MODELING THE FLUID FLOW IN LOW-PERMEABILITY UNCONVENTIONAL RESERVOIRS ACROSS SCALES

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Shanshan Yao, candidate for the degree of Doctor of Philosophy in Petroleum Systems Engineering, has presented a thesis titled, *Modeling The Fluid Flow In Low-Permeability Unconventional Reservoirs Across Scales*, in an oral examination held on March 25, 2019. The following committee members have found the thesis acceptable in form and content, and that the candidate demonstrated satisfactory knowledge of the subject material.

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ABSTRACT

In the last decade low-permeability unconventional reservoirs (i.e., shale and tight formations) become an increasingly important source of oil and especially gas supply in the world. Low permeability reservoirs are characterized with small grain sizes (<\(\mu m\)), low permeability (< 0.1mD), small porosity (<10%) and high total organic carbon (TOC) (0.8-20wt%). The productivity of shale and tight reservoirs heavily depends on the interaction between reservoir rock matrix and multi-stage fractured horizontal wells (MFHWs). To predict and optimize unconventional reservoirs' production behavior, this study tries to model the fluid flow across scales—from the pore scale to the reservoir scale. Shale matrix permeability is important in interpreting permeability measurement experiments as well as modeling the reservoir-scale flow in shale reservoirs. This study utilizes 2D SEM images and the process-based modeling approach to reconstruct 3D multi-scale shale pore networks. When compared with pore models in the literature, the pore network model is advantaged in describing a realistic, wide range of pore size distribution from micrometer (\(\mu m\)) to several nanometers (nm) in a sub-millimeter-sized rock volume. The pore-scale no-slip flow modeling on pore networks provides intrinsic matrix permeabilities under the effect of multi-scale pore structures and different geological-forming processes.

The intrinsic matrix permeability cannot fully represent the gas transport capability of an unconventional reservoir rock when the gas flow velocity at pore surfaces is no longer zero. Unified models are developed for the rarefied gas
flow in single conduits of various cross-sections at elevated pressure. Apparent permeabilities are calculated with running unified models on all throats of pore networks. The relationship among pore space structures, gas pressure and apparent permeability reveals the limitation of Klinkenberg equation in describing the high-pressure rarefied gas flow in shale matrix. This study further develops a new equation of apparent permeability vs. pore pressure.

Hydrocarbon flows out of rock matrix and then flows into hydraulic fractures then to the horizontal wellbore. Models of coupled flow in matrix and hydraulic fractures can be applied to interpret and/or predict the flow rates/pressure at wellbore vs. time. Distinguished from most models in the literature, this study develops a semi-analytical model with considering the dynamic declining rates of hydraulic fracture conductivity vs. increasing effective stress. This study validates that ignoring such fracture stress-sensitivity can underestimate MFHWs’ productivity at late-time stage.

Many low-permeability unconventional reservoirs have a mixture of various conditions, such as rarefied flow, fracture and matrix stress-sensitivity, reservoir heterogeneity and gas adsorption/desorption. In order to easily model multiple flow phenomena, this work develops a composite methodology that combines simple linear flow, radial flow and/or source/sink flow equations. One of this composite method’s applications is validated by the fast and accurate composite modeling of the fluid flow in heterogeneous unconventional reservoirs. In the future work, the composite methodology will be applied in the modeling of gas adsorption/desorption and the rarefied flow in stress-sensitive reservoirs.
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DEDICATION

This work is dedicated to my family, and particularly to Ning.
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LIST OF ABBREVIATIONS, SYMBOLS, NOMENCLATURE

\( C_{ij} = \) elastic stiffness tensor of a layered TI medium, \( m^2/\text{Lt}^2, \text{N/m} \)

\( c_f = \) formation compressibility, \( \text{Lt}/\text{m}, \text{Pa}^{-1} [1\text{Pa}^{-1}=6.895 \times 10^3 \text{psi}^{-1}] \)

\( c_{ij} = \) elastic stiffness tensor of one layer in layered TI medium, \( m^2/\text{Lt}^2, \text{N/m} \)

\( c_t = \) total compressibility, \( \text{Lt}/\text{m}, \text{Pa}^{-1} [1\text{Pa}^{-1}=6.895 \times 10^3 \text{psi}^{-1}] \)

\( d_f = \) hydraulic fracture compressibility, \( \text{Lt}/\text{m}, \text{Pa}^{-1} [1\text{Pa}^{-1}=6.895 \times 10^3 \text{psi}^{-1}] \)

\( D_h = \) hydraulic diameter of a conduit

\( F_c = \) fracture conductivity \( k_w h, L^3, \text{m}^3 [1\text{m}^3=3.32 \times 10^{15} \text{md.ft}] \)

\( F_{ci} = \) fracture conductivity in initial stress field, \( L^3, \text{m}^3 [1 \text{m}^3=3.32 \times 10^{15} \text{md.ft}] \)

\( F_i = \) fracture diffusivity, \( L^2/\text{t}, \text{m}^2/\text{s} [1\text{m}^2/\text{s} =9.3 \times 10^6 \text{ft}^2/\text{d}] \)

\( H = \) pay-zone thickness, \( L, \text{m} [1\text{m}=3.28 \text{ft}] \)

\( k = \) permeability, \( L^2, \text{m}^2 [1\text{m}^2=1.01 \times 10^{15} \text{md}] \)

\( k_i = \) permeability in initial stress field, \( L^2, \text{m}^2 [1\text{m}^2=1.01 \times 10^{15} \text{md}] \)

\( k_F = \) fracture permeability, \( L^2, \text{m}^2 [1\text{m}^2=1.01 \times 10^{15} \text{md}] \)

\( L^{-1} = \) inverse Laplace transformation

\( L_h = \) horizontal wellbore length, \( L, \text{m} [1\text{m}=3.28 \text{ft}] \)

\( L_{\text{ref}} = \) reference length, \( L, \text{m} [1\text{m}=3.28 \text{ft}] \)

\( p, P = \) pressure, \( \text{m/Lt}^2, \text{pa} [1\text{Pa}=1.45 \times 10^{-4} \text{psi}] \)

\( p = \) pressure in Laplace domain, \( \text{m/Lt}^2, \text{pa} [1\text{Pa}=1.45 \times 10^{-4} \text{psi}] \)

\( p_i = \) initial pore pressure, \( \text{m/Lt}^2, \text{Pa} [1\text{Pa}=1.45 \times 10^{-4} \text{psi}] \)

\( p_p = \) pore pressure, \( \text{m/Lt}^2, \text{Pa} [1\text{Pa}=1.45 \times 10^{-4} \text{psi}] \)
\( p_f = \) pressure inside a hydraulic fracture, m/Lt², Pa [1Pa=1.45×10⁻⁴psi]

\( p_F = \) pressure at the hydraulic fracture/wellbore intersection, m/Lt², Pa
  [1Pa=1.45×10⁻⁴psi]

\( p_{wf} = \) flowing bottomhole pressure, m/Lt², Pa [1Pa=1.45×10⁻⁴psi]

\( q = \) flow rate, L³/t, m³/day [1m³/day=3.54×10⁻⁵MMscf/day]

\( \bar{q} = \) flow rate in Laplace domain, L³/t, m³/s [ft³/d]

\( q_{ij} = \) flow rate into hydraulic fracture, L³/t, m³/day [1m³/day=3.54×10⁻⁵MMscf/day]

\( r = \) distance in the radial direction, L, m [1m=3.28ft]

\( r_w = \) hypothetical wellbore radius, L, m [1m=3.28ft]

\( r_e = \) hypothetical radial boundary, L, m [1m=3.28ft]

\( S = \) Source/Sink function

\( S_g = \) gas saturation, fraction

\( S_w = \) water saturation, fraction

\( T = \) Temperature, T, K

\( t, \tau = \) time, t, s [1s=1.16×10⁻⁵day]

\( u, s = \) Laplace variable

\( u^* = \) Laplace variable corresponding to \( \Delta t \)

\( W = \) well spacing, L, m [1m=3.28ft]

\( w_f = \) fracture width, L, m [1m=3.28ft]

\( x = \) distance in the x-direction, L, m [1m=3.28ft]

\( x_f = \) fracture half-length, L, m [1m=3.28ft]

\( y = \) distance in the y-direction, L, m [1m=3.28ft]
Greek Letters

$\alpha =$ Biot’s coefficient

$\beta =$ declining rate of fracture compressibility, Pa$^{-1}$ [1Pa$^{-1}$=6.895×10$^3$psi$^{-1}$]

$\varepsilon =$ strain, L, m [1m=3.28ft]

$\mu =$ fluid viscosity, m/(Lt$^2$), Pa·s [cp]

$\eta =$ dimensionless diffusivity

$\omega =$ storativity ratio

$\lambda =$ flow capacity

$\phi =$ porosity, fraction

$\gamma =$ permeability modulus, Lt$^2$/m, Pa$^{-1}$ [1Pa$^{-1}$=6.895×10$^3$psi$^{-1}$]

$\sigma_1 =$ vertical stress, m/Lt$^2$, Pa [1Pa=1.45×10$^{-4}$psi]

$\sigma_2 =$ maximum horizontal stress, m/Lt$^2$, Pa [1Pa=1.45×10$^{-4}$psi]

$\sigma_3 =$ minimum horizontal stress, m/Lt$^2$, Pa [1Pa=1.45×10$^{-4}$psi]

$\sigma'_1 =$ effective vertical stress, m/Lt$^2$, Pa [1Pa=1.45×10$^{-4}$psi]

$\sigma'_2 =$ effective maximum horizontal stress, m/Lt$^2$, Pa [1Pa=1.45×10$^{-4}$psi]

$\sigma'_3 =$ effective minimum horizontal stress, m/Lt$^2$, Pa [1Pa=1.45×10$^{-4}$psi]

$\Delta \sigma'_1 =$ increment of effective vertical stress, m/Lt$^2$, Pa [1Pa=1.45×10$^{-4}$psi]

$\Delta \sigma'_2 =$ increment of effective maximum horizontal stress, m/Lt$^2$, Pa [1Pa=1.45×10$^{-4}$psi]

$\Delta \sigma'_3 =$ increment of effective minimum horizontal stress, m/Lt$^2$, Pa [1Pa=1.45×10$^{-4}$psi]
Subscripts

\( \infty = \) intrinsic

\( D = \) dimensionless

\( \text{ini} = \) initial condition

\( L = \) left

\( m = \) matrix

\( \max = \) maximum

\( nf = \) natural fracture

\( \text{ref} = \) reference condition

\( R = \) right
CHAPTER 1
INTRODUCTION

1.1 Background

The hydrocarbon production out of low-permeability unconventional reservoirs is a quite complex process. Take the shale gas reservoir as an example. In shale gas reservoirs, the gas is stored in many forms. A major part of gas is stored as free gas in the shale pore space. Another part of the gas is adsorbed on pore walls (Figure 1.1). There is still a part of gas diffused inside the kerogen materials of the matrix. Free gas begins to flow when the pressure difference is applied along the pore space. When the pressure in pore space decreases, the adsorbed gas also starts to desorb off pore surfaces and mix with free gas. Then the gas inside kerogen diffuses off to the pore surface when the adsorbed gas content at pore surfaces decreases during desorption.

There are multi-scale pathways for the hydrocarbon flow from underground to the wellhead (Figure 1.1). The pore space in matrix is the first path when the fluid is driven to flow. The pore sizes of low-permeability unconventional reservoir rocks are typically in micrometer(μm) down to the nanometer(nm). Open natural fractures provide possible pathways for the hydrocarbon flow out of porous rocks. Most natural fractures have apertures (width) larger than 10μm. And many natural fractures are observed to be longer than 10cm. Hydraulic fractures resulted from high-pressure multi-stage fracturing treatments give another pathway for the fluid flow out of matrix pore space and natural fractures.
Figure 1.1 - The multi-scale and multi-mechanism shale gas production process (A multi-stage fractured horizontal well centered in the shale formation).
10 to 40 hydraulic fractures could spread along a 1000m-long horizontal wellbore. The hydraulic fracture width is around 1 to 10mm. And the hydraulic fracture half-length is usually in the range of 30m to 150m.

1.2 Problem Statement

It is concluded that the fluid flow in low-permeability unconventional reservoirs is a multi-scale, multi-mechanism process. To better characterize the flow characteristics and improve the reservoir productivity, several research problems at different length scales need to be identified and solved:

(1) The pore space structure inside shale matrix is multi-scale from \( \mu \text{m} \) to \( \text{nm} \). Many pores in shale matrix are in \( \mu \text{m} \)-scale, which is similar to tight gas reservoirs. But the development of imaging techniques discovers the existence and abundance of \( \text{nm} \)-scale pores in shale matrix. Such multi-scale feature makes it difficult to cover the full range of pore size distribution with single imaging or mathematical reconstruction method. Moreover, the multi-scale pore structure challenges the numerical simulations of fluid flow in extracted or reconstructed pore space.

(2) The gas flow in shale pore space may no longer follow the continuum flow regime. The Darcy’s law is applicable in conventional reservoirs. The reason behind this applicability is the continuum flow in the pore space of those reservoirs. But the continuum flow regime breaks down when gas flows in the \( \text{nm-} \mu \text{m} \)-scale pore space of shale. With decreasing pore size, the gas flow regime in shale moves from continuum flow to slip flow, transition flow and even free molecular flow.
(3) Both natural fractures and hydraulic fractures may be highly stress-sensitive. Natural fractures and hydraulic fractures provide high-conductivity pathways for hydrocarbon flow from unconventional reservoir rock matrix to the horizontal wellbore. During hydrocarbon production, the reservoir pressure will decrease and effective stress inserted on fracture surfaces will increase. Fractures’ conductivities can be reduced if the effective stress becomes larger. The fracture stress-sensitivity will speed up the hydrocarbon production decline of fractured horizontal wells.

(4) Many low-permeability unconventional reservoirs have complex reservoir conditions. For example, heterogeneity, reservoir stress-sensitivity, non-continuum flow and gas adsorption/desorption may coexist in one unconventional reservoir and significantly influence the hydrocarbon production. It would be in high demand if a mathematical model can fast and accurately evaluate the hydrocarbon production behavior for an unconventional reservoir with multiple complex reservoir conditions.

1.3 Objectives

Considering existing problems in low-permeability unconventional reservoir production, this study aims to

(1) Reconstruct the multi-scale pore space in shale matrix to investigate the effect of shale pore space structure on shale’s petrophysical properties (porosity, intrinsic permeability, etc.) and provide information for reservoir-scale/single-well-scale modeling.
(2) Explore the high-pressure rarefied gas flow characteristics in pore space to show the constitutive relationship between apparent permeability, pore structures and gas properties and provide information for reservoir-scale/single-well-scale modeling.

(3) Model the stress-sensitive hydraulic fractures to analyze the influence of fracture stress-sensitivity on a fractured horizontal well’s productivity.

(4) Develop a composite methodology and mathematical composite models that have potentials to evaluate a fractured horizontal well’s productivity under complex reservoir conditions.

1.4 Methodologies

In this study, multiple methodologies are applied to solve problems at different scales:

(1) This study tries to analyze SEM images for grain and pore size distribution (PSD) in shale matrix. With the PSD and process-based modeling approach, pore network models are developed to cover the wide range of pore size distribution from micrometer down to nanometer. No-slip flow simulations are completed on the pore networks for intrinsic permeability estimation.

(2) This study completes numerical simulations and derives simple analytical equations of the rarefied gas flow in a single nm- or µm-scale conduit with various cross-section shapes under high pressure (compared with vacuum conditions). Apparent permeability behavior can be predicted with running the analytical equations on all pores/throats in a pore network.
(3) Non-linear partial differential equations (PDEs) are developed to model the pressure distribution and flow characteristics around a MFHW with stress-sensitive hydraulic fractures. Semi-analytical solutions are obtained based on source/sink functions and fracture discretization.

(4) A composite methodology is developed based on the idea “decomposition”. An unconventional reservoir stimulated with a MFHW would be divided into sets of simple flow regions and the flow/pressure characteristics in each flow region can be easily modeled with linear flow equation, radial flow equation or line source/sink functions.

1.5 Thesis Organization

This thesis comprises six chapters for the clear and concise statement. **Chapter 1** summarizes the research background and provides four research objectives. **Chapters 2, 3, 4 and 5** list details about methods to solve each of the four problems. **Chapter 2** shows the pore network modeling of fluid flow in shale matrix. **Chapter 3** models the high-pressure rarefied flow in a single conduit as well in multi-scale pore networks. **Chapter 4** explains the semi-analytical modeling and analysis of fractured horizontal wells with stress-sensitive hydraulic fractures. **Chapter 5** develops a composite methodology and validate the composite methodology’s applicability in heterogeneous reservoirs. **Chapter 6** summarizes research findings, provides recommendations and lists possible future work.
CHAPTER 2
PORE NETWORK MODELING OF NO-SLIP FLOWS IN
SHALE MATRIX BASED ON PROCESS-BASED
APPROACH

2.1 Background

The hydrocarbon production from shale reservoirs has increased significantly since 2010, driven by improved drilling and completion technologies (horizontal drilling and hydraulic fracturing). However, researchers are still uncertain of the fluid flow inside shales and therefore debates on how to enhance the hydrocarbon recovery continue. Better knowledge of shale pore structure always provides insights into the fluid flow in shale and the reservoir properties—such as permeability and capillary pressure.

2.1.1 Pore Space in Shale

Pore systems in shale matrix are closely related to the sedimentary and diagenetic history during shale formation. During sedimentation, the fine-grained mineral and organic particles transported by turbulent flow in water settle out of suspension when the flow calms. The porosity occurring during sedimentation is the primary porosity. Most pores between grains and particles are primary pores and occur in the open spaces of grains (quartz, carbonate, calcite and dolomite grains, etc.) during sedimentation. A small part of primary pores in shale results from the cavities in biogenic debris particles. In addition, most pores in clay aggregates also occur during the deposition of clay platelets. Any physical,
chemical or biological change of sediments after the initial deposition/sedimentation is considered as part of shale diagenesis process. The diagenesis process includes compaction, cementation, mineral dissolution and hydrocarbon generation, etc. Compaction of shale rocks is driven by large overburden stress and chemical reactions. Mechanical compaction in shale is controlled by the effective stress. Chemical compaction results from dissolution and precipitation of solids. Mechanical and chemical compaction always leads to porosity loss (Laughrey et al. 2011). Katsube and Williamson (1994) concluded that shales at shallow depths tend to have larger pore sizes and higher porosities. During compaction process, the pore volume reduction could reach 88% with several kilometers of burial (Loucks et al. 2012). As the water is expelled out of pores, pore-waters become more concentrated and cement is deposited out of the solution. Secondary pores occur during the diagenesis process. Part of secondary pores inside grains is in association with mineral dissolution. Most pores in organic matter (OM) are secondary pores and occur when hydrocarbon is created in the generative kerogen (Camp 2014). When the temperature is high in deep burial, part of the OM in shale decomposes into hydrocarbons and leaves pores in place. The subspherical nature of OM pores further provides evidence for the post compaction characteristic.

The pore characteristics are vital for the gas storage and gas flow capacity in shales. Pores in shale rocks are much smaller than pores in conventional sandstone and carbonate reservoirs (Clarkson and Bustin, 1996; Dewhurst et al., 1999). Special techniques are required to test or visualize the complex pore size
distribution of shale. High-pressure mercury intrusion (HPMI), low-pressure N\textsubscript{2} adsorption (LP-N\textsubscript{2}GA) and CO\textsubscript{2} adsorption analysis (LP-CO\textsubscript{2}GA), small-angle neutron scattering (SANS) and ultrasmall-angle neutron scattering (USANS) can find the nm-scale pores and provide the PSD in shale samples. In 2008, Bustin et al. collected mercury intrusion porosimetry and adsorption data for four North American shales-Barnett shale, Antrim shale, Devonian shale, and Jurassic shale. Bustin et al. agreed that most above shales have a bimodal distribution of pore diameters with modes of approximately 10 nm and 10000 nm. By using HPMI, LP-N\textsubscript{2}GA and LP-CO\textsubscript{2}GA, Wang et al. (2014) examined 10 samples from the Northwestern Hunan shale gas reservoir, China and summarized three types of PSDs: (1) Bimodal PSD with peaks in micropores (<2 nm, IUPAC) and mesopores (2-50nm, IUPAC); (2) Multi-modal PSDs with peaks in micropores, fine mesopores (2-3nm) and large mesopores (20-50nm); (3) Multimodal PSD with a major peak in macropores (>50nm, IUPAC). Gu et al. (2015) analyzed the SANS and USANS results on Marcellus shale samples. The quantitative PSD from SANS/USANS analysis shows that the shale pore size is in a very broad range from 1nm to about 8 \textmu m.

In addition to pore size measurement, advanced imaging techniques further visualize the structures of small pores in shale rocks. And a better understanding of shale pore structures leads to a strong interpretation of gas storage and transport in shale. The resolution of a scanning electron microscope (SEM) image could reach 1nm at maximum. In 2009, Loucks et al. used SEM to examine 33 ion-milled cut surfaces from Barnett shale samples in the Fort Worth
Basin. Loucks et al. observed most of the pores in the Barnett shale are in nanoscale (<0.75 μm) and most of the nanoscale pores are located in grains of OM. The abundance of such nanopores is also well established in the Eagle Ford, Pearsall, Floyd, Woodford, Marcellus, Horn River and Maquoketa shale systems. In 2012, Loucks et al. further classified pores in shales into three basic types at the SEM resolution scale: (1) Interparticle pores. Interparticle pores are usually found between rigid grains, clay and mineral particles (Figure 2.1a). (2) Intraparticle pores including pores within clay sheets (Figure 2.1b) and between pyrite crystals, etc. (3) Intraparticle pores in OM (Figure 2.1c). In a single OM grain, porosity could reach as high as 50% (Curtis et al. 2010.). Chalmers et al. (2012) examined Barnett, Woodford, Haynesville, Marcellus and Doig shales with transmission electron microscopy (TEM)/SEM imaging techniques. Their results showed that most mesopores and macropores are located and interconnected in aggregates of kerogen, clay and/or carbonates. In addition, a part of macropores is located between grain boundaries as interparticle pores. Focused ion beam scanning electron microscope (FIB-SEM) and Broad Ion Beam scanning electron microscope (BIB-SEM) directly take high-resolution pictures of cut surfaces from shale samples. In FIB-SEM process, successive 2D SEM images after ion-milling generate serial sections for the 3D representation of shale samples. 3D FIB-SEM images from Passey et al. (2010) and Sondergeld et al. (2010) reveal 3D connected nm-scale pore networks inside the organic matter (OM) of Barnett shale samples. Houben et al. (2013) found that pores in clay particles always show elongated wedges or triangular
Figure 2. 1-SEM images with (a) interparticle pores (b) pores in clay and (c) OM pores (original in color).
shapes. In 2015, Klaver et al. (2015) further quantitatively analyzed the pore morphology in clay-rich Haynesville and Bossier shales by means of BIB-SEM and concluded that pore morphology reflects the shale formation history (burial, compaction, etc.). Among their BIB-SEM images, aspect ratios (width/depth) of the pores in fine-grained samples spread over the range (2.8, 3.3), which are larger than those in coarse-grained samples with the range (2.5, 3.3). Gu et al. (2015) analyzed FIB-SEM images of Marcellus shale and concluded that intraparticle pores in clays are sheetlike and roughly parallel to the bedding, while pores in kerogen are bubblelike or irregular in shape.

2.1.2 Pore Space Extraction and Reconstruction

Numerical simulations on the pore space provide insights into the relationship between reservoir characteristics and fluid flow dynamics. According to Sakhaee-Pour and Bryant (2014), the pore spacing modeling methods are classified into non-theoretical and theoretical.

The non-theoretical approach directly extracts the 3D pore space from high-resolution X-ray computed tomography. But even high-resolution X-ray instruments can only provide resolution on the order of a few tens of microns for millimeter-scale to centimeter-scale rock samples, which is not enough for shale samples. In recent years, the nm-scale pore space in shale rocks could be extracted with the FIB-SEM technology. The FIB-SEM images allow the visualization of mesopores and macropores in shales. Meanwhile, the improvement of mathematical methods, such as Lattice-Boltzmann method, makes it possible to run the direct numerical simulations on the complex pore
space. But for shale rocks, pore space extracted with FIB-SEM techniques still have limitations in pore space upscaling and the unconfined state of pore structures. The volume that a 3D FIB-SEM image can cover is typically around $10 \times 10 \times 10 \text{ µm}^3$. Saraji and Piri (2014) suggested that extra caution should be paid when applying the quantitative results of high-resolution 3D images to higher scales.

Because of the extracted pore space’s complex structure, direct numerical simulations on it always take a tremendous amount of time and are deficient in stability. With reducing computational cost as well as ensuring the simulations and predictions are meaningful, researchers have a choice of the simplified pore geometry-the theoretical reconstructed pore space. In 1921, Washburn first proposed that a bundle of tubes could be representative pore space in rocks (Figure 2.2a). Each tube represents a throat in pore space and has a uniform diameter. Considering the fact that pores are interconnected, Fatt (1956a, 1956b) assumed that pores are interconnected by throats and display a regular-lattice pattern (Figure 2.2b). Although regular-lattice models show good predictions of permeabilities for unsaturated soils, those models still cannot reflect the random nature of a porous rock.

