} \Delta T \left[ \frac{2k_o g \alpha k_v C_{vo} t^3}{\phi \Delta S_o h a v_{os, m} \pi} \right] \quad (B.8)
\]
\[
+ \frac{C_v \Delta T h}{h^2} (h^2 + \frac{2k_o g \alpha}{\phi \Delta S_o h a v_{os, m}} t^2) \sqrt{\frac{\phi \Delta S_o V_{os, m} m h \alpha}{2a k_o g}}
\]

The heat loss at a specific time \( t \), can be written as
\[
Q_{inj} = \frac{dH_{inj}}{dt} = h \Delta T C_v \sqrt{\frac{2k_o g \alpha}{\phi \Delta S_o h a v_{os, m}}}
\]
\[
+ 4 \Delta T \left[ \frac{2k_o g \alpha k_v C_{vo} t}{\phi \Delta S_o h a v_{os, m} \pi} \right] + \frac{4 C_v \Delta T a h}{2 \phi \Delta S_o h a v_{os} \alpha k_o g} t \quad \ldots \quad (B.9)
\]
APPENDIX C

DERIVATION OF \( \frac{dP}{dl} \) AND \( \frac{dx}{dl} \)

(1) The derivation of \( \frac{dP}{dl} \) for the vertical wellbore in Section 3.3 is listed as:

In Equation 3.30 of momentum conservation,

\[
\frac{d}{dl} (\rho_m v_m^2) = \frac{1}{A} (v_m \frac{dM_m}{dl} + M_m \frac{dv_m}{dl}), \quad \text{..................................................}(C.1)
\]

Here \( M_m \), the mass flow rate, equals \( \rho_m v_m A \). According to Equation 3.29, Equation C.1 can be written as

\[
\frac{d}{dl} (\rho_m v_m^2) = \frac{1}{A} (-v_m q_{in} + M_m \frac{dv_m}{dl}), \quad \text{..................................................}(C.2)
\]

And the derivative of \( v_m \) over length \( l \) is

\[
\frac{dv_m}{dl} = -\frac{q_{in}}{A \rho_m} + \frac{M_m}{A} \frac{d}{dl} \left( \frac{1}{\rho_m} \right), \quad \text{..................................................}(C.3)
\]

Because of the mist state of steam injection process and large gas flow rate, the gas equation of state is applied. Substituting Equation A.6 into Equation C.3 leads to

\[
\frac{dv_m}{dl} = -\frac{q_{in}}{A \rho_m} + v_m \left( \frac{1}{T} \frac{dT}{dP} - \frac{1}{P} \right) \frac{dP}{dl}, \quad \text{..................................................}(C.4)
\]

Now substituting Equation C.4 into Equation C.2 leads to

\[
\frac{d}{dl} (\rho_m v_m^2) = \frac{M_m v_m}{A} \left( \frac{1}{T} \frac{dT}{dP} - \frac{1}{P} \right) \frac{dP}{dl} - 2 v_m q_{in} A, \quad \text{..................................................}(C.5)
\]

Next step is to put Equation C.5 into Equation 3.30. Equation 3.30 becomes

\[
-\frac{dp}{dl} = \frac{\pi D \rho_m v_m^2}{8A} = \frac{M_m v_m}{A} \left( \frac{1}{T} \frac{dT}{dP} - \frac{1}{P} \right) \frac{dP}{dl} - 2 v_m q_{in} A + \frac{1}{A_h} \left( q_{in} V_{win} \right), \quad \text{..............}(C.6)
\]
(2) The derivation of \( \frac{dx}{dl} \) for the vertical wellbore in Section 3.3 is listed as:

Considering the definition of friction force, Equation 3.32 is written as

\[
- \frac{d}{dl} [M_m (h_m + \frac{v_m^2}{2})] = \frac{\pi D (\rho_m v_m^3)}{8} + q_{in} (h_m + \frac{v_{qin}^2}{2}). \quad \text{.................................(C.7)}
\]

The left side of Equation C.7 is

\[
- \frac{d}{dl} [M_m (h_m + \frac{v_m^2}{2})] = -(h_m + \frac{v_m^2}{2}) \frac{dM_m}{dl} - M_m \frac{dh_m}{dl} - M_m v_m \frac{dv_m}{dl}. \quad \text{..............(C.8)}
\]

According to Equation 3.17, the enthalpy derivation is

\[
\frac{dh_m}{dl} = (h_g - h_i) \frac{dx}{dl} + \left( \frac{dh_g}{dl} - \frac{dh_i}{dl} \right) x + \frac{dh_i}{dl}. \quad \text{.................................(C.9)}
\]

Substituting Equations C.9 and C.4 into Equation C.8 leads to

\[
- \frac{d}{dl} [M_m (h_m + \frac{v_m^2}{2})] = (h_m + \frac{3v_m^2}{2}) q_{in} - M_m (h_g - h_i) \frac{dx}{dl}
\]

\[
- M_m \left( \frac{dh_g}{dl} - \frac{dh_i}{dl} \right) x - M_m \frac{dh_i}{dl} - M_m v_m^2 \left( \frac{1}{T} \frac{dT}{dp} - \frac{1}{P} \frac{dP}{dl} \right)
\]

Furthermore, combining Equation C.6, C.10 and C.7, the final \( \frac{dx}{dl} \) becomes

\[
\frac{dx}{dl} = - \frac{N}{M} x + \frac{Q}{M}, \quad \text{.................................................................(C.11a)}
\]

\[
M = M_m (h_g - h_i), \quad \text{.................................................................(C.11b)}
\]

\[
N = M_m (\frac{dh_g}{dT} - \frac{dh_i}{dT}) \frac{dT}{dp} \frac{dp}{dl} = M_m \left( \frac{dh_g}{dT} - \frac{dh_i}{dT} \right) \frac{dT}{dp} R \frac{S}{S}, \quad \text{.................................(C.11c)}
\]

\[
Q = - \frac{\pi D (\rho_m v_m^3)}{8} - q_{in} \frac{v_{qin}^2}{2} + 3v_m^2 \frac{q_{in}}{2}
\]

\[
- M_m v_m^2 \left( \frac{1}{T} \frac{dT}{dp} - \frac{1}{P} \frac{dP}{dl} \right) R \frac{S}{S} - M_m \frac{dh_i}{dT} \frac{dT}{dp} R \frac{S}{S}, \quad \text{.................................(C.11d)}
\]

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