Considering the random nature of porous rocks, random algorithms started to shine in the reconstruction of pore space in rocks. For better modeling results, 2D SEM images, 2D TEM images and atomic force microscopy (AFM) images (Javadpour et al. 2012) could be input to the random algorithms. Random algorithms commonly include Gauss simulation (Joshi 1974), simulated
Figure 2. 2-(a) Bundle of tubes model. (b) Regular lattice model. (c) The tree-like model (Sakhaee-Pour and Bryant, 2015).
annealing (Hazlett 1997), the process-based simulation method (Bryant and Blunt 1992; Bakke and Øren 1997; Øren and Bakke 2003), multiple point statistics (Okabe and Blunt 2004; Tahmasebi et al. 2015, 2016a, b), the sequential indicator simulation (Keehm 2003), the Markov stochastic reconstruction method (Wu et al. 2006) and various hybrid methods (Hidajat et al. 2002; Liu et al. 2009; Okabe and Blunt 2007; Yao et al. 2013). The process-based modeling starts with the research of Finney (1968). Finney (1968) measured the spatial coordinates of the centers of equal-sized sphere balls packed in a rubber bladder. Later Manson (1972) pointed out that Finney's experiments could be used to extract pore-throats sizes for unconsolidated granular material. It showed that extracted space from sphere packing could represent pore-throat relationship for porous rocks. In 1989, Mellor investigated the capability of Finney's random sphere packing model as a porous medium model. Later based on Mellor's (1989) approach, Bryant et al. (1993) used a completely defined packing of equal spheres from Finney (1968) to predict permeabilities in sandstones. Based on sphere packing models, Bakke and Øren (1997) and Øren and Bakke (2003) proposed a new method to generate homogeneous and heterogeneous 3D sandstone pore-scale models. The approach numerically models the sandstone-forming geological processes. The random sphere packing procedure could mimic the geological sedimentation. And compaction and cementation were modeled with changing sphere locations and radii. Recently, sphere packing models have been extended to the study of tight gas sandstones (Mousavi 2010; Mousavi and Bryant 2012). Mousavi et al.
(2013) further pointed out that the mudstone rocks can be modeled by uniform or disordered sphere packing.

According to shale PSD analysis, shale rocks have multiscale pore sizes and complex pore distribution. For shale reservoirs, the methods in reconstructing multi-scale pore space are desired. In 2012, Bauer et al. developed a Dual Pore Network approach (D-PNM) for carbonate rocks with bimodal PSD. In the D-PNM, the micropore networks are in parallel with the throats in macropore networks which could be resolved by the micro–CT scanning. But the micropore networks in the method are represented by average transport properties rather than extracted or reconstructed networks. In 2013, Mehmani et al. selected part of pores and throats from an unstructured pore network model (extracted from monodisperse sphere packing) and randomly changed their sizes to the nanometer scale. Mehmani et al.'s results provide insights about the influence of nanopores on shale permeability. Later Mehmani and Prodanović (2014) inserted rescaled nanoscale pore networks into the selected pores or grains. This improved pore network considers the interconnectivity of nanoscale pores. In 2015, Peng et al. proposed a conceptual shale pore network. Based on micro-CT scanning, they concluded that in Barnett shale, μm-scale OM particles are connected as a network and each OM particle contains networks of nm-scale pores that can be characterized with the FIB-SEM and HPMI techniques. Sakhaee-Pour and Bryant (2015) observed the non-plateau-like capillary pressure curves in shale samples and developed the tree-like and semi-tree acyclic pore models (Figure 2.2c). Sakhaee and Bryant's
models follow the idea that in shales the access to larger throats is not limited by narrower throats, which reaches a partial consensus with Peng et al. (2015).

2.1.3 Summary

In the literature, different approaches have been applied to modify existing single-scale pore network models for the multiscale extension. These approaches include randomly micropores insertion, tree/semi-tree-like pores and average permeability estimation. Existing multi-scale pore models need of improvement to cover the wide range of PSD and complex pore structures inside shale matrix. Moreover, the influence of geological processes on pore interconnectivity has not been studied yet. The influence of pore interconnectivity in OM and/or clay has not been studied yet. The influence of micropore morphology and distribution has not been studied yet.

This study aims to utilize the process-based approach to develop pore network models and represent the multi-scale pore structure inside shales. The process-based approach is able to describe characteristics of geological processes and μm-scale OM–clay–interparticle pores space distribution in shale. The developed multi-scale pore network could cover the pore sizes from nanometer to micrometer. The model is applied to investigate the relationship between shale multi-scale pore structure and shale permeability, which provides insights for shale production optimization.

2.2 Pore Networks Development

Shale rocks are resulted from complex geological and hydrodynamic processes. 3D rock samples can be reconstructed by simulating the results of
main geological-forming processes. In this study, the geological processes our model simulates include sedimentation and diagenesis (compaction, cementation, generation of OM pores and pores inside clay).

### 2.2.1 Modeling of μm-scale OM-Clay-Interparticle Pores Distribution

Inside shale rocks, interparticle pores, clay agglomerates and OM particles are interconnected as a continuous μm-scale space that is complementary to solid grains (Peng et al. 2015). In this section, the sedimentation, compaction and cementation processes are simulated to provide the μm-scale volume occupied by OM particles, compacted clay agglomerates and interparticle pores.

The process-based modeling starts with sedimentation process. Many siliceous and carbonate particles in clay size (4-5 µm) and silt size (4-6 µm) are transported by turbulent flow in water or air, and deposited as the flow calms. The grain size distribution is an important input for our process-based models. **Fig. 2.3** displays the analyzed equivalent grain diameter vs. fraction for 215 grains in 20 SEM images of shale samples from two shale reservoirs, China. The dominant size of grains is in the range of 5-15µm. In addition to SEM images, any grain size analysis can work as input for the sedimentation process. The sedimentation simulation follows the method of Bakke and Øren (1997) with low-energy condition and randomly packs spheres in a confined cuboid space. The size distribution of packed spheres is set to follow the grain size distribution in SEM images (Fig. 2).
Figure 2. Grain size distribution (proportion of number) from SEM image analysis (black columns) and sphere size distribution in sedimentation process modeling (white columns).
When shale sediment layers are deposited, the space between shale grains/particles is usually filled with organic matter, water and clay minerals. With more sediment layers loaded, the pore space reduces significantly and a large amount of water is expelled because of increasing stresses and more efficient packing of grains. When sediment depth and temperature increases, the chemical compaction further causes grain interpenetration and governs the compaction process. The elongated, flattened interparticle pore morphology in many shale samples is consistent with the heavy compaction history. The compaction process could be modeled by reducing spheres’ center $z$-coordinate values by a ratio $\lambda$:

\[
z = z_0 (1 - \lambda).
\]

Here the ratio $\lambda$ is defined as the compaction factor. Loucks and Ruppel (2007) stated that the compacted lamination around rigid grains in Barnett shale has a compaction factor up to 0.65.

During cementation in shale, dissolved clay minerals precipitate out of pore-waters as stable clay minerals and quartz. Those clay minerals and quartz are concentrated on surfaces of sediment particles and existing cement. In sandstones and carbonates, clay cement can be divided into three broad categories: pore lining cement, pore-filling cement and pore-bridging cement. Cementation in sandstones and carbonates can be modeled by sphere radius increment, random clay particles precipitation and clay particles clustering (Øren et al. 1998). Similar to sandstone and carbonates, the cementation process introduces more heterogeneity to the space inside shale rocks. In this study, the
cement around rigid grain surfaces is simply modeled by uniformly increasing the size of a sedimented sphere:

\[ R = r + l \]

where \( r \) and \( R \) are the original radius and the cemented sphere radius.

**Table 2.1** summarizes the parameters applied in simulations of the sedimentation, compaction and cementation processes for selected shale samples. The cement layer thickness \( l \) in this study is set to be the same for all spheres and determined based on SEM images analysis. The compaction factor \( \lambda \) in **Table 2.1** is determined according to available SEM images and studies of shale compaction (Dræge et al. 2006; Bjorlykke et al. 2009). After sedimentation, compaction and cement, a 300×300×300 grid (2700000 voxels) is applied to the sphere packing and each voxel is 0.28μm×0.28μm×0.2μm in x-, y- and z-direction. The space that is not occupied by sedimented spheres is extracted as voxel combinations from 3D gridding of the sphere packing. The extracted space is in μm-scale and assumed to comprise interparticle pores, porous kerogen and/or porous clay minerals. With different clay content and total organic carbon (TOC), shales fall into organic-rich and clay-rich types. By following the randomness nature of real porous shale, part of the extracted space is randomly chosen as OM particles and clay agglomerates until the predefined clay content and TOC is reached. **Fig. 2.4a** shows one possible distribution of the chaos space inside which OM particles and clay agglomerates occupy 75% volume while interparticle pores account for 25%. The red and black voxels represent porous OM and clay agglomerates, respectively, while the brown part
Table 2. Sedimentation, compaction and cementation parameters for the selected shale sample.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total number of spheres in sedimentation</td>
<td>550</td>
</tr>
<tr>
<td>Mode of sphere diameter distribution, ( \mu m )</td>
<td>11</td>
</tr>
<tr>
<td>Compaction factor ( \lambda )</td>
<td>0.4</td>
</tr>
<tr>
<td>Cement layer thickness ( l )</td>
<td>0.28</td>
</tr>
</tbody>
</table>
Figure 2. (a) μm-scale space composed of interparticle pores (brown), OM particles (red) and clay agglomerates (black). (b) 3D distribution of OM particles (red) and clay agglomerates (black) (original in color, rotation 90° for best presentation).
corresponds to μm-scale interparticle pores. Fig. 2.4b further shows the 3D distribution of OM and clay, which is similar to the extracted OM and clay distribution in micro-CT scanning (Peng et al. 2015) and FIB-SEM images (Saraji and Piri 2014). The μm-scale space (such as Fig. 2.4a) can be simplified to a network of spherical nodes connected by cylindrical throats. Each node’s volume is calculated based on distances from the node center to surrounding surfaces. A node could represent one OM particle, clay agglomerate or interparticle pore. The radius of each cylindrical throat is calculated based on the narrowest constriction measured from the throat center line to surrounding surfaces.

2.2.2 Modeling of nm-scale Pores

When rigid grains sediment, clay minerals also deposit and pores occur between clay minerals. The clay content in many shale rocks is in the range of (20 wt%, 45 wt%), which is higher than sandstones and carbonates. Fig. 2.5a shows the frequency of intraparticle pores inside clay agglomerates with different aspect ratios in SEM images (resolution 5–20nm). In the selected shale samples, the aspect ratios of most pores in clay are smaller than 4. Fig. 2.5b displays the width distribution of these pores. The majority of pores in clay have a width smaller than 100nm.

After most of the compaction has occurred, the temperature and pressure in reservoir increase with deeper burial. With a high temperature, part of the OM in shale decomposes and generates hydrocarbons. The generation of hydrocarbons in OM creates a large number of secondary pores—the OM pores. The subspherical nature of OM pores further provides evidence for the
Figure 2. 5a-The fraction distribution (proportion of number) of pore aspect ratio in clay based on the analysis of SEM Images.

Figure 2. 5b-The fraction distribution (proportion of number) of pore width in clay based on the analysis of SEM Images.
Figure 2. 6-The OM pore diameter distribution (proportion of number) based on the analysis of SEM Images.
post-compaction characteristic. During the hydrocarbon generation, closely spaced pores in OM expand and get interconnected as networks. **Fig. 2.6** shows the equivalent diameter of OM pores from 2D SEM images (resolution 5–20nm).

In the selected shale samples, OM pores' diameters vary from 30nm to 200nm.

Although a sphere packing cannot fully mimic the sedimentation of irregular clay minerals and OM pore generation, the pore networks extracted from random monodisperse sphere packing could still be used to represent the pore space inside clay agglomerates and OM particles in this study. Other kinds of pore networks can also be used to represent the nm-scale pore space. The ratio of a sphere diameter for nm-scale modeling over the mode of sphere diameter distribution in μm-scale modeling is defined as a ratio factor \( f_R \). As shown in **Fig. 2.5**, the \( f_R \) for clay sedimentation is smaller than 0.05. With **Fig. 2.6**, the \( f_R \) for OM pores generation is in the range of (0.01, 0.05). The nm-scale pore space in OM particles has different morphology than that in clay agglomerates. In nm-scale pore networks, the throats inside OM are cylindrical channels while cross sections of throats inside clay agglomerates are set to be rectangular. The aspect ratios of cuboid-shaped throats inside a clay agglomerate can be either constant or follow the distribution from SEM image analysis, such as **Fig.2.5a**.

### 2.2.3 Multi-scale Pore Networks Development

Combination of the continuous μm-scale space with nm-scale pore networks builds a multiscale pore space covering pore sizes from several nanometers to micrometers. There exist a few methods to create multi-scale pore networks in the literature. This study follows the idea of nm-scale networks embedment. A
nm-scale pore network is inserted into a selected spherical node of a \( \mu m \)-scale network simplified from the space such as in \textbf{Fig. 2.4}. The selected node represents a clay agglomerate or OM particle. The center of the nm-scale pore network coincides exactly with the center of the \( \mu m \)-scale spherical node. All the nm-scale pores that are enveloped inside the node will remain in the final pore network. All the nm-scale throats that fall within the spherical node and the connected \( \mu m \)-scale cylindrical throats are also added to the final pore network. If one of two nm-scale pores that a nm-scale throat connects is located in the selected \( \mu m \)-scale spherical node while the other nm-scale pore is in the \( \mu m \)-scale throat, this nm-scale throat will be modified to replace the \( \mu m \)-scale throat. This nm-scale pore network embedment process continues until the predefined nm-scale porosity is reached in the multi-scale pore network. The final pore network can be OM-pore-dominated, clay-pore-dominated or interparticle-pore-dominated. \textbf{Fig. 2.7} shows one multi-scale pore network with only OM pores (red) and interparticle pores (brown) based on the methodology of this study. There are 1040/1318 interparticle pores/throats and 216305/382960 OM pores/throats in \textbf{Fig. 2.7}. The small OM pores are interconnected as well as embedded into a continuous \( \mu m \)-scale network. In this study, a large interparticle pore can be connected with hundreds of nm-scale OM pores, which makes its coordination number very large.
Figure 2. One multi-scale pore network with 1040/1318 large interparticle pores/throats (brown) and 216305/382960 nm-scale OM pores/throats (red) (original in color, rotation 90° for best presentation).
2.3 Network Flow Modeling

When pressure difference exists between the pore network inlet and outlet, fluid flows through pores and throats. The hydraulic conductance of shale pore networks represents the shale matrix permeability. Since the pore body is much larger than connected throats, the pressure difference of single-phase fluid inside pore bodies is negligible. The pressure difference along throats drives the fluid to flow from one pore towards another pore.

For a single cylindrical throat that connects $i$-th and $j$-th pores, the no-slip Hagen-Poiseuille equation gives the volumetric single-phase flow rate $q_{ij}$ through the throat:

$$q_{ij} = \frac{\pi R^4}{8\mu L} (p_i - p_j). \quad \text{.................................................................(2.3)}$$

For a single cuboid throat with width $W$ and height $H$, the volumetric single-phase flow rate $q_{ij}$ becomes

$$q_{ij} = \left(1.0 - \sum_{m=1,2,3, \ldots}^{\infty} \frac{192W}{(2m-1)^5 \pi^5 H} \tanh \left(\frac{(2m-1)\pi H}{2W}\right)\right) \frac{WH^3}{12\mu L} (p_i - p_j). \quad \text{........(2.4)}$$

The fluid density can be regarded as a constant when the single-phase fluid inside the pore network is incompressible. During steady-state flow through the pore network, the pressure distribution inside the network does not change along with time. Under such conditions, the mass conservation for the individual $i$-th pore becomes

$$\sum_j q_{ij} = 0, \quad \text{...........................................................................................................(2.5)}$$
where \( j \) represents any pore connected to the \( i \)-th pore through throats. A system of equations occurs if the mass conservation principle runs over all the pores in the network:

\[
MP = b \tag{2.6}
\]

Here \( M \) is a sparse matrix that comprises hydraulic conductance of throats. The \( P \) is a vector with unknown pressure values at each pore and the \( b \) is a source vector which is essentially zero except for the inlet and outlet boundaries. The solution of Eq. 2.6 gives the pressure distribution inside the pore network.

It is assumed that the fluid flows into and out of the pore network in \( x \)-direction. All pores at \( x \)-direction inlet have constant inlet boundary pressure \( p_{in} \) while pores at the outlet have constant outlet boundary pressure \( p_{out} \). All throats that are connected to the inlet or outlet have the corresponding boundary pressure at one of their two ends. The volumetric flux \( q \) through the pore network is calculated based on Eqs. 2.3 to 2.6 at inlet and outlet boundaries. The intrinsic permeability \( K_\infty \) of the reconstructed digital shale rock is evaluated using Darcy’s equation:

\[
K_\infty = \frac{qul}{A(P_{in} - P_{out})} \tag{2.7}
\]

Where \( A \) is the cross-section area and \( L \) is the \( x \)-direction length of the pore network.

### 2.4 Results and Discussion

Higher intrinsic permeability of shale rocks could increase shale reservoirs productivity. To evaluate the relationship between pore structure and shale intrinsic permeability, this study investigates the effect of nm-scale pore volume,
compaction factor, nm-scale pore connectivity, OM porosity and nm-scale pore morphology on shale permeability.

2.4.1 Effect of nm-Scale Pore Volume

A total of 30 “as received” shale core plugs were analyzed from a shale formation in China. The core plugs have a diameter of 2.54 cm and length 3.26 cm. Helium pycnometry was conducted at unconfined conditions for core plugs porosities. Pulse-decay permeability tests (Finsterle and Persoff 1997; Liang et al. 2001) were conducted under confining pressure 10.3MPa, mean pore pressure 6.9MPa and effective stress 3.4MPa. Fig. 2.8 shows the pulse decay permeability versus measured porosity. Since the pulse-decay permeability was tested with pore pressure 6.9MPa, the slip flow and transition flow effect was greatly reduced for pore/throat size larger than 20 nm. As shown in Fig. 2.8, most of the tested shale samples have permeability smaller than 5μD and porosities smaller than 6%. Typically the larger volume of interconnected pores contributes to higher sample permeability. And usually it is easier for interparticle pores to get interconnected as a network than intraparticle pores from different particles. Fig. 2.8 indicates a predictive positive relationship between shale permeability and porosity, which may reflect the existence of interconnected interparticle pores (Bustin et al. 2008).

To validate our approach and models, simulated permeabilities based on pore networks in this study are compared with the tested data from Fig. 2.8. The μm-scale network is generated according to conditions in Table 2.1. It is assumed that only porous OM particles and interparticle pores constitute a
Figure 2. Pressure pulse-decay permeability versus Helium porosity of 30 shale core plugs from the selected shale formation in China.
continuous $\mu m$-scale space and all nm-scale pores are located inside OM particles. Therefore, all pores are simplified to be spherical and all throats are cylindrical. In the literature of multi-scale pore network development, all nm-scale pore networks in OM particles have same pore size distribution. This always results in a discontinuous pore/throat size distribution with two peaks separated in different zones. To make a continuous, smooth multi-scale pore size distribution, each monodisperse sphere packing for nm-scale pore network extraction has a unique ratio factor in this section. As shown in Fig. 2.5, the ratio factor $f_R$ is randomly selected from a normal distribution in (0.014, 0.1).

Seven multi-scale pore networks are generated with increasing nm-scale porosity and decreasing $\mu m$-scale porosity. Table 2.2 lists the numbers of interparticle pores/throats, numbers of OM pores/throats and pore/throat size range in each pore network. In the No. 1 case, all the pores are interparticle pores. In case 2, 100 interparticle pores are randomly selected and removed out of the $\mu m$-scale network. Then nm-scale pore networks are inserted into the selected locations in the $\mu m$-scale network. And each nm-scale pore network has a unique ratio factor $f_R$. From case 2 to case 7, more $\mu m$-scale pores are randomly removed and then nm-scale pore networks are embedded. Since nm-scale pore volume is smaller than the removed interparticle pores, the multi-scale pore volume decreases while embedded nm-scale pore volume increases.
Table 2. Properties of 7 pore networks developed for selected shale core samples in Section 2.4.1.

<table>
<thead>
<tr>
<th>Case no.</th>
<th>Number of interparticle pores/throats</th>
<th>Number of OM pores/throats</th>
<th>$R_{p, \text{min}}^a$, μm</th>
<th>$R_{p, \text{max}}^b$, μm</th>
<th>$R_{t, \text{min}}^c$, μm</th>
<th>$R_{t, \text{max}}^d$, μm</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1239/1777</td>
<td>0/0</td>
<td>0.16</td>
<td>3.7</td>
<td>0.13</td>
<td>3.4</td>
</tr>
<tr>
<td>2</td>
<td>1139/1538</td>
<td>110,892/196,371</td>
<td>0.0036</td>
<td>3.6</td>
<td>0.0009</td>
<td>3.4</td>
</tr>
<tr>
<td>3</td>
<td>1040/1318</td>
<td>216,305/382,960</td>
<td>0.0034</td>
<td>3.6</td>
<td>0.0009</td>
<td>3.4</td>
</tr>
<tr>
<td>4</td>
<td>839/961</td>
<td>386,463/682,075</td>
<td>0.0034</td>
<td>3.6</td>
<td>0.0009</td>
<td>3.4</td>
</tr>
<tr>
<td>5</td>
<td>640/660</td>
<td>606,509/1,068,678</td>
<td>0.0034</td>
<td>3.6</td>
<td>0.0009</td>
<td>3.4</td>
</tr>
<tr>
<td>6</td>
<td>343/359</td>
<td>888,418/1,559,443</td>
<td>0.0034</td>
<td>3.4</td>
<td>0.0009</td>
<td>3.2</td>
</tr>
<tr>
<td>7</td>
<td>141/190</td>
<td>1,103,833/1,931,523</td>
<td>0.0034</td>
<td>3.4</td>
<td>0.0009</td>
<td>3.2</td>
</tr>
</tbody>
</table>

$^a$Minimum pore radius in the pore network
$^b$Maximum pore radius in the pore network
$^c$Minimum throat radius in the pore network
$^d$Maximum throat radius in the pore network
Figure 2. (a) The simulated no-slip permeability (red dot) and measured permeability (black diamond) versus porosity of seven pore network models with different numbers of interparticle and nm-scale pores. (b) The simulated no-slip permeability versus OM pore volume/total pore volume.
**Fig. 2.9a** gives permeability versus porosity for the 7 multi-scale pore networks. **Fig. 2.9b** provides the permeability versus OM pore volume ratio for the 7 multi-scale pore networks. **Fig. 2.9a** shows that when the pore network porosities reduce into the range of tested porosities, calculated permeabilities also fall into the range of tested permeabilities. It means that pore network models here could simulate key characteristics of the pore structure for selected shale samples. Pore network models based on the process-based approach are verified to be capable to predict shale permeability from shale microscopic origins.

Small pores and throats lead to low permeability. When nm-scale OM pores randomly replace more μm-scale interparticle pores in Cases 1–7, pore network’s permeability keeps decreasing. Conventional reservoir rocks, such as sandstones, always have pore/throat sizes in μm-scale, while shales have lots of nm-scale pores. Therefore, shale rocks usually have lower permeability than conventional rocks, which is in line with the trend from Case 1–7. For Cases 1–5, pore network permeability decreases rapidly with more OM pores and less μm-scale pores. For Cases 6 and 7, nm-scale pores already dominate the pore network and the permeability decrease slows down. **Fig. 2.10** displays the frequency distribution of throats with different sizes in Case 7. The majority of throats has diameters around 200 nm, which is a little bigger than the SEM image analysis in **Fig. 2.6.** **Fig. 2.11** shows the pore volume distribution in Case 7 which covers pore sizes from several nanometers to several micrometers. The volume distribution has a clear bimodal pattern with two modes around 280nm.
Figure 2. Frequency distribution of throat diameters in Case 7 of Section 2.4.1.
Figure 2. 11-Pore volume fraction distribution in Case 7 of Section 2.4.1.
and 4μm. In fact, the pore network models can also be set to be multi-modal if nm-scale pore networks with a different set of ratio factors \( f_R \) are embedded into a μm-scale network.

For the porosity 1–6% in Fig. 2.9, many shale samples have permeabilities higher than 1μD, which exceeds the maximum estimated permeability 0.84μD from Cases 5, 6 and 7. One possible reason could be that nm-scale pores in those shale samples cluster and get interconnected into a continuous nm-scale pore network. The interconnected nm-scale pore network is in parallel connection with interconnected μm-scale pores, which contributes to much higher permeabilities. Another possible reason could be that porosities of porous OM particles in the model are lower than those shale samples.

2.4.2 Effect of Compaction

Shales can be classified as soft shales and hard shales according to the degree of compaction. Usually hard shales have lower porosities than soft shales due to serious compaction. The compaction process significantly reduces the interconnectivity of μm-scale network as well as makes μm-scale interparticle pores smaller. To investigate the influence of compaction, another two sets of μm-scale networks are generated with compaction factor \( \lambda \) as 0.3 and 0.5. For each \( \lambda \), six cases are generated with increasing OM pore volumes and decreasing interparticle pore volumes. Fig. 2.12 shows the calculated permeability of multi-scale pore networks versus porosities under the effect of different compaction factors.

According to the analysis in Section 2.4.1, pore networks’ porosities and
Figure 2. Permeabilities of pore networks with different compaction factors $\lambda$ 0.3, 0.4 and 0.5 (original in color).
Figure 2. Pore size distribution for three cases with different $\lambda$ when (a) effective porosity is around 6% before nm-scale OM pore dominate the network and (b) effective porosity is around 3% after nm-scale OM pores dominate the network (original in color).
permeabilities decrease when more nm-scale pores/throats exist rather than μm-scale pores/throats. Before nm-scale pore volumes dominate in Fig. 2.12, the declining rate of permeability versus porosity is larger for multi-scale pore networks with higher $\lambda$. When total effective porosity is the same in Fig. 2.12, Model 1 with lower $\lambda$ has more nm-scale pores and lower permeability than Models 2 and 3. It means that nm-scale pores heavily influence pore networks permeability regardless of the compaction factor before nm-scale pores dominate. Fig. 2.13a shows pore size distributions of the three models with different $\lambda$ when the effective porosity is around 6% (before OM pores dominate networks). Although only a small fraction of pore volume is contributed by nm-scale OM pores, OM pores have significant influence on permeability. OM pores/throats are patches dispersed in a continuous μm-scale space. Those OM pores break the high interconnectivity among μm-scale pores but have not got interconnected among different patches, which makes the permeability decrease fast.

When nm-scale pores dominate pore networks in Fig. 2.12 (porosities are smaller than 4%), the permeability is closely related to $\lambda$. Model 1 has highest permeability, followed by Models 2 and 3. Smaller compaction leads to a μm-scale space with higher interconnectivity. And such high μm-scale interconnectivity finally contributes to good nm-scale interconnectivity when nm-scale pores dominate a pore network. Fig. 2.13b shows pore size distributions with different $\lambda$ when the effective porosity is around 3% (after OM pores dominate networks). Fig. 2.13b shows that even when nm-scale pores dominate
a multi-scale pore network, the large μm-scale pores may still have larger volume fraction. This explains the phenomenon that many shale samples have high μm-scale pores volume fractions but their permeability is still very low in the range of μD–nD (Bustin et al. 2008; Wang et al. 2014). Moreover, it takes less nm-scale pore volumes to dominate a multi-scale pore network when the compaction factor \( \lambda \) becomes larger.

In addition, in Fig. 2.12 Model 1 with lower \( \lambda \) has higher porosity but almost same range of permeability with Model 2. This finding agrees with the data given by Bustin et al. (2008). It shows that although soft shales usually have higher porosities than hard shales, soft shales may have similar permeability with hard shales.

2.4.3 Effect of nm-Scale Pore Interconnectivity

The interconnectivity of nm-scale pore networks has a significant influence on shale intrinsic permeability. To check the influence of nm-scale pore network interconnectivity, the pore network with porosity 7.1% (Case 4 in Section 2.4.1) is picked out as the first pore network in this section. And 63775 nm-scale throats are randomly removed from this first pore network to create the second pore network with lower interconnectivity. Fig. 2.14 compares number fractions of pores with coordination number 1–6 for the two pore networks. There are 387,302 pores in total for each of two pore networks. The majority of pores in the two networks have a coordination number smaller than 7. And the fraction of pores with coordination number 6–160 is less than 0.003, which is not shown in Fig. 2.14. In Fig. 2.14, the number fraction of pores with the coordination
Figure 2. Comparison of coordination numbers for the two pore networks with different nm-scale pore network interconnectivity (black column for higher interconnectivity and white column for lower interconnectivity).
number 4 in the second network is 0.34, which is much lower than the value of 0.5 in the first network. It matches the idea that the nm-scale pores in the second network have lower interconnectivity with lower coordination numbers. The simulated permeability of the second network is 0.26μD, which is much lower than the permeability of the first network, 10.26μD (Fig. 2.9). It is concluded that the higher nm-scale pore interconnectivity leads to higher shale rock permeability. For the first network, nm-scale OM pores destroy the interconnectivity of μm-scale interparticle pores but have not been interconnected among different OM patches yet (before OM pores dominate the network). If the interconnectivity of pores inside each patch is further reduced, the multi-scale pore network’s permeability can be significantly reduced as in the second network of this section. When a large amount of gas is generated in OM during diagenesis, the left-over pores after gas generation have a higher possibility to be interconnected as a network. Usually, the OM with higher gas generation potential tends to generate more pores and accordingly have more interconnected OM pores inside.

### 2.4.4 Effect of OM Porosity

The abundance and size of OM pores are controlled by the maturity and type of the OM present. The original kerogen is non-porous. Abundant nanopores show up when oil and gas are generated and ejected in mature and over-mature kerogens. In the μm-scale OM–clay–interparticle pores space, higher porosity of porous OM particles makes the total porosity larger. In the literature of multi-scale pore network modeling, a higher OM porosity is always
Table 2. 3 Characteristics of four pairs of pore networks with different OM porosities in Section 2.4.4.

<table>
<thead>
<tr>
<th>Case no.</th>
<th>Number of Pores/throats</th>
<th>Average OM porosity (%)</th>
<th>Total porosity (%)</th>
<th>Permeability (μD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>112,031/197,909</td>
<td>4.9</td>
<td>9.26</td>
<td>136</td>
</tr>
<tr>
<td></td>
<td>166,084/310,569</td>
<td>8.5</td>
<td>9.27</td>
<td>136</td>
</tr>
<tr>
<td>2</td>
<td>217,345/384,278</td>
<td>5.2</td>
<td>8.47</td>
<td>27.3</td>
</tr>
<tr>
<td></td>
<td>348,892/651,787</td>
<td>7.1</td>
<td>8.50</td>
<td>27.8</td>
</tr>
<tr>
<td>3</td>
<td>387,302/683,036</td>
<td>5.1</td>
<td>7.11</td>
<td>10.2</td>
</tr>
<tr>
<td></td>
<td>668,502/1,248,603</td>
<td>7.8</td>
<td>7.17</td>
<td>11.0</td>
</tr>
<tr>
<td>4</td>
<td>888,761/1,559,802</td>
<td>4.8</td>
<td>1.59</td>
<td>0.213</td>
</tr>
<tr>
<td></td>
<td>1,576,079/2,929,888</td>
<td>8.1</td>
<td>1.74</td>
<td>0.638</td>
</tr>
</tbody>
</table>
achieved by increasing OM pore/throat diameters. But it cannot describe the high porosity resulted from nm-scale pore abundance inside OM. This section investigates the influence of OM porosities on permeability by increasing pore/throat numbers in nm-scale pore networks. **Table 2.3** lists the properties of four pairs of multi-scale pore networks. Each pair includes two networks with same pore size distribution and pore interconnectivity. But one has a lower OM porosity and the other one has a higher OM porosity. In the four pairs of cases, the higher OM porosity contributes to higher total porosity and higher permeability. By comparing Case 4 with Cases 1, 2 and 3 in **Table 2.3**, it is concluded that higher OM porosity significantly improves permeability when nm-scale pores in OM particles dominate the multi-scale pore network.

### 2.4.5 Effect of nm-Scale Pore Morphology

Pore space has different morphology in different components of shale rocks (Afsharpoor and Javadpour 2016; Afsharpoor et al. 2017). All pore networks are set to have cylindrical throats in Sections 2.4.1-2.4.4. In this section, multi-scale networks in Section 2.4.1 are reset to have cuboid-shaped throats without changing the pore space volume. And for easy comparison, all throats in one pore network are set to own same aspect ratio. **Fig. 2.15** calculates and compares the permeabilities of pore networks with cylindrical and cuboid-shaped throats. It shows that when the porosity is same, a shale sample with cuboid-shape throats has lower permeability than that with cylindrical throats. Furthermore, shale with cuboid-shaped throats will have lower permeability when the aspect ratio becomes larger. Many cuboid-shape throats are located
Figure 2. Comparison between the permeability $K_{cuboid}$ of pore networks with all cuboid-shaped throats and the permeability $K_{cylindrical}$ of pore networks with all cylindrical throats.
inside clay agglomerates. It is inferred that porous clay in shale tends to have lower permeability than porous OM.

Fig. 2.15 shows that the permeability difference between pore networks with all cylindrical and all cuboid-shaped throats tends to be larger when more nm-scale pores and throats exist. When cylindrical throats are changed to be cuboid-shaped with aspect ratio 2, the pore network with nm-scale pore volume fraction 36% loses 28% of its permeability. Shale rocks with high clay content tend to have more cuboid-shaped pores. It means that shale rocks with higher clay content tend to have lower permeabilities than those with lower clay content.

2.5 Conclusions

Multi-scale pore network models are developed for selected shale samples in this study. The \( \mu \)m-scale continuous OM–clay–interparticle pores space is reconstructed based on simulations of sedimentation, compaction and cementation processes. Nm-scale pore networks inside OM particles and clay agglomerates are reconstructed with monodisperse sphere packing and SEM images analysis. Coupling of \( \mu \)m-scale space and nm-scale pores creates multi-scale pore networks with smooth, continuous pore/throat size distributions from nanometer to micrometer. The calculated permeability on multi-scale pore networks could match the pulse-decay permeability data. Furthermore, the influence of pore space structure on shale permeability is investigated in details.

1. The influence of nm-scale pore volumes on permeability is investigated. Pore networks with more nm-scale pores, such as in shales, have smaller permeability than networks with more \( \mu \)m-scale pores, such as in sandstones.
2. A small volume fraction of nm-scale pores can break the interconnectivity of \( \mu m \)-scale space and make the permeability much lower than that of the \( \mu m \)-scale pore network.

3. In a multi-scale pore network, it is possible for nm-scale pores to have smaller volume than \( \mu m \)-scale pores even when nm-scale pores dominate the network and get interconnected among different OM patches.

4. With same porosity, pore networks with more nm-scale pores have lower permeability than those with less nm-scale pores before nm-scale pores dominate a multi-scale network. After nm-scale pores dominate a pore network, lower compaction factor \( \lambda \) contributes to a higher interconnectivity of porous OM particles and therefore a larger permeability.

5. OM pores have higher interconnectivity when their coordination number is large. Before nm-scale pore networks get interconnected among OM patches, low nm-scale pore interconnectivity significantly reduces the multi-scale pore network permeability.

6. Different shale samples have different OM porosities because of varying kerogen type, composition, maturity, etc. Higher OM porosities can significantly improve the permeability of a multi-scale pore network when OM pore networks get interconnected among OM patches and dominate the multi-scale pore network.

7. When all nm-scale throats are cuboid-shaped rather than cylindrical, the shale matrix tends to have a lower intrinsic permeability. For cuboid-shaped nm-scale throats, higher aspect ratio reduces shale intrinsic permeability.
CHAPTER 3
PORE-SCALE MODELING OF HIGH-PRESSURE RAREFIED GAS FLOW IN SHALE MATRIX

3.1 Background

Characterizing the complex pore space inside shale rocks is challenging. The situation becomes more complicated when the fluid starts to flow inside porous shale. Gas and/or liquid that are stored inside pore bodies may be driven to flow when a pressure difference is applied along the pore space. The modeling of fluid flow in shale pore space could reveal the constitutive relationship among pore space characteristics, fluid properties and shale permeability.

3.1.1 Rarefied Flow

The fluid flow behavior is strongly influenced by interactions between fluid particles as well as between fluid particles and pore surfaces/walls. For the slow, laminar liquid flow (with low Reynolds number) in cylindrical pores or tubes, the Hagen–Poiseuille flow (Poiseuille, 1838; Hagen, 1839) is recognized as the major feature in terms of liquid flow dynamics. Correspondingly, the liquid at the pore center usually has highest velocity while the liquid velocity at pore surfaces is considered as zero. This is the widely-applied no-slip boundary condition (Day, 1990; Deen, 1998; Perrin et al. 2005) to describe the interaction between liquid moleculars and wall/surface in the continuum flow regime. Recently exceptions to the no-slip boundary condition have been found in the experiments
(Majumder et al. 2005; Holt et al. 2006) and molecular simulations (Whitby and Quirke 2007) of liquid flow in carbon nanotubes. When liquid slippage occurs, the liquid flow rates through those nanotubes are higher than expectations based on the Hagen–Poiseuille equation. The liquid flow dynamics in nanotubes implies the possibility of liquid slippage in nm-scale shale pores.

Compared with liquid, the gas flow in narrow pore space has more opportunities to deviate from continuum flow dynamics. The gas flow under pressure difference is in nature intermolecular and gas-molecular-wall collisions. In 1909, Knudsen defined the Knudsen number $K_n$ to differentiate different types of collisions:

$$K_n = \frac{\lambda}{L}$$

Here $\lambda$ is the average distance traveled by a gas molecule between successive collisions, i.e. mean free path. And $L$ is the pore’s characteristic length. As the $K_n$ increases, gas flow in a pore/tube changes from the continuum flow regime ($K_n < 0.001$), slip flow regime (0.001 $< K_n < 0.1$), transition flow regime (0.1 $< K_n < 10$) to free molecular flow (FMF, 10 $< K_n$) (Schaaf and Chambre, 1961). From continuum flow to FMF, the gas-molecular-wall collisions become more frequent than those between gas molecules. In macroscopic view, the gas velocity at boundaries gradually increases above zero and the gas flow inside pore bodies tend to have uniform velocity distribution.

Rarefied flow generally refers to the flow with $K_n$ bigger than 0.001. Considering Eq. 3.1, the value of Knudsen number $K_n$ depends on both the $\lambda$
and characteristic length. The mean free path $\lambda$ is defined as (Chapman et al., 1990; Struchtrup, 2005):

$$\lambda = \left( \frac{2}{\pi N/V d^2} \right)^{-1}, \quad \text{.........................................................}(3.2)$$

where $N$ is the number of molecules, $V$ is the volume occupied by molecules and $d$ is the molecular diameter. For low pressure gas flow, the molecular diameter can be related to the macroscopic gas viscosity (Bird et al. 1960) as the ideal gas and the corresponding mean free path becomes

$$\lambda = \frac{\mu}{P} \sqrt{\frac{\pi R_s T}{2}}. \quad \text{.........................................................}(3.3)$$

In Eq. 3.3 $P$ is the gas pressure, $T$ is the temperature and $R_s$ is specific gas constant. Generally the gas mean free path decreases with increasing gas pressure. At elevated pressure and temperature gas properties may deviate from ideal gas behavior. In 2011, Michel et al. corrected the mean free path $\lambda$ for real gas at high pressure and temperature as

$$\lambda = \frac{\mu}{P} \sqrt{\frac{\pi z R_s T}{2}} = \mu \sqrt{\frac{\pi}{2P \rho}}, \quad \text{.........................................................}(3.4)$$

The $z$ is gas compressibility factor. Fig. 3.1 shows the ratios of the mean free paths in Eq. 3.4 over Eq. 3.3 for methane when the gas pressure and temperature increase. The compressibility factor $z$ is calculated based on the methane bulk phase properties (NIST Chemistry WebBook). Fig. 3.1 further shows the calculated $K_n$ based on the mean free paths of Eqs. 3.3 and 3.4 for the methane flow in a tube with 10nm diameter. It demonstrates that the different
Figure 3.1-The ratio of mean free paths (data points) estimated in Eq. 3.4 over Eq. 3.3 for methane in varying pressure and temperature and the calculated Knudsen number (lines) based on Eqs. 3.4 and 3.3.
definitions of gas mean free paths lead to slightly different values of $K_n$ in the reservoir pressure and temperature.

### 3.1.2 Experiments and Simulations

The rarified gas flow starts with the study of gas flow in large tubes/channels under low-pressure or vacuum condition. Dong (1956) tested the rarefied flow rates of H$_2$, He, CO$_2$ and air in cylindrical (radius: 0.0364m and length: 3.2m) and rectangular (height $h$: 0.324cm, width $w$: 0.2286m and length $L$: 1.27m) cross-sections tubes. The inlet pressure was set in the range of 0.01-2000$\mu$Pa and the ratios of inlet over outlet pressure were measured as between 1.2 and 4.5. As the micro- and nano-technologies mature, more experimental studies begin to focus on the rarefied gas flow under higher pressure in microchannels—shorter mean free path as well as smaller channel sizes (Harley et al. 1995; Arkilic 1997; Zohar et al., 2002; Gad-el-Hak 2006; Yang and Garimella, 2009).

Harley et al. (1995) measured the flow dynamics of N$_2$, He and Ar in 0.01m-long ($L$), 100$\mu$m-width($w$) and 0.5-20$\mu$m depth($h$) channels with the inlet pressure up to 3.4MPa, the atmospheric outlet pressure and the maximum $K_n$ 0.4 at the outlet. Arkilic (1997) further tested the CO$_2$ flow for $K_n$ up to 0.44 in two 0.0075m-long ($L$), 1.33$\mu$m-depth ($h$) channels with width ($w$) as 52.3$\mu$m and 102.3$\mu$m, respectively. In Arkilic’s experiments, the inlet pressure is chosen from (135kPa, 430kPa) and the outlet pressure is in (13kPa, 100kPa). Zohar et al. measured the flow rates and pressure distribution for the gas flow in a channel with 0.004m length ($L$), 40$\mu$m width ($w$) and 0.53$\mu$m depth ($h$). The maximum inlet pressure was set as 0.5MPa. In 2009, Yang and Garimella measured the air flow...
dynamics through several capillary tubes with maximum length 0.095m \((L)\) and minimum radius 10\(\mu\text{m}\) \((r)\). The inlet pressure was the atmospheric pressure and the ratio of inlet to outlet pressure can reach 170. It is summarized that most experiments of rarefied gas flow are in long channels/tubes \((L>>h \text{ or } L>>r)\) with considerable pressure drop along the flow pathway.

Multiple numerical methods have been developed to explain mechanisms behind experiments in the literature. In general, the Boltzmann equation describes the thermodynamics of gas across flow regimes. Navier-Stokes (N-S) equations can be derived from the Boltzmann equation (Chapman and Cowling, 1991) and their solutions well describe the continuum gas flow behavior. In 1938, Kennard demonstrated that Computational Fluid Dynamics (CFD) simulations based on N-S equations can still be applied in slip flow regime when the Maxwell’s slip and von Smoluchowski’s temperature jump conditions were added. And Mobedi et al. (2016) applied CFD simulations in the modeling of gas slip flow in extracted or reconstructed porous media. In addition to N-S equations, Molecular Dynamics (MD) method (Cracknell et al., 1995; Binder et al. 2004; Cai et al., 2008) and Direct Simulation Monte Carlo (DSMC) method (Bird, 1963; Arkilic, 1997; Fang and Liou, 2002; White et al., 2012) were developed to solve Boltzmann equation and calculate flow dynamics in all the rarefied flow regimes for microchannels of simple geometries. In the past few decades, the Lattice Boltzmann method (LBM) appeared as another CFD method for microscale rarefied gas flow modeling based on discrete Boltzmann equations (Frisch et al. 1986; Ohwada et al. 1989; Loyalka and Hamoodi 1990;
Niu et al, 2007). Compared with conventional CFD simulations, the MD, DSMC and LBM simulations are always intensive in computation as well as time resources even for simple geometries. Currently it is quite difficult to directly apply these simulations in a complex multi-scale shale pore space.

Many analytical equations have been developed for the rarefied gas flow in a single tube/channel. Liu and Tai (1995) and Wang (1995) derived analytical solutions of N-S equations with Maxwell’s slip boundary condition for the 2D slip flow in long cylindrical tube and rectangular ducts with large aspect ratio, respectively. Duan and Muzychka (2007) further developed analytical solutions of N-S equations for 3D slip flow in rectangular, elliptical and annular ducts. For the Knudsen diffusion flux of FMF, Livesey (1998) summarized analytical equations of the gas transport in long and short ducts with various cross section shapes under low to vacuum pressure conditions. The simple expressions of Hagen–Poiseuille flow, slip flow and FMF flow in regular-shaped conduits inspired many researchers to create a unified, analytical model for the whole rarefied gas transport in regular-shaped conduits. Adzumi (1937), Mason and Malinauskas (1983) and Ertekin et al. (1986) proposed that the mass rate of rarefied gas flow through a cylindrical tube was a summary of the Hagen–Poiseuille flow equation and the Knudsen diffusion equation with constant weight coefficients. Thompson and Owens (1975) and Javadpour (2009) modeled the rarefied gas mass flow rate in cylindrical tubes as a summary of slip flow equations and Knudsen diffusion equations. The weight coefficients for slip flow and Knudsen diffusion equations were selected from (0, 1) based on low-
pressure experiments or numerical simulations. Karniadakis et al. (2005) tried to develop a unified flow model that could not only predict the mass flow rate but also the velocity profile vs. pressure drop along long channels, pipes, and arbitrary aspect ratio rectangular ducts in the entire rarefied flow regime. In Karniadakis et al.’s work, a general slip boundary condition was first proposed to match the velocity profile beyond the slip flow regime in a conduit. Then a rarefaction coefficient $C_r$ was added to match the flow rate in the transition and FMF regimes. The idea and equations of Karniadakis et al. were widely applied in the pore-scale modeling of gas transport in shale. In 2009, Civan utilized an inverse power-law expression for the $C_r$ in Karniadakis et al.’s equation and derived an apparent permeability equation for gas flow inside shale. In 2012, Sakhaee-Pour and Bryant directly applied Karniadakis et al.’s equations to calculate the apparent permeability of shale when the gas flow in shale is in transition flow regime. Wu et al (2015) coupled the unified model with Knudsen diffusion equation to describe the gas transport in rectangular-shaped microfractures of shale. The weight coefficients are selected based on varying ratios of intermolecular and molecule-wall collisions when the pore pressure changes.

3.1.3 Summary

In the literature, the Helium (He) and Nitrogen (N$_2$) gas apparent permeabilities were measured with the crushed shale rock pressure-decay or core plug pulse-decay methods. And empirical equations of apparent permeabilities vs. pressure with various coefficients have been applied to match
the experimental data (Ghanizadeh et al. 2015; Kazemi and Takte-Borujeni 2015; Zhu et al. 2007). But He and N₂ permeabilities cannot represent the apparent permeability for the methane flow in shale reservoirs because of different molecular structure and pressure conditions. Moreover, the physics behind empirical permeability equations need yet to be uncovered.

Gas flow simulations can reveal the constitutive relationship among shale permeability, pore structure and gas properties. Shale rocks have pore sizes from μm down to nm. In addition, pores in shale may have subspherical, triangular or slit-like morphologies. The complex pore structure makes the gas flow behavior more complex. Under same reservoir pressure, gas flow in μm-scale pores/throats may stay in continuum flow ($K_n < 0.001$) while the flow in nm-scale pores/throats is in transition flow ($0.1 < K_n < 10$). It is difficult to find direct simulation methods that are applicable to the entire range of Kn. When it is difficult to run direct simulations on pore space, an alternative way is to combine the modeling/equations of rarefied gas flow in a single conduit with multi-scale shale pore networks. And limited works have been completed to model gas flow on multi-scale pore networks and summarize the apparent permeability behavior vs. multi-scale pore structure in shale.

In my opinion, analytical equations for rarefied gas flow in simple conduits need improvement before their application in estimating shale apparent permeability. First of all, most of unified models that for low-pressure rarefied gas flow experiments and/or simulations should be validated for the high-pressure rarefied gas flow in nm- or μm--scale conduits. The low-pressure
rarefied flow in long conduits results in a pressure drop that is comparable with inlet/outlet pressure. Many equations and coefficients in the literature were based on ideal gas properties at average pressure of inlet and outlet. But for the rarefied gas flow in a single pore/throat, the pressure difference between the inlet and outlet can be negligible compared with the reservoir pressure. And real gas properties, gas mean free path ($\lambda$) and the Knudsen number $K_n$, hardly change along one pore/throat. Secondly, most of the unified models are only developed cylindrical tubes/pipes and ducts. No unified models or coefficients have been developed for conduits with triangular cross-sections while abundant pores/throats of shale have elongated wedges or triangular shapes, especially in clay-rich shales. Moreover, the workflow of Karniadakis et al.’s approach can be improved by introducing correct slip flow equations of gas flow in regular conduits.

This study aims to develop a unified model for high-pressure rarefied methane flow in a short single nano- or micro-conduit with circular, equilateral triangular and rectangular cross sections for the entire range of $K_n$, then apply the unified model on all pores/throats of shale multi-scale pore networks and finally calculate the shale apparent permeabilities.

### 3.2 Modeling of High-Pressure Rarefied Gas Flow in a Single Conduit

In this study the conduits with uniform circular, triangular and rectangular cross sections are considered. Methane $\text{CH}_4$ flows through a conduit at constant
temperature. The first mission is to derive a unified model for the high-pressure gas flow in a single conduit. The derivation of a unified model for high-pressure rarefied gas flow in a single conduit follows the Karniadakis et al.’s procedure.

3.2.1 The Unified Model for a Cylindrical Conduit

Karniadakis et al. (2005) stated that N-S equations with a general slip boundary condition can still be applied as a basis for the unified model derivation. For a finite-length conduit, the gas flow always results in small pressure drop $\Delta P$ along the pathway. Therefore the gas properties can be considered as constant along the conduit. The N-S equations for isothermal, steady-state, incompressible fluid flow in a conduit with the uniform cross-section are

$$\nabla \cdot \mathbf{v} = 0,$$

$$\nabla \cdot (\mathbf{v} \mathbf{v}) + \nabla P - \nabla \cdot (\mu (\nabla \mathbf{v} + (\nabla \mathbf{v})^T)) = 0.$$ (3.5a) (3.5b)

Here $\mathbf{v}=(u,v,w)$ is the velocity field in $x$-, $y$- and $z$-directions, respectively. $P$ is the pressure and $\mu$ is the dynamic viscosity. Consider a circular conduit with radius $r$ and axial length $L$ in the $z$-direction in Fig. 3.2. The transverse velocity components $u$ and $v$ are zero and Eqs. 3.5a and 3.5b are reduced as
Figure 3.2 - The cylindrical conduit with radius $r$ and length $L$ (Original in color).
At the boundary \( x^2 + y^2 = r^2 \), the Maxwell slip boundary condition (first-order slip boundary condition) can be applied

\[
\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = \frac{1}{\mu} \frac{dP}{dz}, \quad -r \leq x \leq r, -r \leq y \leq r
\] (3.6)

where \( w_s \) is the gas velocity at the wall and \( w_w \) is the wall velocity (\( w_w = 0 \) for stationary walls). Here \( n \) is the outward normal direction to the wall. The tangential accommodation coefficient \( \sigma_v \) represents the tangential momentum exchange of gas molecules with walls. The case pf \( \sigma_v = 0 \) means specular reflection of gas molecules while the case \( \sigma_v = 1 \) is the diffusive reflection.

Karniadakis et al. (2005) proposed a general slip boundary condition:

\[
w_s - w_w = \frac{2 - \sigma_v}{\sigma_v} \lambda \frac{\partial w}{\partial n} \frac{2K_n}{1 - 2\beta K_n} \theta(n/r)
\] (3.8)

The Knudsen number \( K_n \) for the gas flow inside is defined as \( K_n = \frac{\lambda}{2r} = \frac{\mu}{2r} \sqrt{\frac{\pi}{2\rho}} \).

The coefficient \( \beta \) is to be determined with varying \( K_n \) and conduit shape. When the pressure drop \( \frac{dP}{dz} \) is considered as a known variable, the solution of Eqs. 3.6 and 3.8 for a cylindrical conduit is

\[
w = -\frac{r^2}{4\mu} \frac{dP}{dz} \left( 1 - \frac{x^2 + y^2}{r^2} \right) \left( 1 - 2\frac{\sigma_v}{\sigma_v - 2K_n} \frac{2K_n}{1 - 2\beta K_n} \right).
\] (3.9)

The term \( 1 - \frac{x^2 + y^2}{r^2} \) describes the parabolic distribution of velocities in radial direction in the entire range of \( K_n \). The \( 2\frac{\sigma_v}{\sigma_v - 2K_n} \frac{2K_n}{1 - 2\beta K_n} \) term can be regarded as the enhancement of velocity values in the rarefied flow. It is concluded that in a cylindrical conduit, the analytical solution follows the idea that the rarefied flow
does not change the velocity distribution but increases velocities. Karniadakis et al. (2005) extended this idea from the cylindrical conduit to the rectangular conduit in the derivation of unified models. I will test the idea’s applicability in conduits with other kinds of cross-sections in Sections 3.2.2 and 3.2.3.

According to Eq. 3.9, the volumetric rate \( Q \) out of the cylindrical conduit is

\[
Q = -\frac{\pi r^4}{4\mu} \frac{dP}{dz} \left( \frac{1}{2} + 2 \frac{\sigma_v}{\sigma_v} \frac{2Kn}{1 - 2\beta Kn} \right) \tag{3.10}
\]

Although Eq. 3.10 with constant coefficient \( \beta \) works well in the slip flow regime, it gives a lower estimation of volumetric flow rates in transition and free molecular flow regimes. Karniadakis et al. (2005) proposed a unified equation based on Eq. 3.10 for rarefied flow in slip, transition and FMF regimes:

\[
Q = -\frac{\pi r^4}{4\mu} \frac{dP}{dz} \left( \frac{1}{2} + 2 \frac{\sigma_v}{\sigma_v} \frac{2Kn}{1 - 2\beta Kn} \right) (1 + \alpha(\overline{K_n})) \tag{3.11}
\]

Here \( \overline{K_n} \) is evaluated at the average value \( \overline{P} \) of inlet \( (P_{in}) \) and outlet pressure \( (P_{out}) \). The coefficient \( \alpha \) increases with bigger \( \overline{K_n} \) values. Karniadakis et al. (2005) determined \( \alpha \) with fitting numerical simulations and/or experiments. Those numerical simulations and experiments focused on the low-pressure rarefied gas flow in long, large cylindrical conduits rather than high-pressure rarefied flow in nm- or \( \mu \)m-scale cylindrical conduits. For low-pressure gas flow in long conduits, the pressure drop along the conduit is comparable with inlet/outlet pressure and the \( K_n \) could show a change of one order of magnitude. It is summarized that the \( \alpha \) values in Karniadakis et al.’s work is strongly related by the \( K_n \) distribution along the long cylindrical conduit. Therefore the \( \alpha \) in Karniadakis et al.’s work cannot represent the case with constant \( K_n \) along a conduit. The gas flow inside
pores of shale is high-pressure rarefied gas flow. The rarefied gas flow through a single throat results in a pressure drop of mPa, which can be negligible when compared with reservoir pore pressure of MPa. Therefore the $K_n$ can be regarded as constant along the throat.

In this study, the volumetric rate out of a cylindrical conduit in high-pressure rarefied flow is assumed as

$$Q = -\frac{\pi r^4}{4\mu} \frac{dP}{dz} \left(\frac{1}{2} + 2\frac{2-Kn}{\sigma_v} \frac{2K_n}{1-2\beta K_n}\right) \alpha(K_n) \quad (3.12)$$

The $\sigma_v$ is set as 1 for the diffusive reflection of gas molecules. The $\alpha$ and $\beta$ are to be determined based on the curve-fitting with numerical simulations. Table 3.1 and Fig. 3.3 give the cylindrical conduit sizes and gas properties in numerical simulations. Two types of cylindrical conduits with different radii are considered. The thermophysical properties of methane are based on methane bulk phase properties in the NIST Chemistry WebBook. In the numerical modeling of slip flow, N-S equations with Maxwell slip boundary condition are solved with finite element method. The Boltzmann equation with Bhatnagar-Gross-Krook (BGK) collision models are solved with lattice Boltzmann method for the numerical modeling of transition flow. The angular coefficient method is applied in the numerical modeling of FMF in the conduit. All these numerical simulations are completed in the software COMSOL. And the capability of COMSOL in rarefied flow modeling is validated by Dong’s experimental data (1956) and the Sturnfield’s work (Sturnfield, 2015).
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<table>
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<tr>
<td><strong>Table 3. 1</strong> The cylindrical conduit sizes and gas properties.</td>
<td></td>
</tr>
<tr>
<td><strong>Radius</strong> $r$, nm</td>
<td>5, 50</td>
</tr>
<tr>
<td><strong>Length</strong> $L$, nm</td>
<td>60</td>
</tr>
<tr>
<td><strong>Maximum</strong> $P_{out}$, MPa</td>
<td>70</td>
</tr>
<tr>
<td><strong>Minimum</strong> $P_{out}$, MPa</td>
<td>0.1</td>
</tr>
<tr>
<td>$P_{in}$</td>
<td>$P_{out}$+0.002Pa</td>
</tr>
<tr>
<td><strong>$\Delta P$, Pa</strong></td>
<td>0.002</td>
</tr>
<tr>
<td><strong>Temperature</strong> $T$, K</td>
<td>338</td>
</tr>
<tr>
<td><strong>Molar Mass</strong> $M_n$, g/mol</td>
<td>16.04</td>
</tr>
</tbody>
</table>
Figure 3. The methane density (solid line) and viscosity (dash line) change with pressure in the range of (100kPa, 70000kPa) at constant temperature T=338K.
There are multiple ways to obtain $\alpha$ and $\beta$ by fitting numerical simulations. This study sets $\beta$ as a constant and gets the best $\beta$ value and curves of $\alpha$ vs. $K_n$ based on numerical simulations. Another way is to set $\alpha$ as unit and obtain curves of $\beta$ vs. $K_n$, which works the same as N-S equations with higher-order slip boundary condition for high-$K_n$ flow. This method of determining $\alpha$ and $\beta$ will be completed in a future study.

Fig. 3.4a shows the volumetric flow rates $Q$ out of the $r=5$nm cylindrical conduit calculated by numerical simulations, Eq. 3.12 with $\alpha=1$ and $\beta=0$, and Eq. 3.12 with $\alpha=1$ and $\beta=-1$. The outlet pressure decreases from 70MPa to 0.1MPa, which corresponds to the $K_n$ range (0.03, 7.4). It is concluded that the flow of natural gas inside pores with diameter larger than 10nm is in the slip and transition flow regimes under reservoir pressure condition. If the effect of $\alpha$ is absent ($\alpha=1$), Eq. 3.12 with $\beta=0$ predicts the increasing trend but lower values of $Q$ when $K_n>0.2$ while Eq. 3.12 with $\beta=-1$ approaches a upper limit which is much smaller than numerical simulations. Corresponding to Fig. 3.4a, Fig. 3.4b displays the mass flow rates out of the conduit. The Knudsen minimum phenomenon is well captured by the numerical simulations. The minimum mass flow rate is $2.5\times10^{-23}$ kg/s and the corresponding $K_n$ is approximately 0.8. According to numerical simulations, the gas mass rate (or volumetric rate in standard condition) out of the conduit increases when the pressure decreases from 70MPa ($K_n=0.032$) to 40MPa ($K_n=0.035$). Then the gas mass rate decreases if the pressure continues to decreases towards 1MPa ($K_n=0.75$). Afterwards, the gas mass rate would increase slowly towards the FMF mass rate.
Volumetric flow rate, m³/s

Mass flow rate, kg/s

(a) $r=5\text{nm}$, Eq. 3.12 with $\alpha=1$, $\beta=-1$

(b) $r=5\text{nm}$, Eq. 3.12 with $\alpha=1$, $\beta=0$

Numerical simulations
Figure 3. 4-\(a\) The volumetric flow rate, \(b\) The mass flow rate, \(c\) normalized flow rate distribution calculated in numerical simulations, analytical equation with \(\alpha=1\) and \(\beta=-1\), analytical equation with \(\alpha=1\) and \(\beta=0\) for the conduit with radius \(r=5\text{nm}\) (original in color).
limit even the pressure keeps decreasing. **Fig. 3.4c** shows the normalized velocity in the radial direction at $L=30\text{nm}$. The normalized velocity is defined as

$$
\bar{w} = \frac{w(x,y)}{w_{\text{avg}}} = \frac{w(x,y)}{\left(\frac{1}{4} \int_{-r}^{r} \int_{r-x}^{r+x} w(x,y) \, dx \, dy\right) / \pi r^2} = \frac{1}{\frac{1}{2} + \frac{1}{2} \frac{2 - \sigma_v}{\sigma_v} \frac{2Kn}{1 - 2\beta Kn}} \cdot \frac{\frac{x^2 + y^2}{r^2} + 2 \frac{2 - \sigma_v}{\sigma_v} \frac{2Kn}{1 - 2\beta Kn}}{\frac{1}{2} + \frac{1}{2} \frac{2 - \sigma_v}{\sigma_v} \frac{2Kn}{1 - 2\beta Kn}}. \quad (3.13)
$$

In **Fig. 3.4c**, the velocity at the conduit center is much higher than the velocity at boundaries when $K_n=0.03$. When the $K_n$ becomes larger than 1, the velocity distribution tends to be homogeneous. **Fig. 3.4c** shows that in the slip flow regime, both $\beta=0$ and $\beta=-1$ can match the velocity distribution in numerical simulations. When $K_n$ becomes much larger than 1, the $\beta=-1$ (i.e. $\beta=-1$) in **Eq. 3.13** approaches to 1 and the gas flow would have a constant velocity distribution in high $K_n$, which deviates from the uniform velocity distribution in high-$K_n$ flow. Generally **Eq. 3.13** with $\beta=0$ provides a better match with numerical simulations than $\beta=-1$. Therefore in this study $\beta$ is set as 0 for a cylindrical conduit.

The $\alpha(K_n)$ is determined by matching numerical simulations with **Eq. 3.12**. **Fig. 3.5** shows the variation of $\alpha$ as a function of the $K_n$ for the conduit with $r=5\text{nm}$. To validate the applicability of $\alpha$ on cylindrical conduits with different radii, **Fig. 3.5** also displays the $\alpha$ vs. $K_n$ for a conduit with $r=50\text{nm}$. Since **Eq. 3.12** is valid for the slip flow regime, the value of $\alpha$ is 1 when $K_n < 0.1$ for cylindrical conduits with different radii. It can be inferred that $\alpha$ will converge to a constant when $Kn >> 1$. The relationship of $\alpha$ vs. $K_n$ for a cylindrical conduit regardless of its size can be represented as
\[
\alpha = \begin{cases} 
\alpha_0 \frac{2}{\pi} \arctan[\alpha_1 (K_n - 0.1)^{\alpha_2}] + 1, & K_n \geq 0.1 \\
1, & K_n < 0.1 
\end{cases} \tag{3.14}
\]

where \(\alpha_0 = 1.22\), \(\alpha_1 = 1.4\) and \(\alpha_2 = 1.0\).
Figure 3. 5-Variation of $\alpha$ as a function of $Kn$ for conduits with $r=5\text{nm}$ (diamonds) and 50nm (circles) and the fit to this variation based on Eq. 3.14 (dash line).
3.2.2 The Unified Model for a Triangular Conduit

Many pores/throats in shales have elongated wedges or triangular shapes, especially in clay-rich shales. **Fig. 3.6** shows a equilateral triangular conduit with side length $a$ and length $L$. The pressure, temperature and gas properties are same as in Section 3.2.1 unless otherwise stated. The N-S equation can be reduced as

$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = \frac{1}{\mu} \frac{dP}{dz}. \hspace{1cm} (3.15)$$

At the bottom boundary $x = \frac{a}{2\sqrt{3}}$, the general slip boundary can be applied

$$w_x - w_y = \frac{2 - \sigma_v}{\sigma_v} \frac{2K_n}{1 - 2\beta K_n} \frac{\partial w}{\partial (n/2)} \frac{\partial}{\partial (\frac{a}{2\sqrt{3}})}. \hspace{1cm} (3.16)$$

Here $K_n$ is defined as $K_n = \frac{\sqrt{3} \lambda}{a} = \frac{\sqrt{3} \mu}{a} \sqrt{\frac{\pi}{2P \rho}}$. The $\sigma_v$ can be set as 1 for the diffusive reflection of gas molecules. The other two sides have the same boundary condition as **Eq. 3.16**. The solution of **Eq. 3.15** is

$$w = -\frac{a^2}{144\mu} \frac{dP}{dz} \left( -3x^2 - 3y^2 - \frac{x^3 - 3xy^2}{1 + \frac{2 - \sigma_v}{\sigma_v} \frac{2K_n}{1 - 2\beta K_n}} + f(\beta, K_n) + 4 \right),$$

$$f(\beta, K_n) = \frac{2 - \sigma_v}{\sigma_v} \frac{12K_n}{1 - 2\beta K_n} + \frac{2 - \sigma_v}{\sigma_v} \frac{4K_n}{1 + \frac{2 - \sigma_v}{\sigma_v} \frac{2K_n}{1 - 2\beta K_n}} \frac{2 - \sigma_v}{\sigma_v} \frac{2K_n}{1 - 2\beta K_n}. \hspace{1cm} (3.17)$$

Following the idea in Section 3.2.1, the unified equation for a triangular conduit is

$$w = -\frac{a^2}{144\mu} \frac{dP}{dz} \left( -3x^2 - 3y^2 - \frac{x^3 - 3xy^2}{1 + \frac{2 - \sigma_v}{\sigma_v} \frac{2K_n}{1 - 2\beta K_n}} + f(\beta, K_n) + 4 \right) \alpha(K_n). \hspace{1cm} (3.18)$$

The volumetric flow rate out of the conduit is

$$Q = \frac{a^4}{144\mu} \frac{dP}{dz} g(\beta, K_n) \alpha(K_n),$$

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Figure 3. 6-The triangular conduit with side length $a$ and length $L$ (Original in color).
Accordingly the normalized velocity $\bar{w}$ is defined as
\[
\bar{w} = \frac{w(x,y)}{w_{avg}} = \frac{-\sqrt{3} \left( -3x^2 - 3y^2 - \frac{x^2 - 3xy^2}{1 - \frac{2 - \sigma_v}{1 - 2\beta K_n}} + f(\beta, K_n) + 4 \right)}{4g(\beta, K_n)}. \quad \text{............................(3.20)}
\]

The $\beta$ and $\alpha$ can be determined based on fitting with numerical simulations.

**Figs. 3.7a** shows the volumetric flow rates $Q$ out of a triangular conduit ($a=16$nm, $L=60$nm) calculated by numerical simulations, **Eq. 3.19** with $\alpha=1$ and $\beta=0$, and **Eq. 3.19** with $\alpha=1$ and $\beta=-1$. In terms of flow rates, the relationship among numerical simulations and analytical equations with varying $\alpha$ and $\beta$ for triangular conduits is similar to that of cylindrical conduits. The choice of $\beta=0$ makes the analytical calculation in Eq. 3.19 same trend but smaller values when compared with numerical simulations beyond the slip flow regime. When $\beta$ equals to -1, the volumetric flow rates of Eq. 3.19 will converge to a constant value. **Fig. 3.7b** gives the mass flow rates calculated by numerical simulations, **Eq. 3.19** with $\alpha=1$ and $\beta=0$, and **Eq. 3.19** with $\alpha=1$ and $\beta=-1$. The Knudsen minimum occurs around $K_n=0.6$ in numerical simulations. When Kn is larger than 1, the mass rate increases slowly towards the FMF mass rate limit. The mass flow rate of $\beta=0$ converges to a constant value while that with $\beta=-1$ keeps decreasing when the $K_n$ increases. **Fig. 3.7c** displays the normalized velocity in the $-x$-direction from the bottom towards the vertex of a triangular cross-section calculated by numerical simulations, **Eq. 3.20** with $\beta=0$, and **Eq. 3.20** with $\beta=-1$. Numerical
Volumetric flow rate, m$^3$/s

Numerical simulations
Eq. 3.19 with alpha=1, beta=-1
Eq. 3.19 with alpha=1, beta=0

Mass flow rate, kg/s

Numerical simulations
Eq. 3.19 with alpha=1, beta=-1
Eq. 3.19 with alpha=1, beta=0
(c) **Figure 3. 7-(a)** The volumetric flow rate, (b) The mass flow rate, (c) normalized flow rate distribution (from bottom towards the vertex in \(x\)-direction) calculated in Eq. 3.20 with \(\alpha=1\) and \(\beta=-1\), Eq. 3.20 with \(\alpha=1\) and \(\beta=0\) and in numerical simulations for the triangular conduit with \(a=16\text{nm}\) (original in color).
simulations demonstrate that the velocity distribution in the triangular cross-section is not centrosymmetric. Thus the simple addition of a velocity enhancement term in a non-slip solution (i.e. Eq. 3.9) cannot describe the velocity profile in a conduit with triangular cross-sections. Numerical simulations in Fig. 3.7c also shows that the gas around cross-section vertexes become mobile with higher $K_n$ number, which contributes to a higher volumetric flow rate in rarefied flow than in the no-slip (continuum) flow. Fig. 3.7c further validates that analytical solutions with $\beta=0$ give better match with numerical simulations in terms of velocity distribution in the entire range of $K_n$. Therefore in this study $\beta$ is set as 0 for a equilateral triangular conduit.

The $\alpha(K_n)$ is determined by matching numerical simulations with Eq. 3.19. Fig. 3.8 shows the variation of $\alpha$ as a function of the $K_n$ for the triangular conduit. Since Eq. 3.19 is accurate for the slip flow regime, the value of $\alpha$ is 1 when $K_n <0.1$. It can be inferred that $\alpha$ will converge to a constant when $Kn >>1$. The relationship of $\alpha$ vs. $K_n$ for a equilateral triangular conduit regardless of its size can still be represented with Eq. 3.14 when $\alpha_0=2.05$, $\alpha_1=0.63$ and $\alpha_2=1.0$.  


Figure 3.8-Variation of $\alpha$ as a function of $K_n$ for triangular conduits (diamonds) and the fit to this variation based on Eq. 3.20 (dash line).
3.2.3 The Unified Model for a Rectangular Conduit

Rectangular conduits can resemble the nm-scale pores/throats with multiple edges. In addition, many shale rocks has μm-scale pore space with rectangular or slit-like morphology that can be regarded as microfractures. Those microfractures can also be well described by rectangular conduits. **Fig. 3.9** shows a conduit with the rectangular cross-section (a: width; b: height/depth; L: length). The pressure, temperature and gas properties are same as in Section 3.2.1 unless otherwise stated. The N-S equation for the steady-state gas flow in a rectangular conduit under high pressure can be reduced as

\[ \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = \frac{1}{\mu} \frac{dP}{dz} \] ..........................................................(3.21a)

with general slip boundary conditions as

\[ W_s - W_w = \frac{2 - \sigma_v}{\sigma_v} \frac{K_n}{1 - \beta K_n} \frac{D_h}{a/2} \frac{\partial w}{\partial (n/2)} \left. \right|_x = \pm \frac{a}{2} \] ..........................................................(3.21b)

\[ W_s - W_w = \frac{2 - \sigma_v}{\sigma_v} \frac{K_n}{1 - \beta K_n} \frac{D_h}{b/2} \frac{\partial w}{\partial (n/2)} \left. \right|_y = \pm \frac{b}{2} \] ..........................................................(3.21c)

\[ \frac{b}{a} = \varepsilon. \] ..........................................................(3.21d)

Here \( K_n \) is defined as \( K_n = \frac{\lambda}{D_h} = \frac{\mu}{D_h} \sqrt{\frac{\pi}{2Pp}} \) and \( D_h \) is the hydraulic diameter \( D_h = \frac{2b}{1+\varepsilon} \). The \( \sigma_v \) can be set as 1 for the diffusive reflection of gas molecules. The analytical solution of **Eq. 3.21** is
Figure 3.9: The rectangular conduit with width $a$, height $b$ and length $L$ (Original in color).
Following the procedure in Sections 3.2.1 and 3.2.2, the unified equation for a triangular conduit is

\[ w = -\frac{b^2 dP}{4\mu dz} G(x, y, K_n, \beta, \varepsilon), \]

\[ G(x, y, K_n, \beta, \varepsilon) = \sum_{i=1}^{\infty} \frac{\sin(\delta_i) \cos(\delta_i \varepsilon)}{\delta_i^2 (\delta_i + \sin \delta_i \cos \delta_i)} \left( 1 - \frac{\cosh(\delta_i \varepsilon)}{\cosh(\delta_i \varepsilon)} \right)^{-4} \frac{4 - \sigma_v}{2 - \sigma_v} \left( \frac{4}{1+\varepsilon} \frac{4}{1-\beta K_n} \right). \]

\[ \delta_i \tan \delta_i = \frac{4 \sigma_v}{1-\beta K_n} \frac{1-\beta K_n}{K_n}. \]  

..(3.22)

The volumetric flow rate out of the rectangular conduit is

\[ w = -\frac{b^2 dP}{4\mu dz} G(x, y, K_n, \beta, \varepsilon) \alpha(K_n). \]  

..(3.23)

The volumetric flow rate out of the rectangular conduit is

\[ Q = -\frac{ab^2 dP}{4\mu dz} F(K_n, \beta, \varepsilon) \alpha(K_n), \]

\[ F(K_n, \beta, \varepsilon) = \sum_{i=1}^{\infty} \frac{\varepsilon \sin^2(\delta_i)}{\delta_i^4 (\delta_i + \sin \delta_i \cos \delta_i)} \left( \frac{\delta_i \varepsilon}{\sin(\delta_i \varepsilon)} \frac{4 - \sigma_v}{2 - \sigma_v} \frac{4}{1+\varepsilon} \frac{4}{1-\beta K_n} \right). \]  

..(3.24)

The normalized velocity \( \bar{w} \) in the rectangular cross-section is defined as the ratio of velocity over the average velocity:

\[ \bar{w} = \frac{w}{w_{avg}} = \frac{G(x, y, K_n, \beta, \varepsilon)}{F(K_n, \beta, \varepsilon)}. \]  

..(3.25)

The \( \beta \) and \( \alpha \) can still be determined based on fitting with numerical simulations. **Fig. 3.10** shows the volumetric flow rates of four rectangular conduits with length \( L=60\text{nm} \), cross-section area \( 100\text{nm}^2 \) and aspect ratios \( \varepsilon \) (1, \( \frac{1}{2}, \frac{1}{3}, \frac{1}{4} \)) calculated by numerical simulations, Eq. 3.24 with \( \alpha=1 \) and \( \beta=0 \), and Eq. 3.24 with \( \alpha=1 \) and \( \beta=-1 \). In the entire range of \( K_n \) numbers, the relationship among volumetric flow rates out of rectangular conduits calculated by numerical simulations and analytical equations follow same behaviors of cylindrical and
Figure 3. 10-The volumetric flow rates out of rectangular conduits with length $L=60\text{nm}$, cross-section area $100\text{nm}^2$ and aspect ratios ($1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}$) calculated by numerical simulations (solid lines), Eq. 3.24 with $\alpha=1$ and $\beta=0$ (dash lines), and Eq. 3.24 with $\alpha=1$ and $\beta=-1$ (dot dash lines) (Original in color).
triangular conduits. For rectangular conduits with constant cross-section area, the smaller aspect ratio \( \varepsilon \) results in lower conduit conductivities and volumetric flow rates under same pressure gradient \( \frac{dP}{dz} \) and Knudsen number \( K_n \). In Fig. 3.10, two set of flow rates are selected (marked with red dot dash lines) from conduits with different aspect ratios. The first set of flow rates correspond to rectangular conduits with aspect ratio 1, \( \frac{1}{2} \), \( \frac{1}{3} \), \( \frac{1}{4} \) under same pressure \( P=20\text{MPa} \) from upper left towards lower right direction. The second set of flow rates correspond to rectangular conduits with aspect ratio 1, \( \frac{1}{2} \), \( \frac{1}{3} \), \( \frac{1}{4} \) under same pressure \( P=0.3\text{MPa} \) from the left to right direction. First set of rates data validate that rectangular conduits with larger aspect ratios \( \varepsilon \) have higher conductivity for gas flow than those with small aspect ratios in the slip flow under the same pressure and same cross-section area. But as shown by the second set of rates, conduits with same cross-section areas under same gas pressure would have same conductivity regardless of the aspect ratios \( \varepsilon \) when the gas flow is in transition flow regime. It can be concluded that the rarefied flow mechanism (with high \( K_n \)) tends to eliminate discrepancies of conductivities among rectangular conduits with different aspect ratios.

Fig. 3.11 shows the mass flow rates of rectangular conduits with same cross-section area and different aspect ratios \( \varepsilon \) calculated with numerical simulations and analytical equations. The analytical equation Eq. 3.24 with \( \alpha=1 \) predicts the lower mass rates than numerical simulations. Furthermore, Eq. 3.24 with \( \alpha=1 \) and \( \beta=-1 \) predicts lower mass rates than that with \( \alpha=1 \) and \( \beta=0 \). The Knudsen minimum phenomenon can be observed at approximately \( K_n=0.3 \) for
Figure 3. 11-The mass flow rates out of rectangular conduits with length $L=60\text{nm}$, cross-section area $100\text{nm}^2$ and aspect ratios $(1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4})$ calculated by numerical simulations (solid lines), Eq. 3.24 with $\alpha=1$ and $\beta=0$ (dash lines), and Eq. 3.24 with $\alpha=1$ and $\beta=-1$ (dot dash lines) (Original in color).
rectangular conduits with various aspect ratios $\varepsilon$. Similar to Fig. 3.10, mass flow rates of conduits with high $\varepsilon$ in Fig. 3.11 can be higher than those with low $\varepsilon$ in the slip flow and early-stage transition flow regimes under same gas pressure. But the mass rates difference resulted from aspect ratios become negligible in the late-stage transition flow regime.

Fig. 3.12 shows the normalized velocity distribution along the distance from the center point towards the lower-left vertex (green dash line in Fig. 3.9) in cross-sections with aspect ratio 1 (Fig. 3.12a), $\frac{1}{2}$ (Fig. 3.12b), $\frac{1}{3}$ (Fig. 3.12c), and $\frac{1}{4}$ (Fig. 3.12d). When the pressure or $K_n$ is constant, the velocity distribution along the distance in numerical simulations becomes closer to a parabolic shape when the aspect ratio $\varepsilon$ decreases from 1 to $\frac{1}{4}$. For a conduit with fixed aspect ratio $\varepsilon$, the velocity distribution tends to be uniform along the distance in the late-stage transition flow regime. In the slip flow regime, conduits with different aspect ratios $\varepsilon$ have similar normalized slip velocities at the vertex under same gas pressure. But the difference between velocities at center and vertex locations becomes larger when the aspect ratio $\varepsilon$ increases. In addition, Figs. 3.12a to d demonstrate that either $\beta=0$ or $\beta=-1$ predicts velocity distribution that match well with numerical simulation results in the slip flow regime. For the transition flow regime, neither $\beta=0$ nor $\beta=-1$ gives good match with numerical simulations. But velocity profiles of analytical cases with $\beta=0$ are closer to numerical results for the four kinds of rectangular conduits. It is recommended that $\beta=0$ should be chosen in Eq. 3.25, which is consistent with cylindrical and triangular conduits.
Aspect ratio 1, $P=70\text{MPa}$, $Kn=0.03$ in Eq.3.25 with $\beta=0$

Aspect ratio 1, $P=70\text{MPa}$, $Kn=0.03$ in numerical simulations

Aspect ratio 1, $P=7\text{MPa}$, $Kn=0.1$ in Eq.3.25 with $\beta=0$

Aspect ratio 1, $P=7\text{MPa}$, $Kn=0.1$ in Eq.3.25 with $\beta=-1$

Aspect ratio 1, $P=7\text{MPa}$, $Kn=0.1$ in numerical simulations

Aspect ratio 1, $P=0.3\text{MPa}$, $Kn=2.2$ in Eq.3.25 with $\beta=0$

Aspect ratio 1, $P=0.3\text{MPa}$, $Kn=2.2$ in Eq.3.25 with $\beta=-1$

Aspect ratio 1, $P=0.3\text{MPa}$, $Kn=2.2$ in numerical simulations

(a)
(b)
Aspect ratio 1/3, $P=70\text{MPa}$, $Kn=0.037$ in Eq.3.25 with $\beta=0$

Aspect ratio 1/3, $P=70\text{MPa}$, $Kn=0.037$ in numerical simulations

Aspect ratio 1/3, $P=7\text{MPa}$, $Kn=0.11$ in Eq.3.25 with $\beta=0$

Aspect ratio 1/3, $P=7\text{MPa}$, $Kn=0.11$ in numerical simulations

Aspect ratio 1/3, $P=0.3\text{MPa}$, $Kn=2.5$ in Eq.3.25 with $\beta=0$

Aspect ratio 1/3, $P=0.3\text{MPa}$, $Kn=2.5$ in numerical simulations
Figure 3. 12-The normalized velocity distribution along the distance from the center point towards the lower-left vertex in the rectangular cross-section with aspect ratios $\varepsilon$ as (a) 1 (b) $1/2$, (c) $1/3$, and (d) $1/4$ (Original in color).
The \( \alpha(K_n) \) is determined by matching numerical simulations with Eqs. 3.14 and 3.24. Fig. 3.13 shows the variation of \( \alpha \) as the function of the \( K_n \) for rectangular conduits with different aspect ratios based on fitting with numerical simulations (data points in the figure). Since Eq. 3.24 is accurate for the slip flow regime, the value of \( \alpha \) is 1 when \( K_n < 0.1 \). It can be inferred that \( \alpha \) will converge to a constant when \( Kn >> 1 \) for a rectangular conduit. The relationships of \( \alpha \) vs. \( K_n \) for rectangular conduits can be represented with Eq. 3.14 (lines in Fig. 3.13) when appropriate \( \alpha_0 \), \( \alpha_1 \), and \( \alpha_2 \) are chosen. Table 3.2 summarizes the parameters of Eq. 3.14 for rectangular conduits with aspect ratios 1 to \( 1/10 \). Table 3.2 further includes the parameters for cylindrical and triangular conduits. Fig. 3.13 and Table 3.2 show that the \( \alpha_0 \) for rectangular conduits increases while the \( \alpha_2 \) decreases when the aspect ratio \( \varepsilon \) of a rectangular conduit decreases from 1 to \( 1/10 \). Fig. 3.14 further shows the trends of \( \alpha_0 \) and \( \alpha_2 \) for those rectangular conduits. A linear correlation can be inferred for \( \alpha_0 \) vs. \( 1/\varepsilon \) and a power-law relationship can be obtained for \( \alpha_2 \) vs. \( 1/\varepsilon \). The \( \alpha_0 \) is restricted by the FMF rates for conduits with different aspect ratios \( \varepsilon \). When the aspect ratio \( \varepsilon \) becomes smaller, the velocity distribution in the rectangular conduit approaches the condition in a 2D channel. Accordingly the \( \alpha_2 \) tends to be a constant for rectangular conduits with large width/height ratios.

In this study, three types of cross-section shapes are considered, i.e., circular, triangular and rectangular. The \( \alpha_0 \) is related to the conduit’s FMF rate limit. The \( \alpha_1 \) and \( \alpha_2 \) strongly depend on the cross-section shapes. According to
Figure 3. Variations of $\alpha$ as a function of $K_n$ for rectangular conduits with various aspect ratios based on numerical simulations (data points) and the fit to the variations based on Eq. 3.14 (lines).
Table 3.2 Parameters in Eq. 3.14 for cylindrical conduits, equilateral triangular conduits and rectangular conduits with various aspect ratios $\varepsilon$.

<table>
<thead>
<tr>
<th></th>
<th>$\alpha_0$</th>
<th>$\alpha_1$</th>
<th>$\alpha_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cylindrical conduit</td>
<td>1.22</td>
<td>1.4</td>
<td>1</td>
</tr>
<tr>
<td>Equilateral triangular conduit</td>
<td>2.05</td>
<td>0.63</td>
<td>1</td>
</tr>
<tr>
<td>Rectangular conduit</td>
<td>$\varepsilon=\frac{1}{1}$</td>
<td>1.76</td>
<td>0.8</td>
</tr>
<tr>
<td></td>
<td>$\varepsilon=\frac{1}{2}$</td>
<td>1.95</td>
<td>0.8</td>
</tr>
<tr>
<td></td>
<td>$\varepsilon=\frac{1}{3}$</td>
<td>2.33</td>
<td>0.8</td>
</tr>
<tr>
<td></td>
<td>$\varepsilon=\frac{1}{4}$</td>
<td>2.68</td>
<td>0.8</td>
</tr>
<tr>
<td></td>
<td>$\varepsilon=\frac{1}{5}$</td>
<td>2.93</td>
<td>0.8</td>
</tr>
<tr>
<td></td>
<td>$\varepsilon=\frac{1}{6}$</td>
<td>3.21</td>
<td>0.8</td>
</tr>
<tr>
<td></td>
<td>$\varepsilon=\frac{1}{7}$</td>
<td>3.55</td>
<td>0.8</td>
</tr>
<tr>
<td></td>
<td>$\varepsilon=\frac{1}{8}$</td>
<td>3.85</td>
<td>0.8</td>
</tr>
<tr>
<td></td>
<td>$\varepsilon=\frac{1}{9}$</td>
<td>4.08</td>
<td>0.8</td>
</tr>
<tr>
<td></td>
<td>$\varepsilon=\frac{1}{10}$</td>
<td>4.3</td>
<td>0.8</td>
</tr>
</tbody>
</table>
Figure 3. The $\alpha_0$ and $\alpha_2$ for rectangular conduits with aspect ratio $\varepsilon (1, \frac{1}{10})$ in data points and the best-match equations in lines.

$$\alpha_0 = \frac{0.2904}{\varepsilon} + 1.4714$$

$$\alpha_2 = 1.0111\varepsilon^{0.195}$$
Table 3.2, it can be inferred that the $\alpha_1$ should have a constant value for conduits with a certain type of cross-section shape. Furthermore, one major type of cross-section shapes can be divided into several sub-types. And the $\alpha_2$ is influenced by sub-types of cross-sections. For example, triangular cross-sections may be equilateral, isosceles or scalene. Conduits with equilateral, isosceles and scalene triangle cross-sections should have different $\alpha_2$ values. Table 3.2 also indicates that the $\alpha_2$ should be 1 if a conduit has the same velocity distribution at all the walls/surfaces.

3.3 Modeling of High-Pressure Rarefied Gas Flow on Pore Networks

The high-pressure rarefied gas flow modeling on pore networks is an efficient way to explore the constitutive relationship between shale permeability and shale pore structures. In this section methane CH$_4$ will flow through a pore network at a constant temperature when a pressure difference exists between the pore network inlet and outlet. In this study the temperature is kept as 338K (in Table 3.1). There are three assumptions that make the network rarefied gas flow modeling possible and reliable:

(i) The high-pressure rarefied gas flow is in steady-state at pore scale, i.e., the network inlet and outlet pressures ($P_{in}$, $P_{out}$) keep constant.

(ii) The difference of gas pressure inside a pore body is negligible.

(iii) The pressure difference at the network inlet/outlet is small when compared with the gas pressure. Since the shale matrix space that a pore network in this study can cover is usually in the scale of 10$\mu$m (x-direction)×
10μm (y-direction) × 10μm (z-direction), the inlet/outlet pressure difference could be in the range of 0.01~1Pa.

The procedure of rarefied flow modeling in shale pore networks is similar to that of no-slip flow modeling in Section 2.4.4. For a single throat that connects i-th and j-th pores, the volumetric flow rate $q_{ij}$ through the throat:

$$q_{ij} = C_{ij}(P_i - P_j).$$ ..............................................................(3.26)

Here the $C_{ij}$ is the throat conductivity, $P_i$ and $P_j$ represent the pressure in i-th and j-th pores, respectively. The throat conductivity can be defined as

$$C = \frac{\pi r^4}{4\mu L} \left( \frac{1}{2} + 2 \frac{2-\sigma_v}{\sigma_v} \frac{2Kn}{1-2\beta Kn} \right) \alpha(K_n) \text{ for a cylindrical throat,}$$ ..............................................(3.27a)

$$C = \frac{a^4}{144\mu L} g(\beta, K_n) \alpha(K_n) \text{ for a triangular throat,}$$ ..................................................(3.27b)

$$C = \frac{ab^3}{2\mu L} F(K_n, \beta, \varepsilon) \alpha(K_n) \text{ for a rectangular throat.}$$ ..................................................(3.27c)

Different from Section 2.4.4.4, the throat conductivity depends on not only throat sizes but also the gas pressure. The gas density can be regarded as a constant when the pressure difference along the pore network is very small. The mass conservation for the i-th pore is satisfied

$$\sum_j q_{ij} = 0,$$ ......................................................................................(3.28)

where $j$ represents all pores connected to the i-th pore through throats. A system of equations occurs if the mass conservation principle runs over all the pores in the network. The final solution gives the pressure distribution inside the pore network. The apparent permeability $K_{app}$ of the reconstructed pore network is evaluated using Darcy’s equation:
\[ K_{\text{app}} = \frac{q \mu l}{A(P_{\text{in}} - P_{\text{out}})^q} \]  

Where \( l \) is the length of the pore network along the gas flow direction, \( A \) is the cross-section area and \( q \) is the volumetric flow rate out of the pore network.

### 3.3.1 Effect of Multi-scale Pore Size Distribution

As discussed in Chapter 2, many shale rocks have a wide range of pore size distribution. The mixture of nm-scale and \( \mu \)m-scale pore space in a shale rock has significant influence on its apparent permeability behavior. Five sets of pore networks are developed by following the procedure in Section 2.2. All the throats in the pore networks are cylindrical and pores are spherical. Fig. 3.15a shows that the pore networks have two modes at approximately 300nm and 4\( \mu \)m, respectively. Table 3.3 includes these pore networks’ no-slip (intrinsic) permeabilities \( K_{\infty} \). From No. 1 case to the No. 5 case the nm-scale pore volume is increasing and become dominant in the multi-scale pore networks in No. 4 and 5 cases. From No. 5 to the No. 1 case, the pore networks with higher \( \mu \)m-scale pore volume fractions tend to have higher intrinsic permeabilities.

Different from intrinsic permeability, these pore networks’ apparent permeabilities \( K_{\text{app}} \) change when the gas pressure varies. Fig. 3.15b displays the ratios \( K_{\text{app}}/K_{\infty} \) vs. gas pressure \( P \) (in log-log plot) when the gas pressure inside those pore networks decreases from 70MPa towards 0.1MPa. The behavior of \( K_{\text{app}}/K_{\infty} \) vs. \( 1/P \) can be regarded as a two-phase process: in Phase I the apparent permeabilities increase slowly as the pressure decreases while in Phase II the apparent permeabilities have significant increase vs. further pressure drop. The Phase I is controlled by the slip flow in most pores/throats and can be matched...
Table 3.3 Intrinsic and apparent permeabilities of pore networks with different pore sizes and pore/throat morphologies in Sections 3.3.1, 3.3.2 and 3.3.3.

<table>
<thead>
<tr>
<th>Description</th>
<th>$K_{\infty}$, $\mu D$</th>
<th>$a$ in 2nd-order correlation, MPa$^2$</th>
<th>In linear or 2nd-order correlation, MPa</th>
<th>$b$ in linear or 2nd-order correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>No.1 pore network in Section 3.3.1</td>
<td>41</td>
<td>—</td>
<td>0.098</td>
<td>1.001</td>
</tr>
<tr>
<td>No.2 pore network in Section 3.3.1</td>
<td>6.2</td>
<td>—</td>
<td>0.15</td>
<td>1.002</td>
</tr>
<tr>
<td>No.3 pore network in Section 3.3.1</td>
<td>0.39</td>
<td>—</td>
<td>0.65</td>
<td>1.008</td>
</tr>
<tr>
<td>No.4 pore network in Section 3.3.1</td>
<td>0.077</td>
<td>0.10</td>
<td>0.91</td>
<td>1.025</td>
</tr>
<tr>
<td>No.5 pore network in Section 3.3.1</td>
<td>0.062</td>
<td>0.20</td>
<td>0.92</td>
<td>1.026</td>
</tr>
<tr>
<td>No.5.1 pore network in Section 3.3.2</td>
<td>0.015</td>
<td>0.70</td>
<td>1.8</td>
<td>1.046</td>
</tr>
<tr>
<td>No.5.2 pore network in Section 3.3.2</td>
<td>0.0039</td>
<td>1.9</td>
<td>3.9</td>
<td>1.100</td>
</tr>
<tr>
<td>The pore network with rectangular throats and $\varepsilon=1$ in Section 3.3.3</td>
<td>0.014</td>
<td>0.53</td>
<td>2.0</td>
<td>1.0576</td>
</tr>
<tr>
<td>The pore network with rectangular throats and $\varepsilon=1/4$ in Section 3.3.3</td>
<td>0.0034</td>
<td>3.3</td>
<td>6.8</td>
<td>2.1213</td>
</tr>
<tr>
<td>The pore network with triangular throats in Section 3.3.3</td>
<td>0.011</td>
<td>0.59</td>
<td>2.3</td>
<td>1.0637</td>
</tr>
<tr>
<td>The pore network with mixed throats morphologies in Section 3.3.3</td>
<td>0.013</td>
<td>0.63</td>
<td>2.2</td>
<td>1.0628</td>
</tr>
</tbody>
</table>
Figure 3. 15-(a) The pore size distributions of the five pore networks with two modes at approximately 300nm and 4μm. (b) The ratios $K_{app}/K_\infty$ vs. gas pressure $P$ in the pressure range (0.1MPa, 70MPa).
with a straight line in the Cartesian plot, i.e. the linear correlation \( \frac{K_{\text{app}}}{K_\infty} = \frac{c}{P} + d \). The Phase II is influenced by the transition flow in the majority of pores/throats and does not follow the linear trend of \( \frac{K_{\text{app}}}{K_\infty} \) vs. \( \frac{1}{P} \). The dash line in Fig. 3.15b separates phases for calculated permeabilities of the five pore networks. It demonstrates that with increasing nm-scale pore volume fractions, it becomes easier for the transition flow to be dominant in pore space at high pressure (low reciprocal pressure).

It becomes important to find a correlation of \( \frac{K_{\text{app}}}{K_\infty} \) vs. \( \frac{1}{P} \) in the reservoir pressure condition, which could be applied in reservoir-scale modeling. Figs. 3.16a-3.16e gives the ratios \( \frac{K_{\text{app}}}{K_\infty} \) vs. reciprocal pressure \( \frac{1}{P} \) for the five pore networks when the gas pressure \( P \) is in the reservoir pressure range (1MPa, 30MPa). Only Phase I can be identified in Figs. 3.16a-3.16c. In reservoir pressure condition, it is quite difficult for the Knudsen number \( K_n \) of methane to go beyond 0.1 in the \( \mu \text{m} \)-scale pores/throats. When the \( \mu \text{m} \)-scale pores/throats dominate the pore space (in Figs. 3.16a-3.16c), the linear correlation can be applied to describe the apparent permeability behavior, which works similarly to the Klinkenberg correlation \( \frac{K_{\text{app}}}{K_\infty} = \frac{b}{P} + 1 \). In addition, the slope \( b \) value is still influenced by nm-scale pores/throats. As shown in Table 3.3, the slope \( c \) becomes larger with larger nm-scale pore volume and smaller average pore sizes from No.1 to No.3 cases. Compared with \( \mu \text{m} \)-scale pores/throats, the nm-scale pores/throats provide more chances for the methane flow to achieve \( K_n \) higher than 0.1 under reservoir pressure condition. When the nm-scale pore/throats dominate the pore networks, both Phases I and II appear in
Case 1: Linear Relationship Matching

Case 2: Linear Relationship Matching

Reciprocal Pressure $1/P$, MPa$^{-1}$

$K_{app}/K_\infty$
1. Linear Relationship Matching in Case 3

2. Linear Relationship Matching in Case 4

3. 2nd-order Polynomial Relationship Matching in Case 4
Figure 3. The ratios of apparent permeability over intrinsic permeability change along with the gas pressure (data points) and analytical correlations of the ratios vs. inverse pressure (lines) for the (a) Case 1, (b) Case 2, (c) Case 3, (d) Case 4 and (e) Case 5.
**Figs. 3.16d and 3.16e.** The linear correlation from the Klinkenberg equation cannot fully describe the gas permeability behavior. A 2nd-order correlation is applied to describe the behavior of gas apparent permeability vs. pressure under reservoir pressure conditions when the Klinkenberg relationship is not applied:

\[
\frac{K_{\text{app}}}{K_{\infty}} = f \left( \frac{1}{p} \right)^{1/2} (a \frac{1}{p} + c) + d = a \frac{1}{p^2} + c \frac{1}{p} + d. \]

\[
(3.30)
\]

**Figs. 3.16d and 3.16e** show that results of Eq. 3.30 (solid curves) match well with the calculated permeabilities. The applicability of **Eq. 3.30** in describing apparent permeabilities behavior would be further tested in the following sections. In this study, the Eq. 3.30 is validated to be applicable for the isothermal shale gas flow in the slip flow and transition flow regimes under reservoir pressure (1MPa, 30MPa). When the pressure is lower than 1MPa, late-stage transition flow or FMF may occur and Eq. 3.30 may no longer describe the behavior of \( K_{\text{app}}/K_{\infty} \) vs. \( p \). As shown in **Table 3.3**, both \( a \) and \( c \) increase with higher nm-scale pore volume fractions while \( d \) keeps close to 1 for cases in **Section 3.3.1**. It can be inferred that \( a \) and \( c \) are closely related to the pore structure.

**3.3.2 Effect of nm-Scale Pore Sizes**

Most shale rocks have abundant nm-scale pores/throats. When the nm-scale pores/throats dominate a pore space of rock matrix, the matrix apparent permeability heavily depends on the sizes of those nm-scale pores/throats. To check the influence of nm-scale pores/throats sizes, the No. 5 pore network in **Section 3.3.1** is selected. The other two networks with smaller nm-scale pores/throats, named as No. 5.1 and 5.2 cases, are generated for clear
comparison. All the throats are cylindrical in the three pore networks. Fig. 3.17a shows the PSDs and apparent permeabilities are calculated for the three pore networks in Fig. 3.17b. Since nm-scale pores/throats dominate the pore networks, the \( \mu \)m-scale pores/throats have negligible influence on apparent permeabilities. Table 3.3 lists the pore networks’ intrinsic permeabilities. It demonstrates that smaller nm-scale pores/throats lead to lower intrinsic (no-slip) permeabilities. Fig. 3.17b gives the calculated gas permeabilities ratios of the pore networks in the pressure range (0.1MPa, 70MPa). In Fig. 3.15, the No.5 pore network can have an apparent permeability which is ten times higher than the intrinsic permeability at \( P=0.1 \)MPa. The \( K_n \) can be larger when the methane is flowing in smaller nm-scale pores/throats. Here in Fig. 3.17b, the No. 5.2 pore network can even have an apparent permeability (at \( P=0.1 \)MPa) that is almost one hundred times higher than the intrinsic permeability. For pore networks with smaller pores, their high \( K_{app}/K_\infty \) ratios can partially make up for the fluid transport difficulty caused by the low intrinsic permeability.

In the reservoir pressure condition, the apparent permeability of No. 5 case in Section 3.3.1 could be more than twice its intrinsic permeability in Fig. 3.16e. Fig. 3.18 further examines the apparent permeability behavior of No. 5.1 and 5.2 pore networks in reservoir pressure range (1MPa, 30MPa). It shows that the ratios of \( K_{app}/K_\infty \) of the two networks could reach 3 and 7 respectively, which is much higher than that in No. 5 case. The linear Klinkenberg equation that considers the slip flow can only be applicable when the pressure is very high. The Eq. 3.30, by contrast, is applicable to match the pore networks’
Figure 3.17a-The pore size distributions of the selected three pore networks. The modes of nm-scale pores are 300nm for the No. 5 case, 150nm for the No. 5.1 case and 60nm for the No. 5.2 case.

Figure 3.17b-The calculated permeability ratios $K_{\text{app}}/K_\infty$ of the three pore networks in pressure range (0.1MPa, 70MPa).
Figure 3. 18-The ratios of apparent permeability over intrinsic permeability change along with the gas pressure (data points) and 2nd-order polynomial correlations of the ratios vs. reciprocal pressure (lines) for No. 5.1 and 5.2 pore networks in Section 3.3.2 in the reservoir pressure range (1MPa, 30MPa).
permeabilities in Fig. 3.18. Table 3.3 gives the best-match $a$, $c$ and $d$ values in Eq. 3.30. It is summarized that both $a$ and $c$ increase when the nm-scale pores/throats become smaller.

### 3.3.3 Effect of Pore/Throat Morphology

As discussed in Chapter 2, pore space has different morphology in different components of shale rocks. In this section, four different pore networks are utilized to examine the influence of pore/throat morphology on apparent permeabilities. These pore networks in this section are variations of the No. 5.1 case pore network in Section 3.3.2. For every throat in the No. 5.1 pore network, its circular cross-section can be modified as a rectangular or equilateral triangular shape without changing its cross-section area. In the first pore network, all the throats are set as rectangular throats with aspect ratio $\varepsilon$ as 1. And all the throats in the second pore network are rectangular with aspect ratio $\varepsilon$ as $1/4$. The third pore network has all the throats with equilateral triangular cross-sections. In the fourth pore network, cylindrical, rectangular and triangular throats each contributes one third of total number of throats. The aspect ratios $\varepsilon$ of the rectangular throats vary in the range of $(1/4, 1)$.

Table 3.3 lists the intrinsic permeabilities of these four pore networks. It shows that in this section, pore networks with rectangular throats have lower intrinsic permeabilities than those of pore networks with cylindrical throats. And the smaller aspect ratio $\varepsilon$ is, the lower intrinsic permeabilities become. For the pore network with triangular throats, its intrinsic permeability is lower than those of pore networks with cylindrical throats and rectangular throats of $\varepsilon=1$. Same as
the conclusion in Section 2.4.5, porous clay in shale with irregular-shaped throats tends to have lower intrinsic permeability than porous OM with cylindrical throats. The intrinsic permeability of the pore network with mixed throats lies between those of pore networks with full cylindrical or triangular throats.

Apparent permeabilities are calculated with considering the slip and/or transition flow in the pore networks under pressure (0.1MPa, 70MPa). Fig. 3.19 shows the apparent permeability \(K_{app}/K_\infty\) ratios of the four pore networks in this section and the No. 5.1 case pore network for reference. The permeability ratios increase along with decreasing gas pressure. In Fig. 3.19, pore networks with cylindrical, triangular or cylindrical throats \(\varepsilon=1\) have similar trends of \(K_{app}/K_\infty\) vs. \(P\). The permeability ratios starts almost from 1 at 70MPa and increases towards 40 when the pressure decreases to 0.1MPa. But the ratio \(K_{app}/K_\infty\) can reach 2.35 even when the gas pressure is 70MPa for the pore network with rectangular throats \(\varepsilon=1/4\). So the permeability measured at high pressure may still not be the intrinsic one for pore networks dominated by rectangular throats with small \(\varepsilon\).

Fig. 3.20 further shows the behavior of \(K_{app}/K_\infty\) vs. the reciprocal pressure in the reservoir pressure range (1MPa, 30MPa). The Eq. 3.30 is still applicable to match the pore networks' permeabilities and Table 3.3 gives the best-match \(a\), \(c\) and \(d\) values. The dash line in Fig. 3.20 shows the linear part of the 2\(^{nd}\)-order Eq. 3.30, i.e. \(\frac{c}{P}+d\), matches the permeability ratios when \(\frac{1}{P}<0.2\text{MPa}^{-1}\) for the case with aspect ratio \(\varepsilon=1/4\). In addition the \(K_n\) of dominant rectangular throats \((D_h=35\text{nm})\) is between (0.01, 0.07) corresponding to the pressure range
Figure 3. 19- The calculated permeability ratios $K_{app}/K_{∞}$ of pore networks with cylindrical throats (Case 5.1 pore network), rectangular throats and triangular throats in pressure range (0.1MPa, 70MPa).
Figure 3. 20-The ratios of apparent permeability over intrinsic permeability (data points) and the linear (for slip flow under high pressure) and 2nd-order polynomial correlations of the ratios vs. reciprocal pressure (lines) for pore networks with rectangular or triangular throats in the reservoir pressure range (1MPa, 30MPa).
(30MPa, 3MPa). Therefore the slip flow is dominant when $1/p < 0.2\text{MPa}^{-1}$ in Fig. 3.20. Moreover it should be noted that the $d$ value here is 2.1213, which is significantly larger than 1. In fact, the $d=1.1$ of No.5.2 pore network in Section 3.3.2 already deviates from 1. It demonstrates that the classical Klinkenberg correlation $K_{app}/K_{\infty} = \frac{b}{p} + 1$ is no longer valid even for slip flow. Such deviation results from real gas properties and small pore sizes. In the Klinkenberg correlation, the parameter $b$ is derived based on the slip flow in a single cylindrical conduit:

$$b = \frac{4ClP}{r} = \frac{4Cl}{r} \sqrt{\frac{\pi z R_s T}{2}},$$

(3.31)

where $C$ is the Adzumi constant and $r$ is the pore radius. For conventional rocks, the $b$ can be considered as a constant for a specific gas flow at a specific temperature because the slip flow in conventional rocks occur at the medium-to-low pressure (<5MPa) where the variations of $\mu \sqrt{z}$ vs. $P$ is very small. But the slip flow occur at high-to-medium pressure (>5MPa) for pores with tens of nm diameter. Meanwhile the $\mu \sqrt{z}$ and $b$ in Eq. 3.31 varies significantly in the high-to-medium pressure. Therefore the linear correlation $K_{app}/K_{\infty} = \frac{c}{P} + d$ is only an approximation of the slip flow behavior. More specifically, the linear Klinkenberg correlation $K_{app}/K_{\infty} = \frac{b}{P} + 1$ is an approximation of the gas slip flow behavior in the medium-to-low pressure range.

**3.3.4 Apparent Permeability vs. Intrinsic Permeability**

The relationship between apparent and intrinsic permeabilities is of great interest for researchers that focus on the tight formation and shale rocks. In Eq.
3.31, the pore radius $r$ is inversely proportional to the intrinsic permeability. Therefore, the constant $b$ has been widely applied in the literature to represent conventional rocks’ intrinsic permeability. For low-permeability unconventional rocks, Heid et al. (1950) and Jones and Owens (1980) provided an empirical correlation of $b$ in atm vs. intrinsic permeability $K_\infty$ based on the measured permeabilities from several hundreds of sandstones samples with $K_\infty$ in the range (0.0001 mD, 10 mD):

$$b = d_1 K_\infty^{-d_2}.$$

(3.32)

where $d_1=0.777$, $d_2=0.39$ in Heid et al.’s work and $d_1=0.86$, $d_2=0.33$ in the study of Jones and Owens. In Jones and Owens (1980)’s work, the $b$ values were obtained by connecting an experiment data point at high pressure $P=6.9$ MPa with the other one at low pressure $P=0.7$ MPa with the Klinkenberg correlation. The problem is that the Klinkenberg correlation is invalid for the sample rocks under the pressure range (0.7 MPa, 6.9 MPa). The calculated $b$ should be higher than the true $b$ value of slip flow under the influence of real gas properties and transition flow regime.

It is necessary to check the relationship of coefficients in Eq. 3.30 vs. intrinsic permeabilities $K_\infty$ when the high-pressure rarefied flow mechanisms are considered in low-permeability unconventional rocks. This section summarizes the $a$ vs. $K_\infty$ (solid diamonds in Fig. 3.21) and $c$ vs. $K_\infty$ (solid squares in Fig. 3.21) from all cases in Table 3.3 in log-log plot (Fig. 3.21). In general, both $a$ and $c$ follow the power-law relationships as
Figure 3.21- The parameters $a$ and $c$ vs. intrinsic permeability $k_\infty$ (data points) in the 2nd-order correlation or linear correlation for all cases in Sections 3.3.1-3.3.3, the power-law equations that match the $a$ and $c$ (solid lines) and the $b$ vs. intrinsic permeability $k_\infty$ in Eq. 3.32.
\[ a = d_1 K_\infty^{-d_2}, \]  
\[ c = d_3 K_\infty^{-d_4}. \]

Fig. 3.21 also shows the trend of \( b \) vs. \( K_\infty \) in Eq. 3.32 for reference. Coupling Eqs. 3.30 and 3.33, the apparent permeability of isothermal methane flow in shale at a reservoir pressure \( P \) in the range of (1MPa, 30MPa) can be inferred as:

\[ K_{\text{app}}(K_\infty, P) = d_1 K_\infty^{1-d_2} \frac{1}{p^2} + d_3 K_\infty^{1-d_4} \frac{1}{p} + d K_\infty. \]

The coefficients \( d, d_1, d_2, d_3 \) and \( d_4 \) depend on the type of gas and the temperature. In this study \( d_1 = 0.0099(\text{MPa}^2), d_2 = 0.981, d_3 = 0.1794(\text{MPa}), \) and \( d_4 = 0.596 \) when the temperature is 338K and the flowing gas is methane. And the \( d \) could be considered as 1 for shale rocks with cylindrical, triangular or rectangular pores/throats (with dominant nm-scale pore size larger than 50nm and \( \varepsilon \) close to 1). With Eq. 3.34, the apparent permeability at pressure \( P \) for a certain type of gas can be calculated with intrinsic permeabilities that are measured with He or \( \text{N}_2 \). When the influence of gas types is ignored or the target reservoir temperature is close to 338K, the coefficients in this study could also be used to predict apparent permeabilities behavior at a different temperature. Eq. 3.34 can be applied in the reservoir-scale or single-well-scale modeling to include the effect of rarefied flows in reservoir conditions.

### 3.4 Conclusions

1. The gas flow in the pressure range (0.1MPa, 70MPa) is located in the slip flow and transition flow regimes for pores with diameters from several nanometers to several micrometers. When the gas pressure decreases from
70MPa, the mass flow rate out of a single conduit would increase first, then decreases to the Knudsen minimum and finally increase slowly.

2. The rarefied flow mechanism with high $K_n$ tends to eliminate the conductivities/apparent permeabilities discrepancies among rectangular conduits/throats with different aspect ratios.

3. New unified models/equations are developed for high-pressure rarefied gas flow in nm-scale conduits with negligible pressure drop (compared with the gas pressure) along the conduit. The equations are applicable for conduits with circular, rectangular and equilateral triangular cross-sections. The aspect ratio $\varepsilon$ of rectangular cross-sections can be any number from $(0, 1)$.

4. In shale pore networks, the ratios of apparent permeability over intrinsic permeability increase along with decreasing gas pressure because of the slip and transition flow.

5. When the nm-scale pore volume fraction increases, the ratios of apparent permeabilities over intrinsic permeabilities become larger. The permeability ratios also increase when the nm-scale pore sizes become smaller or rectangular throats have lower aspect ratios $\varepsilon$. In the same porosity condition, shale pore networks with cylindrical throats have lower permeability ratios than those with triangular or rectangular throats.

6. The Klinkenberg correlation is conditionally valid for the gas slip flow in nm-scale pores. A 2nd-order equation is developed and validated to describe the permeability ratios vs. pressure in the pressure range (1MPa, 30MPa).
7. Power-law relationships are found in the shale intrinsic permeability vs. coefficients of the 2\textsuperscript{nd}-order equation, which can be applied in the reservoir-scale modeling.
CHAPTER 4

SEMI-ANALYTICAL MODELING OF FLOW IN THE UNCONVENTIONAL RESERVOIR WITH STRESS-SENSITIVE HYDRAULIC FRACTURES

4.1 Background

In some unconventional reservoirs, the loss of fractured horizontal wells’ productivity is observed with decreased bottomhole flowing pressure. The contributing factors include the reservoir permeability reduction and the fracture conductivity loss (Ostensen, 1986; Clarkson et al., 2013). The effect of stress-sensitive reservoir permeability and porosity on pressure transient responses was experimentally studied by Dadmohammadi et al. (2016). This chapter focuses on the stress-sensitive hydraulic fractures.

For tight formations and shale reservoirs, hydraulic fractures provide high-conductivity pathways for fluid flow from the reservoir matrix to a horizontal wellbore (Fig. 4.1a). When the pore pressure decreases in production, the geomechanical interaction between fracture proppants and in-situ rocks could crush and embed the proppants, which leads to the fracture conductivity loss (Fig. 4.1b). Friedel et al. (2007), Abass et al. (2009) and Zhang et al. (2014) measured the hydraulic fracture conductivity changes vs. stress increase in experiments. Their results suggested that stress-sensitive characteristics of hydraulic fractures vary significantly in different fracturing treatments. In 2015, Shelley et al. list two cases of hydraulic fracture conductivity loss during field
Figure 4. 1-(a) The multi-stage fractured horizontal well. (b) Fracture proppant crush and embedment because of pore pressure decrease during production (original in color).
production in Utica shale.

During production, a change in reservoir pore pressure always results in a change in the stress field (Rutqvist et al., 2002). Governing equations of the fluid flow inside reservoir matrix can be explicitly (Settari and Walters, 2001; Minkoff et al., 2003; Wang et al., 2015), iteratively (Settari and Mourits, 1994; An et al., 2017) or fully (Lewis and Sukirman, 1993) coupled with reservoir geomechanics equations (Dean et al., 2006). The three coupling techniques can produce similar results. The explicit coupling method allows flow dynamics computation first and updates geomechanical properties sequentially during same time step. Compared with the iteratively- and fully-coupled methods, the explicit coupling scheme is advantaged in fast calculation and not restricted by different solving methods of flow equations and geomechanics equations. Furthermore, a significant improvement to the explicit coupling scheme can be achieved by obtaining analytical expressions of stress tensor $\sigma_{ij}$ vs. pore pressure $p_p$. Rudnicki (1985) provided analytical relationship between the stress tensor $\sigma_{ij}$, pore pressure $p_p$ and the elastic strain tensor $\varepsilon_{ij}$ for isotropic poroelastic rocks. Many low-permeability unconventional reservoirs are highly anisotropic because of partial alignment of anisotropic minerals and bedding-parallel lamination of rock materials (Vernik and Nur, 1992; Sondergeld and Rai 2011; Misra et al., 2016). Ignoring the rock anisotropy leads to an incorrect estimation of stress changes. In 2013, Sayers incorporated shale anisotropy into the influence of pore pressure on stress changes based on Rudnicki’s work.
Principal stresses and pore pressure have different declining trends during hydrocarbon production, which increases the effective stress exerted against proppants inside hydraulic fractures. The correlations between effective stress change $\Delta \sigma'$ and hydraulic fracture conductivities $F_C$ is seldom discussed in the literature while several equations for the matrix and/or natural fracture permeability vs. effective stress have been provided. Jones (1975) derived the relationship between rock effective permeability and stress for a fractured carbonate rock sample based on experiments. A linear correlation of cubic root of permeability vs. logarithm of confining pressure was put forward. In 1988, Jones summarized that a rock’s matrix permeability decreases with increasing confining stress but the decrease becomes progressively gentle at higher stresses. Meanwhile, Pedrosa (1986) defined the permeability modulus $\gamma$ of stress-sensitive reservoir matrix permeability $k$ as $\frac{1}{k} \frac{\partial k}{\partial p}$. Raghavan and Chin (2004) proposed the exponential equation $k = k_i e^{-\Delta \sigma'_m}$ for the Type-I rock. Here $\Delta \sigma'_m$ is the change of effective mean stress. In 2013, Cho et al. applied Raghavan and Chin’s (2004) Type-I correlation to describe the natural fracture permeability and defined $d_f$ as the permeability modulus of stress-sensitive natural fractures. For hydraulic fractures, the fracture conductivity $F_c$ that incorporates fracture permeability $k_f$ and aperture $w_f$ is more important for hydrocarbon production than the $k_f$ itself. In this study, the hydraulic fracture compressibility $d_f$ is defined in terms of $F_c$:

$$d_f = -\frac{1}{F_c \Delta \sigma'_2}, \quad \text{.................................................................(4.1)}$$
where $\sigma_3^*$ is the effective minimum stress inserted on hydraulic fractures.

The stress-sensitive conductivity makes flow equations highly non-linear. Numerical models (Moinfar et al., 2013; Eshkalak et al., 2014; Aybar et al., 2015; Dadmohammadi et al., 2017) can solve non-linear flow equations and update conductivities of stress-sensitive hydraulic fractures and/or natural fractures at each time step. But numerical models have disadvantages in increased computational requirements, implicit functional relationships of key parameters and the inconvenience in iterative applications (Brown et al. 2011). Several analytical approaches were also proposed to deal with the non-linearity. A number of researchers, such as Vairogs et al. (1971) and Raghavan et al. (1972), defined different kinds of pseudopressure functions which incorporated the stress-dependent rock properties into flow equations. In 1994, Celis et al. used the modulus $\gamma$ to analytically model the transient and pseudo-steady-state transfer between matrix and stress-sensitive natural fractures in a radial natural fractured reservoir. In 2000, Poe proposed the pseudotime and pseudopressure integral transformation to linearize the non-linear flow equations of gas flow in stress-sensitive hydraulic fractures in infinite reservoirs. Clarkson et al. (2013) proposed a time-dependent skin factor in flow equations to consider the effects of stress-dependent hydraulic fracture conductivities in transient flow regimes. Cho et al. (2013) and Aybar et al. (2014) incorporated the stress-sensitive natural fracture permeability into the pseudopressure definition for a trilinear flow model. Wang (2013) applied source functions, perturbation methods and Laplace transformation to semi-analytically solve the flow equations of stress-

Lots of efforts have been made in considering the geomechanical influence for stress-sensitive matrix or natural fractures in isotropic reservoirs. To date, the influence of anisotropic geomechanics on fluid dynamics is not well understood for stress-sensitive hydraulic fractures. In addition, the dynamic declining rates of hydraulic fracture conductivity $F_c$ vs. increasing effective stress are not considered in fluid flow models. In view of above problems this chapter develops a new model considering the geomechanical effect of anisotropic low-permeability unconventional reservoir rocks and dynamic fracture compressibility. This chapter improves the semi-analytical approach developed by Yao et al. (2016) to solve the model for transient rate analysis of MFHWs and focuses on the transient rate behavior of MFHWs in single- and dual-porosity reservoirs rather than the transient pressure behavior under the influence of stress-sensitive hydraulic fractures. This chapter will consider the phenomena that hydraulic fracture's conductivities decrease with increasing effective stress but the decrease can be progressively gentle at higher stresses and show the
difference between pore pressure change and effective stress change that is related to the anisotropic rock geomechanics.

4.2 Mathematical Model

The modeling of stress-sensitive hydraulic fractures requires coupling fluid flow dynamics and geomechanical responses. In this study, the semi-analytical modeling of MFHWs with stress-sensitive fractures consists of three parts: The first part is the geomechanical modeling for fluid-saturated anisotropic shales; the second part is the modeling of stress-sensitive hydraulic fracture conductivities and the last part is the fluid flow modeling in reservoir. The three parts are coupled according to the relationship of effective stress changes $\Delta \sigma'$ vs. pore pressure changes $\Delta p_p$.

4.2.1 Geomechanical Equations

According to SEM images (Fig. 4.2a) and petrographic observations (Vemik and Landis, 1996), shales can be regarded as layers of shaley material and kerogen. In 1997, Vernik and Liu assumed that the anisotropic shale is transversely isotropic (TI) with an axis of rotational symmetry perpendicular to the layers (Fig. 4.2b). The assumption of layered TI medium can still be applied to tight sandstones. The elastic stiffness coefficients $C_{ij}$ of a layered TI medium can be easily calculated by using the volume-weighted average of the elastic stiffness $c_{ij}$ ($i, j=1, 2 \ldots 5$) from all layers. Based on this assumption, research of Rice and Cleary (1976), Rudnicki (1985) and Sayers (2013) revealed equations of principal stress $\sigma$ vs. pore pressure $p_p$ for the TI poroelastic rock:

$$\sigma' = \sigma - \alpha_2 p_p = \frac{c_{12}}{c_{11}} (\sigma_1 - \alpha_1 p_p) + \left( C_{23} - \frac{c_{13} c_{12}}{c_{11}} \right) \epsilon_3 + \left( C_{22} - \frac{c_{12}^2}{c_{11}} \right) \epsilon_2, \ldots (4.2)$$
Figure 4.2 (a) A SEM image showing the layering texture of shales in Sichuan Basin, China. (b) A layered transversely isotropic (TI) medium.
and
\[ \sigma'_3 = \sigma_3 - \alpha_3 p_p = \frac{C_{13}}{C_{11}} (\sigma_1 - \alpha_1 p_p) + \left( C_{33} - \frac{C_{13}^2}{C_{11}} \right) \varepsilon_3 + \left( C_{23} - \frac{C_{13} C_{12}}{C_{11}} \right) \varepsilon_2. \] ......(4.3)

In Eqs. 4.2 and 4.3, \( \sigma_1 \), \( \sigma_2 \) and \( \sigma_3 \) represent the vertical stress, maximum horizontal stress and minimum horizontal stress, respectively. The \( \varepsilon_i \) is a principal strain and \( \alpha_i \) is a Biot’s coefficient \((i=1, 2, 3)\). Here \( i \) is the index for the direction of the \( i \)-th principal stress. Eqs. 4.2 and 4.3 shows the effective horizontal stresses \( \sigma'_2 \) and \( \sigma'_3 \) are related to the effective vertical stress \( \sigma'_1 \). Under the uniaxial strain condition \((\varepsilon_2=\varepsilon_3=0)\), the effective horizontal stresses \( \sigma'_2 \) and \( \sigma'_3 \) satisfy
\[ \sigma'_2 = \sigma'_3 = K_0 \sigma'_1, \quad K_0 = \frac{C_{13}}{C_{11}} = \frac{C_{12}}{C_{11}}. \] .................................................................(4.4)

In Eq. 4.4, the effective vertical stress, \( \sigma'_1 \), is equal to \( \sigma_1 - p_p \) when \( \alpha_1 \) is assumed as unit for overburden rocks. And the change of effective vertical stress, \( \Delta \sigma'_1 \), is the same as pore pressure change \( \Delta p_p \). The change of effective minimum horizontal stress, \( \sigma'_3 \), is proportional to the change of effective vertical stresses, \( \Delta \sigma'_1 \), by factor \( K_0 \). Sayers (2013) thought that \( K_0 \) in organic-rich shale is closely related to the kerogen content and distribution.

**4.2.2 Fracture Conductivity Equation**

For stress-sensitive matrix and natural fractures, one common equation that describes the stress-dependent permeability is as below:
\[ \frac{k}{k_1} = e^{-d_f^* (p_i-p_p)} = e^{-d_f^* \Delta p_p}, \] .................................................................(4.5)

where \( k \) is the permeability, \( k_1 \) is the initial permeability at initial pore pressure \( p_i \), and \( d_f^* \) is the compressibility defined as \(-\frac{1}{k} \frac{dk}{d\sigma_m}\). The effective mean stress
increment, $\Delta \sigma'_m$, is directly related to the matrix or natural fractures’ stress-sensitivity. And $\Delta \sigma'_m$ is close to the pore pressure change, $\Delta p_p$, for many reservoirs. It’s reasonable to use $\Delta p_p$ instead of $\Delta \sigma'_m$ in Eq. 4.5 for matrix and natural fractures. But for hydraulic fractures in reservoirs, the effective minimum horizontal stress $\sigma'_3$ is inserted on hydraulic fractures. The $\Delta \sigma'_3$ is different from $\Delta p_p$. It introduces errors if Eq. 4.5 is applied to stress-sensitive hydraulic fracture conductivities by simply replacing $k$ and $d_f^*$ with $F_c$ and $d_f$ (defined in Eq. 4.1), respectively. More errors are created if the $d_f$ is regarded as constant. These two assumptions significantly restrict Eq. 4.5’s applicability in stress-sensitivity modeling.

The first step to correct Eq. 4.5 is to apply the effective stress changes. For hydraulic fractures, the effective minimum horizontal stress $\sigma'_3$ has a significant influence on the fracture conductivity. With Eq. 4.4, the Eq. 4.5 is modified as

$$\frac{F_c}{F_{ci}} = e^{-d_f(\sigma'_3-\sigma'_3)} = e^{-d_fK_0\Delta p_p}, F_{ci} = k_f w_f,$$

(4.6)

where $F_{ci}$ is the initial fracture conductivity and $d_f$ is still assumed as a constant. In addition, hydraulic fracture conductivity loss is an irreversible process and Eq. 4.6 is not applicable when $\Delta p_p$ decreases. In order to check the applicability of Eq. 4.6, nine sets of fracture conductivities are calculated from listed experiments in the literature (Friedel et al., 2007; Abass et al., 2009, Zhang et al., 2014; Palisch et al., 2007; Weaver et al., 2010). Those experiments’ conditions vary significantly and Table 4.1 gives the detailed proppant conditions. Fig. 4.3a shows the comparison of Eq. 4.6 calculation vs. experimental data. It demonstrates that in six out of the nine cases, Eq. 4.6 underestimates fracture
Table 4.1 Experimental conditions for propped fractures (Friedel et al., 2007; Palisch et al., 2007; Abass et al., 2009; Weaver et al., 2010; Zhang et al., 2014).

<table>
<thead>
<tr>
<th>$d_0$ (Pa$^{-1}$)</th>
<th>5.5×10$^{-8}$ (Case 1)</th>
<th>1.1×10$^{-7}$ (Case 4)</th>
<th>3×10$^{-7}$ (Case 6)</th>
<th>1.7×10$^{-7}$ (Case 3)</th>
<th>1.5×10$^{-7}$ (Case 9)</th>
<th>1.4×10$^{-7}$ (Case 5)</th>
<th>1.5×10$^{-7}$ (Case 2)</th>
<th>3.1×10$^{-8}$ (Case 7)</th>
<th>5.2×10$^{-8}$ (Case 8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proppant Type</td>
<td>Sand</td>
<td>Sand</td>
<td>Sand</td>
<td>Sand</td>
<td>Sand</td>
<td>Sand</td>
<td>Bauxite Proppant</td>
<td>Intermediate Strength Proppant</td>
<td>Ceramic Proppant</td>
</tr>
<tr>
<td>Proppant size (mesh)</td>
<td>40/70</td>
<td>40/70</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>20/40</td>
<td>100</td>
<td>—</td>
</tr>
<tr>
<td>Proppant loading (kg/m$^2$)</td>
<td>0.49</td>
<td>0.29</td>
<td>0.14</td>
<td>0.29</td>
<td>0.49</td>
<td>0.73</td>
<td>9.76</td>
<td>—</td>
<td>9.76</td>
</tr>
</tbody>
</table>
Figure 4. 3a-Experimental data and the matching results based on Eq. 4.6 with constant $d_f$. 
Fig. 4. 3b—Experimental data and the matching results based on Eq. 4.8 with $d_{fi}$ and $\beta$. 

Case1: $d_{fi} = 5.5 \times 10^{-4}$ Pa$^{-1}$, $\beta = 1.0 \times 10^{-4}$ Pa$^{-1}$
Case2: $d_{fi} = 1.5 \times 10^{-4}$ Pa$^{-1}$, $\beta = 6.0 \times 10^{-4}$ Pa$^{-1}$
Case3: $d_{fi} = 1.7 \times 10^{-4}$ Pa$^{-1}$, $\beta = 5.9 \times 10^{-4}$ Pa$^{-1}$
Case4: $d_{fi} = 1.1 \times 10^{-4}$ Pa$^{-1}$, $\beta = 9.0 \times 10^{-4}$ Pa$^{-1}$
Case5: $d_{fi} = 1.4 \times 10^{-4}$ Pa$^{-1}$, $\beta = 8.8 \times 10^{-4}$ Pa$^{-1}$
Case6: $d_{fi} = 9.0 \times 10^{-5}$ Pa$^{-1}$, $\beta = 2.3 \times 10^{-5}$ Pa$^{-1}$
Case7: $d_{fi} = 3.1 \times 10^{-6}$ Pa$^{-1}$, $\beta = 6.3 \times 10^{-6}$ Pa$^{-1}$
Case8: $d_{fi} = 5.2 \times 10^{-6}$ Pa$^{-1}$, $\beta = 1.1 \times 10^{-7}$ Pa$^{-1}$
Case9: $d_{fi} = 1.5 \times 10^{-6}$ Pa$^{-1}$, $\beta = 7.7 \times 10^{-7}$ Pa$^{-1}$
conductivities when the effective stress increases. Therefore, a more accurate and flexible modeling of $F_C$ vs. $\Delta \sigma'$ is required.

In 1986, Zimmerman et al. showed that measured compressibilities of Berea and Bandera sandstones exponentially decrease with increasing effective stress. McKee et al. (1988) and Chen et al. (2015) applied the stress-sensitive compressibility equation to define the stress-dependent permeability of coals and shales. This section extends the compressibility definition from Zimmerman et al. (1986) and McKee et al. (1988) to the stress-sensitive hydraulic fracture conductivities. The $d_f$ in this study is defined as

$$d_f = \frac{d_\beta}{\beta (\sigma_3' - \sigma_y')} \left(1 - e^{-\beta (\sigma_i' - \sigma_y')}ight)$$

Here $\beta$ is the declining rate of fracture compressibility. The $d_f$ is no longer a constant but influenced by stress changes. Combining Eqs. 4.6 and 4.7 gives the relationship between fracture conductivity and pore pressure changes:

$$\frac{F_C}{F_a} = e^{-d_f \frac{1 - e^{-\beta \Delta pp}}{\beta}}$$

Fig. 4.3b shows the close matching between Eq. 4.8 calculation and experimental data over the entire range of effective stress change. Eq. 4.8 is validated to be more flexible and practical in describing fractures’ stress-sensitive behavior. As shown in Table 4.1, proppant strength, size, and loading in fracturing treatments would change the $d_f$ and $\beta$ values.
Figure 4. 4a-Discretizing multiple hydraulic fractures into segments (original in color).
Fig. 4.4b-Discretizing the fracture conductivity in time space.
4.2.3 Fluid Flow Modeling

A Semi-analytical scheme is applied to model the fluid flow in a low-permeability unconventional reservoir stimulated by a m-stage fractured horizontal well. The fluid flow in this study includes: (1) the resulted fluid flow in the reservoir; (2) the fluid flow inside hydraulic fractures.

First of all, each hydraulic fracture is divided into n segments (Fig. 4.4a). The fluid flows into each fracture segment from adjacent fracture segments and/or formation. Meanwhile, the time domain is also divided into time steps. Each fracture segment’s conductivity is considered as constant during each time step (Fig. 4.4b). The dimensionless flow equation for the i-th fracture segment \((i=1, 2\cdots n)\) of j-th hydraulic fracture \((j=1, 2\cdots m)\) in k-th time step is

\[
\frac{\partial}{\partial y_D} \left( \frac{\partial p_{jD,i,j}}{\partial y_D} \right) + \frac{q_{jD,i,j}}{F_{C,i,j}} = \frac{1}{F_{g,i,j}} \frac{\partial p_{jD,i,j}}{\partial t_D}, \quad \Delta t_D = t_D^k - t_D^{k-1}, \quad y_D \leq y_D \leq y_D, \quad i=1,2,\cdots n, \\
j=1,2\cdots m.
\]

The dimensionless definitions can be found in Appendix A. The complementary solution of Eq. 4.9 in Laplace domain gives the pressure \(p_f(x, y, t)\) distribution:

\[
\overline{p^*_c}_{jD,i}(y_D,u^*) = E_{i,j} \overline{q}_{jD,i} + F_{i,j} \overline{q}_{jD^{k-1},i} + G_{i,j} \overline{q}_{jD,ki}, \quad \text{.............................}(4.10)
\]

The detailed definition of \(E_{i,j}, F_{i,j}\) and \(G_{i,j}\) and particular solutions of Eq. 4.9 are given in Appendix B.

For the fluid flow in reservoir, the dimensionless governing equation is

\[
\frac{\partial^2 p_D}{\partial x_D^2} + \frac{\partial^2 p_D}{\partial y_D^2} = \frac{\partial p_D}{\partial t_D}, \quad \text{.............................}(4.11)
\]
Source/Sink functions can be applied to solve Eq. 4.11 with Laplace transformation (Yao et al., 2013, 2016). When each segment is regarded as a plane source with source strength \( q_{rfDi,j} (i=1, 2, \cdots n; j=1, 2, \cdots m) \) the pressure \( p(x, y, t) \) in the formation can be written as

\[
\overline{p_D}(x_D, y_D, u) = \sum_{j=1}^{m} \sum_{i=1}^{n} q_{rfDi,j} \int_{0}^{\infty} S_{xj} S_{yi} e^{-u t} dt_D, \quad \text{ .......................................................... (4.12)}
\]

The \( S_{xj} \) and \( S_{yi} \) (Appendix B) are source/sink functions for \( i \)-th segment of \( j \)-th hydraulic fracture. When the influence of natural fractures is considered, the transient dual-porosity modification can be introduced to Eq. 4.12 with replacing the Laplace variable \( u \) by \( s \):

\[
 s = u \left(1 + \sqrt{\frac{\lambda \omega}{3u}} \tan \left( \sqrt{\frac{3 \lambda u}{\lambda}} \right) \right), \quad \text{ .......................................................... (4.13)}
\]

Serra et al. (1983) and Brown et al. (2011) give detailed definitions of the storativity ratio \( \omega \) and flow capacity ratio \( \lambda \).

### 4.2.4 Equations Coupling

The pressure and production rate are continuous at interfaces of fracture/reservoir, fracture/wellbore and fracture segments. The coupling of Eqs. 4.10 and 4.12 at interfaces leads to the solution of the whole system-a cuboid-shaped reservoir stimulated by a MFHW. Fig. 4.5 shows the coupling scheme of this semi-analytical model. The key procedure in the equations coupling includes:

1. At the fracture segment/reservoir interface, the pressure in reservoir (Eq. 4.12) is equal to that in fracture segment (Eq. 4.10), which creates Eq. C-6 in Appendix C.
Figure 4. 5-Flow chart of the semi-analytical solution procedure for fluid flow in reservoirs with multiple stress-sensitive hydraulic fractures.
2. At the interface of adjacent $i$-th and $i+1$-th fracture segments, the flow rate out of $i$-th segment is the same as that into $i+1$-th segment. The pressure of $i$-th segment (Eq. 4.10) is equal to that of $i+1$-th segment at the interface of the two adjacent segments, which creates Eq. C-7 in Appendix C.

3. At each fracture segment/wellbore interface, the pressure is same as bottomhole pressure and the rate out of all hydraulic fractures is the MFHW’s production rate (Eq. C-8 in Appendix C).

4. The effective stress changes (Eq. 4.4) and new fracture conductivities (Eqs. 4.7 and 4.8) are calculated based on the pressure in each fracture segment at the end of each time step.

5. The new conductivities are applied and Steps 1-5 are repeated until the maximum time step is reached.

4.2.5 Model Validation

The numerical software Ecrin (Topaz Modules) is applied to validate the semi-analytical model. The flow rate of constant-pressure production and the bottomhole flowing pressure of constant-rate production are calculated and then compared with Ecrin’s numerical results. The basic condition is that a MFHW is at the center of a cuboid-shaped shale reservoir. Table 4.2a lists the reservoir characteristics and fractured horizontal well’s properties. This semi-analytical model is applicable for MFHWs with heterogeneous completion. But for simplicity, all hydraulic fractures in the validation case are assumed to have same properties. The fracture compressibility is assumed as a constant $d_f=10^{-7}$ Pa$^{-1}$ ($6.895\times10^{-4}$ psi$^{-1}$). Since Ecrin doesn’t consider the influence of rock
Table 4. 2a Single-porosity reservoir and fractured horizontal well’s characteristics in the Model Validation (Fig. 4.6).

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reservoir lateral length (m)</td>
<td>1400</td>
</tr>
<tr>
<td>Horizontal well length (m)</td>
<td>1170</td>
</tr>
<tr>
<td>Well spacing (m)</td>
<td>300</td>
</tr>
<tr>
<td>Pay zone thickness (m)</td>
<td>100</td>
</tr>
<tr>
<td>Ko</td>
<td>1</td>
</tr>
<tr>
<td>Porosity</td>
<td>0.1</td>
</tr>
<tr>
<td>Formation compressibility $c_f$ (Pa$^{-1}$)</td>
<td>$1.45 \times 10^{-10}$</td>
</tr>
<tr>
<td>Formation permeability $k$ (m$^2$)</td>
<td>$10^{-18}$ (0.001md)</td>
</tr>
<tr>
<td>Viscosity (Pa.s)</td>
<td>0.001 (1cp)</td>
</tr>
<tr>
<td>No. of fractures</td>
<td>10</td>
</tr>
<tr>
<td>Hydraulic fracture conductivity $k_{wf}$ (m$^3$)</td>
<td>$4 \times 10^{-15}$ (13 md-ft)</td>
</tr>
<tr>
<td>Constant Hydraulic fracture compressibility $d_f$ (Pa$^{-1}$)</td>
<td>$10^7$</td>
</tr>
<tr>
<td>Hydraulic fracture half-length (m)</td>
<td>100</td>
</tr>
<tr>
<td>Hydraulic fracture spacing (m)</td>
<td>130</td>
</tr>
<tr>
<td>Pressure drawdown $\Delta p$ (MPa)</td>
<td>40</td>
</tr>
</tbody>
</table>
Table 4.2b Transient dual-porosity reservoir and fractured horizontal well's characteristics in the Model Validation (Fig. 4.6).

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reservoir matrix compressibility $c_f$ (Pa$^{-1}$)</td>
<td>$1.45 \times 10^{10}$</td>
</tr>
<tr>
<td>Reservoir matrix permeability $k$ (m$^2$)</td>
<td>$10^{22}$</td>
</tr>
<tr>
<td>Reservoir matrix porosity</td>
<td>0.1</td>
</tr>
<tr>
<td>Matrix block thickness (m)</td>
<td>2.5 (8.2 ft)</td>
</tr>
<tr>
<td>Number of matrix blocks</td>
<td>40</td>
</tr>
<tr>
<td>Hydraulic fracture conductivity $k_{hf}$ (m$^3$)</td>
<td>$4 \times 10^{15}$ (13 md-ft)</td>
</tr>
<tr>
<td>Constant Hydraulic fracture compressibility $\delta_f$ (Pa$^{-1}$)</td>
<td>$10^7$</td>
</tr>
<tr>
<td>Hydraulic fracture half-length (m)</td>
<td>100</td>
</tr>
<tr>
<td>Hydraulic fracture spacing (m)</td>
<td>130</td>
</tr>
<tr>
<td>Hydraulic fracture porosity</td>
<td>0.5</td>
</tr>
<tr>
<td>Natural fracture permeability (m$^2$)</td>
<td>$4 \times 10^{13}$ (400md)</td>
</tr>
<tr>
<td>Natural fracture porosity</td>
<td>0.45</td>
</tr>
<tr>
<td>Natural fracture compressibility (Pa$^{-1}$)</td>
<td>$1.45 \times 10^{10}$</td>
</tr>
<tr>
<td>Natural fracture thickness (m)</td>
<td>0.0003048 (0.001 ft)</td>
</tr>
<tr>
<td>Number of natural fractures</td>
<td>40</td>
</tr>
<tr>
<td>storativity ratio $\omega$</td>
<td>1823</td>
</tr>
<tr>
<td>flow capacity ratio $\lambda$</td>
<td>2.5</td>
</tr>
<tr>
<td>Pressure drawdown $\Delta \mu$ (MPa)</td>
<td>40</td>
</tr>
</tbody>
</table>
Figure 4. Model verification for constant-bottomhole-flowing-pressure production in single-porosity and transient dual-porosity reservoirs.
anisotropy, the coefficient $K_0$ in the validation case is set to be 1. Here the effective stress change is equal to the pore pressure change.

**Fig. 4.6** shows the production rates calculated by the semi-analytical model and Ecrin Topaz numerical model. The circles/triangles and solid/dash lines in **Fig. 4.6** present calculated rates from Ecrin and the semi-analytical model, respectively. The close match between numerical results and this study validates our semi-analytical model's applicability and accuracy. The dashed line in **Fig. 4.6** gives the production rate calculated by this model when there is no fracture stress-sensitivity during production. It shows that stress-sensitive fracture conductivities can reduce production rates significantly at early-stage production.

In addition to above single-porosity reservoir, the transient dual-porosity reservoir condition is also applicable in the semi-analytical model. **Table 4.2b** lists matrix, natural fractures and hydraulic fractures properties of the transient dual-porosity reservoir. **Fig. 4.6** displays the production rates from the dual-porosity reservoir with storativity ratio $\omega$ 1823 and flow capacity ratio $\lambda$ 2.5. It shows that the MFHW's transient rate behavior in the dual-porosity case could be quite similar to that in single-porosity reservoir with higher matrix permeability.

This chapter will only focus on the influence of stress-sensitive hydraulic fractures on transient rate behavior. The stress-sensitive matrix and natural fracture permeability will be completed in the future work.

**4.3 Results and Discussion**

The effects of different parameters on production behavior are investigated in sensitivity analysis. All reservoir and wellbore characteristics used in
Figure 4. 7-The fracture conductivity ratio \( F_c/F_{ci} \) vs. pore pressure drop under the influence of constant \( d_f \) (i.e. \( \frac{F_c}{F_{ci}} = e^{-d_f \beta \Delta p_p} \)), \( d_f \cdot K_0 \) (i.e. \( \frac{F_c}{F_{ci}} = e^{-d_f K_0 \Delta p_p} \)) and dynamic \( d_f \) in Eq. 4.8 (i.e. \( \frac{F_c}{F_{ci}} = e^{-d_f \frac{1-e^{-K_0 \beta \Delta p_p}}{\beta}} \)) when (a) \( K_0=0.1 \), (b) \( K_0=0.3 \) and (c) \( K_0=0.5 \).
sensitivity analysis are the same as in Table 4.2a unless otherwise stated. The key parameters include the initial fracture compressibility $d_{fi}$, the declining rate of fracture compressibility $\beta$, rock anisotropy indicator $K_0$ and bottomhole flowing pressure. In all the following cases, the values of $d_{fi}$, $\beta$ and $K_0$ are set as $1.0\times10^{-7}$ Pa$^{-1}$ (6.895×10$^{-4}$ psi$^{-1}$), $1.0\times10^{-10}$ Pa$^{-1}$ (6.895×10$^{-7}$ psi$^{-1}$) and 1.0 respectively unless otherwise specified.

4.3.1 The Fracture Compressibility

It’s of great importance to compare the fracture compressibility equations in the literature and in this chapter. Fig. 4.7 gives the fracture conductivity ratio $F_c/F_{ci}$ vs. pore pressure drop under the influence of dynamic $d_f$ in Eq. 4.8

\[ \frac{F_c}{F_{ci}} = e^{-d_f(1-e^{-\beta K_0 \Delta P_p})} \] (i.e. $\frac{F_c}{F_{ci}} = e^{-d_f(1-e^{-\beta K_0 \Delta P_p})}$), constant $d_f$ (i.e. $\frac{F_c}{F_{ci}} = e^{-d_f(1-e^{-\beta K_0 \Delta P_p})}$) and $d_{fi} \cdot K_0$ (i.e. $\frac{F_c}{F_{ci}} = e^{-d_f(1-e^{-\beta K_0 \Delta P_p})}$). Constant $d_f$ makes the fracture conductivity decrease fastest, followed by $d_{fi} K_0$ and dynamic $d_f$ comes last. When $K_0$ becomes smaller, the conductivity difference resulted from $d_{fi} K_0$ and constant $d_f$ becomes bigger. When $\beta$ is larger, the conductivity difference resulted from $d_{fi} K_0$ and dynamic $d_f$ becomes bigger. It is concluded that ignoring $K_0$ leads to the underestimation of fracture conductivities under same pore pressure drop. It also shows that both constant $d_f$ and $d_{fi} K_0$ are only two of many cases described by the dynamic conductivity in Eq. 4.8. Analytical and numerical models with constant $d_f$ and $d_{fi} K_0$ are limited in simulating stress-sensitive fractures. It may be argued that numerical software can receive discrete data of $F_c$ vs. $P$ as a table, which is as flexible as the dynamic $d_f$. But the problem is that a table of $F_c$ vs. $P$ cannot
reveal the mathematical relationship between $F_c$ and $P$. Therefore, it is difficult to use the table of $F_c$ vs. $P$ in type curve matching and production prediction.

4.3.2 The Initial Fracture Compressibility $d_{fi}$

The initial fracture compressibility $d_{fi}$ is an important indicator for fracture stress-sensitivity. As suggested in Fig. 4.3b, the $d_{fi}$ ranges from $5 \times 10^{-8}$ Pa$^{-1}$ ($3.45 \times 10^{-4}$ psi$^{-1}$) to $3 \times 10^{-7}$ Pa$^{-1}$ ($2.07 \times 10^{-3}$ psi$^{-1}$). Fig. 4.8a demonstrates the effect of $d_{fi}$ on the transient production rate of the MFHW. As one key step in production analysis workflow, the straight line analysis in square-root-of-time plots is helpful in identifying linear flow and obtaining reservoir/fracure characteristics (Clarkson 2013). Accordingly Figs. 4.8b and 4.8c analyze the inverse rate $1/q$ vs. $\sqrt{t}$.

The effect of $d_{fi}$ is different for fractures with different initial conductivities. Fig. 4.8a shows the influence of $d_{fi}$ when initial fracture conductivities are 80 md.ft and 13 md.ft. It is concluded that larger $d_{fi}$ value makes the production rate $q$ lower. Typically the strength, size and loading of proppants influence the initial compressibility $d_{fi}$. Stronger and larger proppants with bigger loading usually contribute to lower $d_{fi}$. Although fractures are stress-sensitive, the reduction of production rates is negligible when the initial conductivity is high (80 md.ft) and the initial compressibility $d_{fi}$ is low ($5 \times 10^{-8}$ Pa$^{-1}$). Moreover, the slope of production rate $q$ vs. $t$ is closely related to the $d_{fi}$ value. For stress-sensitive hydraulic fractures with high initial conductivities, the slope of linear flow is no longer 0.5. When $d_{fi}$ increases from 0 to $3 \times 10^{-7}$ Pa$^{-1}$($2.07 \times 10^{-3}$ psi$^{-1}$), the slope decreases from 0.5 to 0.24 for $F_{ci}$=80 md.ft in Fig.8a. In Fig. 8b a straight line
Figure 4. 8a-Effect of initial fracture compressibility $d_f$ on the transient rate behavior of MFHWs with different initial fracture conductivities $F_{ci}$ in log-log plot.
Figure 4.8b—Square-root time analysis for MFHWs with $F_{ci}=80\, \text{md}\cdot\text{ft}$ and different $d_{fi}$.

Figure 4.8c—Square-root time analysis for MFHWs with $F_{ci}=13\, \text{md}\cdot\text{ft}$ and different $d_{fi}$.
with zero intercept can be inferred for linear flow when hydraulic fractures have no stress-sensitivity. When the $d_{fi}$ becomes larger, it becomes more difficult to find straight lines to cover the $1/q$ vs. $\sqrt{t}$ during linear flow. And the intercepts and slopes of best-matching straight lines become larger when the $d_{fi}$ becomes larger. It is summarized that stress-sensitive hydraulic fractures work like dynamic skin factors. For cases with $F_{ci}=13$md.ft, the slope in Fig. 8a decreases from 0.38 to 0.22 when $d_{fi}$ increases from 0 to $3 \times 10^{-7}$Pa$^{-1}$ ($2.07 \times 10^{-3}$ psi$^{-1}$). The slopes and intercepts of straight lines for $F_{ci}=13$md.ft in Fig. 8c are larger than those in Fig. 8b with same $d_{fi}$.

**Fig. 8a** shows that the well loses 15% of its production at the first day when non-stress-sensitive hydraulic fractures $F_{ci}=80$md.ft become stress-sensitive with $d_{fi}=5 \times 10^{-8}$Pa$^{-1}$ ($3.45 \times 10^{-4}$psi$^{-1}$). The percentage of rate loss increases from 15% to 84% when the $d_{fi}$ increases from $5 \times 10^{-8}$Pa$^{-1}$ ($3.45 \times 10^{-4}$psi$^{-1}$) to $3 \times 10^{-7}$Pa$^{-1}$ ($2.07 \times 10^{-3}$ psi$^{-1}$). For $F_{ci}=13$md.ft cases, the percentage of rate loss at first day increases from 27% to 90%. It is concluded that the percentage of rate loss for fractures with low initial conductivities is higher than that with high initial conductivities under the same $d_{fi}$. Such comparison in **Fig. 8** supports the statement that production rates from fractures with high initial conductivity appear to be more resistant to the loss of fracture conductivities. Typically in hydraulic fracturing, larger, stronger and evenly distributed proppants support higher fracture initial conductivities. Increasing volumetric injection of fluid mixed with proppants contributes to longer fracture length. The multi-stage fracturing designs that aim at high conductivities should work well for the low-permeability
formations. And the fracture designs with large-volume sand but low conductivities may result in poor production performance. Shelley et al. (2015) also concluded that some small-volume fracture designs with shorter fracture length but improved conductivity result in better production than large-volume low-conductivity designs.

4.3.3 The declining rate of fracture compressibility $\beta$

The declining rate of fracture compressibility, $\beta$, is the other indicator for fracture stress-sensitivity. According to the experimental data of Fig. 4.3, the declining rate ranges between $1 \times 10^{-10}$ Pa$^{-1}$ (6.895$\times 10^{-7}$ psi$^{-1}$) and $1 \times 10^{-7}$ Pa$^{-1}$ (6.895$\times 10^{-4}$ psi$^{-1}$). Fig. 4.9a shows the effect of $\beta$ on the transient production rate. Fig. 4.9b gives the analysis of $1/q$ vs. $\sqrt{t}$ with different $\beta$. The well data and reservoir information are also same as the model validation case unless otherwise stated.

The $\beta$ also changes the slope of $q$ vs. $t$ in Fig. 4.9a. The slope of early-stage production rate reduces from 0.36 to 0.325 when $\beta$ increases from $10^{-10}$ to $10^{-7}$ Pa$^{-1}$, which is similar to the effect of $d_f$. In Fig. 4.9b, the slopes and intercepts of best-matching straight lines becomes smaller when the $\beta$ increases. Different from the $d_f$ in Fig. 4.8a, the production rate with larger $\beta$ is higher than that with smaller $\beta$ at the early-stage production. The reason is simple. Larger $\beta$ means that the fracture compressibility $d_f$ decreases faster with the increase of effective stress. In other words, large $\beta$ will weaken the influence of fracture stress-sensitivity on MFHW’s production rates. In addition to high initial conductivity and low initial compressibility $d_{fi}$, high $\beta$ should be another criterion in multi-stage
**Figure 4.9a** - Effect of declining rate of fracture compressibility $\beta$ on the transient rate behavior of MFHWs in log-log plot.

**Figure 4.9b** - Square-root time analysis for MFHWs with $F_c=13$ md.ft and different $\beta$. 

1/3

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fracturing designs. The proppant strength and size will influence the $\beta$. According to data in Table 4.1, it is summarized that stronger and smaller proppants lead to higher $\beta$.

Furthermore, the transient production behavior in Fig. 4.9 shows that in most cases the effect of $\beta$ appears gradually as the production time goes on. It is easy to miss the influence of fracture stress-sensitivity in the analysis of early-stage production data with single type of plots. My suggestion is to examine and compare the behavior of production rates in multiple types of plots (such as Figs. 4.9a and 4.9b). When the stress-sensitivity of hydraulic fractures cannot be identified, it is always good to apply both constant and stress-sensitive models for production prediction.

### 4.3.4 Shale Anisotropy

According to Eq. 4.4, the stress change is equal to the product of pore pressure changes and factor $K_0$. According to Sayers (1995, 2013), the factor $K_0$ can also be written as

$$K_0 = 1 - \frac{2C_{55}}{C_{11}} + \chi, \quad \chi = \frac{C_{13} + 2C_{55} - C_{11}}{C_{11}}.$$

Dipole sonic data from a well parallel to the symmetry axis help to calculate the $C_{55}/C_{11}$. But $\chi$ cannot be determined using sonic data in field analysis. Therefore, $\chi$ is always ignored in Eq. 4.14 and $K_0$ is underestimated as 0.24-0.34. In 2013, Sayers considered that kerogen content will influence the $K_0$ value and the range of $K_0$ was estimated as 0.33-0.37 for organic-rich shales. In this section, the $K_0$ is chosen from 0.2-1.0.
Fig. 4.10 shows the influence of factor $K_0$ on the production rate of a MFHW in shale reservoir. Larger $K_0$ makes the production rate lower at early-stage production. The $K_0$ corresponds to the ratio of effective stress change and pore pressure change around fractures. When $K_0$ becomes larger, the hydraulic fracture loses more conductivity due to the larger effective stress change. If $K_0$ is underestimated, the influence of fracture stress-sensitivity will be underrated. When $K_0$ is overestimated as 1, the pore pressure change is considered as the effective minimum horizontal stress change (Eq. 4.4). This assumption will amplify the increment of effective stresses during gas and oil production. Therefore, reasonable $K_0$ should be determined base on elastic properties of shale.

The $K_0$ of shale is related to the volume fraction of kerogen in shale. Kerogen acts to increase $K_0$ and accordingly increases the change of effective horizontal stress under a given pore pressure change. It could be inferred that the hydraulic fractures in organic-rich shales are subject to serious stress-sensitivity during production. In addition to the kerogen volume fraction, the existence of clay-rich layers in VTI shale rocks may also contribute to higher $K_0$ (Mokhtari et al., 2016).

Eqs. 4.2, 4.3, 4.4 and 4.14 reasonably link the effective stress change to the pore pressure. But these equations do have their limitations. Those equations are based on the assumption of poroelastic shale rocks. However, laboratory studies show that shales have time-dependent deformation that cannot be fully predicted by linear elasticity (Dudley et al., 1998; Chang and Zoback, 2009;
Figure 4. 10-Effect of $K_0$ on the transient rate behavior of MFHWs in log-log plot.
Sone and Zoback, 2014). Understanding the time-dependent inelasticity of shales is important to accurately model the long-term stress response inside shale reservoirs over time, which will be the focus of our future work.

**4.3.5 Drawdown Management**

For stress-sensitive fractures, an interesting question comes out: under the constant-pressure condition, is it possible for stress-sensitive fractures to have lower rates when the bottomhole pressure is smaller? Research from Rosen et al. (2014) and Tabatabaie et al. (2015) suggests that increasing the drawdown could reduce production rates because of stress-sensitive fracture conductivity reduction.

This section investigates the effect of drawdown on the MFHW’s performance in Fig. 4.11a when the initial fracture compressibility $d_{fi}$ is $10^{-7}$ Pa$^{-1}$ ($6.895 \times 10^{-4}$ psi$^{-1}$). Each curve in Fig. 4.11a shows the calculated rate $q$ when the bottomhole pressure $p_{wf}$ is constant during production. Fig. 4.11a indicates that when the hydraulic fracture conductivity is stress-sensitive, increasing the drawdown always results in higher production rate. Such conclusion is in contrast with the results from Rosen et al. (2014) and Tabatabaie et al. (2015). Fig. 4.11b further summarizes the cumulative production of MFHWs with different $d_{fi}$ when the drawdown increases from 0 to 90MPa ($1.305 \times 10^4$ psi). Higher $d_{fi}$ makes the cumulative production lower. Although increasing the drawdown could achieve higher cumulative production, the increment of cumulative production tends to be smaller with larger drawdown. There should exist a point for each curve in Fig. 4.11b, beyond which the cumulative
Figure 4. 11a-The effect of $\Delta p=p_i-p_{wf}$ on the transient rate behavior of MFHWs with $F_{ci}=13\text{md.ft}$, $d_f=1\times10^{-7}$ Pa$^{-1}$, $\beta=1\times10^{-10}$ Pa$^{-1}$ and $K_0=1$ in log-log plot.

Figure 4.11b-The cumulative production vs. pressure drop $\Delta p=p_i-p_{wf}$ for MFHWs with $F_{ci}=13\text{md.ft}$, $\beta=1\times10^{-10}$ Pa$^{-1}$ and $K_0=1$. 
production increment is negligible.

**Figs. 4.11a and 4.11b** don’t mean that reducing flowing bottomhole pressure as low as possible is feasible in drawdown management. There are many other factors that should be considered in field production, such as production schedules, contracts, economics and regulatory (Tabatabaie et al., 2016). Moreover, a combination of stress-sensitive matrix and fracture could result in more complex production behavior, which is also included in the future plan.

In order to mitigate the effect of stress-sensitivity, many producers reduce the \( p_{wf} \) gradually rather than keep a low, constant \( p_{wf} \). This section also investigates the influence of different drawdown schemes on the cumulative production. **Fig. 4.12a** gives six bottomhole pressure profiles when the \( p_{wf} \) decreases from 50MPa (7252psi) to 10MPa (1450psi) in 10 years. The pressure curves 1 and 2 show that the \( p_{wf} \) reduces slowly in the first two years and drops rapidly in the next four years. Pressure curves 4 and 5 mean that the \( p_{wf} \) drops rapidly in the first two years and reduces slowly in the next four years. The \( p_{wf} \) in curve 3 gradually reduces to 10 MPa. The pressure profiles are applied to MFHWs with different \( d_f \) values and **Figs. 4.12b-4.12d** compare the cumulative productions. The best drawdown scheme balances the production efficiency and final cumulative rates. Accordingly, the constant \( p_{wf} \) is the best choice for the MFHW with \( d_f \) as \( 5 \times 10^{-8} \) Pa\(^{-1}\). But in **Figs. 4.12c and 4.12d**, pressure profiles 5 and 4 are the most appropriate drawdown profiles for stress-sensitive fractures with \( d_f \) as \( 1 \times 10^{-7} \) Pa\(^{-1}\) and \( 3 \times 10^{-7} \) Pa\(^{-1}\), respectively.
Figure 4.12a-The six types of bottomhole flowing pressure profiles.

Figure 4.12b-The cumulative production rate of MFHWs with $F_{ci}=13\text{md.ft}$, $d_{f}=5 \times 10^{-8} \text{ Pa}^{-1}$, $\beta=1 \times 10^{-10} \text{ Pa}^{-1}$ and $K_0=1$ when different bottomhole flowing pressure profiles are applied.
Figure 4.12c-The cumulative production rate of MFHWs with $F_{ci}=13\text{md.ft}$, $d_{fi}=1 \times 10^{-7}$ Pa$^{-1}$, $\beta=1 \times 10^{-10}$ Pa$^{-1}$ and $K_0=1$ when different bottomhole flowing pressure profiles are applied.

Fig. 12d-The cumulative production rate of MFHWs with $F_{ci}=13\text{md.ft}$, $d_{fi}=3 \times 10^{-7}$ Pa$^{-1}$, $\beta=1 \times 10^{-10}$ Pa$^{-1}$ and $K_0=1$ when different bottomhole flowing pressure profiles are applied.
It is concluded that for MFHWs with negligible stress-sensitivity, the constant low $p_{wf}$ is a better option than other drawdown schemes. But for MFHWs with serious stress-sensitivity, a quick bottomhole pressure drop followed by a late-stage slowdown can achieve higher productivity in the long term. The best drawdown scheme is dependent on the parameters of stress-sensitive fractures. Although a large pressure drop gradient is desired at early-stage production, the gradient for fractures with larger $d_{fl}$ should be smaller than fractures with smaller $d_{fl}$.

4.3.6 Linear Flow Analysis with Corrections

When hydraulic fractures have high conductivities, linear flow is usually the longest-acting transient flow regime for shale reservoirs. The plot of $(p_i-p_{wf})/q$ vs. $\sqrt{t}$ during linear flow always follows a straight line with $m$ and $b$ as the slope and intercept. Stress-sensitive hydraulic fractures result in deviation from straight lines in the square-root-of-time plot (Figs. 4.8b and 4.8c). An easy way to account for such deviation is to apply a time-dependent $b(t)$ rather than constant $b$ (Clarkson et al., 2013). Our rigorous model works as basis for investigating $b(t)$ patterns during linear flow. Fig. 4.13 gives the calculated $b$ vs. $t$ in the first 200 days’ production for constant-bottomhole-pressure cases listed in Figs. 4.8a and 4.9a. It shows that power equations $b=nt^c$ could match most curves of $b$ vs. $t$, which makes a new equation for linear flow:

$$\frac{p_i-p_{wf}}{q} = mt^{\frac{1}{c}} + nt^c.$$

Pseudopressure should be used in Eq. 4.15 for gas production. In field production analysis, the $m$, $n$ and $c$ are estimated by matching field production
Figure 4. 13a-The calculated $b$ vs. $t$ (data points) based on cases in Fig. 4.8a and best-matching power equations (dash lines) with coefficient of determination $R^2$ (original in color).
The calculated $b$ vs. $t$ (data points) based on cases in Figs. 4.8a and 4.9a and best-matching power equations (dash lines) with coefficient of determination $R^2$ (original in color).

\begin{align*}
y &= 1.8174x^{0.1878} \\
R^2 &= 0.9933 \\
y &= 0.177x^{0.2556} \\
R^2 &= 0.9999 \\
y &= 0.1721x^{0.2333} \\
R^2 &= 0.9997 \\
y &= 0.1544x^{0.13} \\
R^2 &= 0.9922 \\
y &= 0.1303x^{0.0289} \\
R^2 &= 0.8591 \\
y &= 1.8174x^{0.1878} \\
R^2 &= 0.9933 \\
y &= 0.177x^{0.2556} \\
R^2 &= 0.9999 \\
y &= 0.1721x^{0.2333} \\
R^2 &= 0.9997 \\
y &= 0.1544x^{0.13} \\
R^2 &= 0.9922 \\
y &= 0.1303x^{0.0289} \\
R^2 &= 0.8591
\end{align*}
data with Eq. 4.15. Preliminary analysis in Fig. 4.13 shows that with same $d_f$ and $\beta$, higher $F_{ci}$ results in smaller $n$ and $c$. With same $F_{ci}$, larger $d_f$ and smaller $\beta$ make $n$ larger.

4.3.7 Field Example

This case is from Clarkson et al. (2013) for a MFHW in the Haynesville shale gas reservoir. The well was completed in 18 stages with 8 perforations spaced on average 27 ft apart in each stage. Fig. 4.14a shows the bottomhole pressure and rates data during 730 days’ production. Different from No. 1 field case, the flowing pressure here continuously decreased over time. Considering the changing $p_{wf}$, Fig. 4.14b uses rate-normalized pseudopressure difference $\Delta m(p)/q$ vs. $\sqrt{t}$ to examine the linear flow regime. In fact, Fig. 4.14b shows no straight line can be inferred after 25 days’ production because the $\Delta m(p)$ increases very fast. Clarkson et al. summarized that such nonlinearity may be caused by hydraulic fracture stress-sensitivity and matrix permeability degradation.

The effect of stress-sensitive hydraulic fractures is investigated in this study. As discussed in the Drawdown Management section, different flowing pressure profiles cause different production behaviors even for same stress-sensitive fractures. Therefore pressure-normalized rate analysis techniques may introduce extra errors for stress-sensitive fractures. In this study the flowing pressure profile in Fig. 4.14a works as input for our model. And the $p_{wf}$ is assumed to be constant as 13MPa after 760 days’ production. Since the geomechanics data is unknown, the $K_0$ is set to be 1. Clarkson et al. (2013) estimated the fracture half-
Figure 4. 14- (a) Gas rates, water rates and the calculated flowing bottomhole pressure of the MFHW. (b) Rate-normalized pseudopressure vs. square root of time (original in color).
length as 340ft (104m) and the initial permeability of stress-sensitive matrix as 0.001md. Fig. 4.15a gives our model results with fracture half-length 328ft (100m) and matrix permeability 0.00048 md. The combination of $d_f = 1.7 \times 10^{-7}$ Pa$^{-1}$ (1.1$\times 10^{-3}$ psi$^{-1}$) and $\beta = 1 \times 10^{-8}$ Pa$^{-1}$ (6.9$\times 10^{-5}$ psi$^{-1}$) is the best matching curve. During the first 10 days’ production, the gas rate increases with decreasing flowing pressure $p_{wf}$ although fractures’ conductivities keep reducing. Later gas rates decrease because of depleted reservoir pressure and stress-sensitive fractures. If a constant $p_{wf} = 13$ MPa is chosen (long dash lines in Fig. 4.15a) during the whole production history, the cumulative rate after 730 days’ production is only 85% of the cumulative amount with dynamic $p_{wf}$ (solid lines in Fig. 4.15a). Fig. 4.15b further gives rate-normalized pseudopressure vs. $\sqrt{t}$ for simulated cases without stress-sensitivity. Production out of the MFHW exhibits clear linear flow regime (green solid line in Fig. 4.15b) when hydraulic fractures have constant conductivities during production.

4.4 Conclusions

The study of stress-sensitive hydraulic fractures is crucial for optimizing and monitoring fracturing treatments, especially in shale reservoirs. This chapter contributes to the semi-analytical modeling of transient pressure and rate behavior for fractured horizontal wells with considering stress-dependent fractures and shale anisotropy. This chapter focuses on the analysis of MFHWs’ transient rate behavior. Non-constant bottomhole pressure profiles can be input of the semi-analytical model. For simplicity, all hydraulic fractures are assumed to have same stress-sensitive characteristics and fully penetrate the pay zone.
Figure 4.15-(a) Gas rate data vs. semi-analytical model results in log-log plot and semi-analytical model cumulative rates vs. time in Cartesian coordinate. (b) Rate-normalized pseudopressure data vs. semi-analytical model results in square-root-of-time plot (original in color).
And all boundaries of the reservoir are assumed to be closed. Based on results and discussion, several findings are obtained:

1. Compared with constant \( d_f \) assumption in the literature, the relationship of dynamic \( d_f \) vs. \( \Delta p \), in this study is more practical and flexible in describing the stress-sensitive behavior of hydraulic fractures.

2. With stress-sensitive hydraulic fractures, the slope of \( q \) vs. \( t \) in log-log plot during the linear flow regime is no longer 0.5. It also becomes difficult to find straight lines to match \( 1/\sqrt{q} \) vs. \( \sqrt{t} \) in the square-root time analysis during linear flow regime.

3. Higher initial fracture compressibility \( d_{fi} \) and smaller compressibility declining rate \( \beta \) tend to aggravate fracture conductivity loss and reduce production rates.

4. For the same \( d_{fi} \) and \( \beta \), the percentage of rate loss resulted from stress-sensitivity is smaller for fractures with higher initial conductivities. The fracture designs with improved initial conductivities are more resistant to production loss caused by fracture conductivity reduction.

5. Ignoring the \( K_0 \) means ignoring the difference between pore pressure change \( \Delta p_p \) and effective minimum horizontal stress increment \( \Delta \sigma' \). When shale is considered as layered TI medium, higher kerogen content tend to raise the \( K_0 \) and increase stress changes when pore pressure decreases during production.

6. The best drawdown scheme for MFHWs is selected according to the hydraulic fracture stress-sensitivity. When hydraulic fractures have \( d_{fi} \) smaller than \( 5 \times 10^{-8} \) Pa\(^{-1} \) (3.45 \times 10^{-4} \) psi\(^{-1} \), the constant \( p_{wf} \) is the best option for drawdown
management. When hydraulic fractures have $d_f$ larger than $5 \times 10^{-8} \text{ Pa}^{-1}$ ($3.45 \times 10^{-4} \text{ psi}^{-1}$), it’s better to choose a quick pressure drop at early-stage production followed by a slow pressure drop later.

7. When stress-sensitive hydraulic fractures are assumed to have constant conductivities, the common transient rate analysis may underestimate the MFHW’s productivity at late stage.
CHAPTER 5

COMPOSITE MODELING OF FLOW IN THE
UNCONVENTIONAL RESERVOIR STIMULATED WITH A
FRACTURED HORIZONTAL WELL

5.1 Introduction

At present, combining horizontal well drilling and multi-stage hydraulic fracturing treatments becomes a common way to stimulate the low-permeability unconventional reservoirs. When a pressure difference is applied between the horizontal wellbore and reservoir, the hydrocarbon stored in naturally-fractured/intact porous rocks flows into hydraulic fractures then converges in the horizontal wellbore that is connected to the land surface. The resulted hydrocarbon production is a combination of flow in hydraulic fractures, in the matrix and/or in natural fractures. Modeling of such complex fluid flow is critical for production evaluation and optimization in low-permeability unconventional reservoirs.

Low-permeability unconventional reservoirs have complex characteristics. Complex rock mechanics, such as time-dependent inelasticity (Sone and Zoback, 2014) and stress-sensitivity (Moghadam and Chalaturnyk 2016), and intricate transport phenomena, such as gas adsorption/desorption (Pang et al. 2017) and slip and transition flow (Javadpour, 2009) may coexist in many low-permeability unconventional reservoirs. In addition, many unconventional formations are highly heterogeneous in reservoir properties, including porosity,
permeability, rock compressibility, etc. (Zawila et al., 2015). In multi-stage hydraulic fracturing, the heterogeneous formation reacts differently at different fracturing stages and the generated fracture network is also highly heterogeneous. Johri and Zoback (2013) showed a multifractured horizontal well (MFHW) with heterogeneous completion in a heterogeneous reservoir. The hydraulic fractures’ contributions to production are different at different stages.

Thorough consideration of these complex reservoir conditions is extremely important for unconventional reservoir development. Existing numerical models (Mayerhofer et al. 2006; Medeiros et al. 2008; Freeman et al. 2013) are versatile in considering multiple aforementioned phenomena. But considering such complex conditions significantly increases the computational cost/time of those numerical models. And the convergence problem and big calculation errors are much more likely to occur in numerical models. Numerical models still have other disadvantages such as implicit functional relationships of key parameters and the inconvenience in iterative applications (Brown et al. 2011). A plurality of analytical/semi-analytical models (Chen and Raghavan 1997; Yao et al. 2013; Huang et al. 2015) have also been developed to characterize the flow dynamics in reservoir and the production behavior of MFHWs. Analytical models always provide fast and accurate calculations but are rarely applicable in complex reservoir conditions such as heterogeneity, stress-sensitivity and slip flow. So a model would be in demand if it can shorten calculation time, provide explicit functional relationships as well as keep the key flow characteristics under complex reservoir conditions.
The idea of decomposing a reservoir into discrete parts with independent reservoir properties provides possibilities for executing the vision. In the literature, radial composite analytical/semi-analytical models consider the formation as a radial composite system that consists of two zones—the stimulated reservoir volume (SRV) and the outer intact zone (Zhao et al., 2014; Ren and Guo, 2017). The MFHW is located at the center of radial SRV zone (Fig. 5.1a). In fact, the hydrocarbon produced out of a MFHW mainly comes from the SRV (Mayerhofer et al. 2010). And the formation characteristics around a MFHW have significant influence on the MFHW production behavior. But the radial composite analytical/semi-analytical models in the literature cannot describe the complex reservoir properties of SRV. In 1981, Cinco-Ley and Samaniego-V and Guppy et al. first showed the idea of regarding a stimulated reservoir as combinations of different flow regions. They developed a bilinear analytical model which combines two rectangular linear flow regions (Fig. 5.1b). Each flow region can have different characterises, such as permeability, porosity, etc. In 1986, Lee and Brockenbrough further developed a trilinear flow model for the fractured vertical well by coupling one fracture linear flow region and two formation linear flow regions. Ozkan et al. (2009) and Brown et al. (2011) proposed that the trilinear model also preserves the key characteristics of fluid flow towards a MFHW in unconventional reservoirs (Fig. 5.1c). Stalgorova and Mattar (2013) extended the trilinear flow model to a five-region model (Fig. 5.1d). In Fig. 5.1d, Region 1 (adjacent to the hydraulic fracture region) usually has higher permeability than Regions 2-4. Both trilinear and five-region models
Figure 5. (a) Radial composite model with a multifractured horizontal well (MFHW), simulated reservoir volume (SRV) and outer zone. (b) Bi-linear flow model for a fractured vertical well (Cinco-Ley and Samaniego-V, 1981). (c) Tri-linear flow model (Brown et al. 2011). (d) Five-region flow model (Stalgorova and Mattar, 2013).
assume that the production behavior of each hydraulic fracture is the same as other fractures along the wellbore. Zeng (2018) built a seven-region model for hydraulic fractures that partially penetrate the target formation. Wang et al. (2015) and Yuan et al. (2015) improved the trilinear and five-region models with considering fractal permeability, stress-sensitivity and non-Darcy flow.

With simple flow equations for each region, above multi-linear models have advantages in fast calculations and applicability in complex reservoir conditions. But those multi-linear models in the literature are still limited in two aspects. Most of the multi-linear models assume that the production behavior of each fracture stage is the same. But for example, in heterogeneous reservoirs hydraulic fractures’ contributions to productivity may be different at different stages. In addition, the multi-linear models all assume that the fluid flow in each region is linear flow. But the validity of assuming the linear flow beyond hydraulic fracture tips depends on the geometry of reservoirs and hydraulic fractures. In fact, fluids tend to converge toward fracture tips when a high-density network of hydraulic fractures is created along a horizontal wellbore in the “Simulfrac” and “Zipperfrac” treatments (Sesetty and Ghassemi 2015). Existing multi-linear models become inapplicable in describing the flow behavior in formations beyond fracture tips, which further weakens their capability in considering pressure/stress-dependent phenomena (such as stress-sensitive reservoir permeability) in the future.

To overcome the limitations as well as reserve the advantages, this chapter extends and generalizes the multi-linear models with the idea of
“decomposition”. The reservoir is regarded as combinations of several sub-systems and each sub-subsystem is further composed of simple flow regions. The fluid flow in regions can be linear flow, radial flow or flow from continuous source/sink. By coupling solutions of the fluid flow in each region, this study finally gives solutions of the production from a MFHW.

5.2 Mathematical Modeling

5.2.1 Background

Fig. 5.2 gives an example of a MFHW. Typically the reservoir which contributes to production is simplified as a cuboid with dimension $L \times W \times H$. The MFHW is at the center line of the reservoir. All hydraulic fractures are assumed to be bi-wing and fully penetrate the target formation. The $i$-th hydraulic fracture has fracture half-length $x_{fi}$, width $w_{fi}$, and conductivity $k_{fi}w_{fi}$. Hydraulic fractures are not equally spaced along the wellbore. Table 5.1 lists the details of well and reservoir characteristics.

Fig. 5.3 displays the dimensionless pressure $p_D$ (Appendix A) distribution when the dimensionless time $t_D$ (Appendix A) is equal to 0.0025, 0.025, 0.1 and 0.5 when the MFHW of Fig. 5.2 is producing in a homogeneous reservoir. The fluid in reservoir flows in the direction of pressure gradients. The pressure distribution shows that each hydraulic fracture controls a part of the reservoir in which the fluid only flows towards the hydraulic fracture. The part of the reservoir that a hydraulic fracture controls is defined as a sub-system with all-closed boundaries. Usually the sub-system boundaries are curved. However, dashed straight lines in Fig. 5.3 can represent key parts of the boundaries. The
Figure 5. A 6-stage Fractured Horizontal Well in a low-permeability unconventional reservoir with length $L$, width $W$ and pay zone thickness $H$. 
Table 5.1 Reservoir and wellbore parameters for the MFHW in Fig. 5.2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reservoir lateral length $L$ (m)</td>
<td>560</td>
</tr>
<tr>
<td>Well Spacing $W$ (m)</td>
<td>160</td>
</tr>
<tr>
<td>Pay zone thickness $H$ (m)</td>
<td>50</td>
</tr>
<tr>
<td>Reservoir permeability $k$ (m$^2$)</td>
<td>$10^{-17}$ (0.01 md)</td>
</tr>
<tr>
<td>Reservoir compressibility $c_r$ (pa$^{-1}$)</td>
<td>$1.45 \times 10^{-10}$ ($10^{-6}$ psi$^{-1}$)</td>
</tr>
<tr>
<td>Reservoir porosity $\phi$</td>
<td>0.1</td>
</tr>
<tr>
<td>Viscosity $\mu$ (Pa·s)</td>
<td>0.001 (1 cp)</td>
</tr>
<tr>
<td>Fracture Stages $n$</td>
<td>6</td>
</tr>
<tr>
<td>Fracture half-length $x_f$ (m)</td>
<td></td>
</tr>
<tr>
<td>Stage 1: 70</td>
<td>Stage 4: 62</td>
</tr>
<tr>
<td>Stage 2: 40</td>
<td>Stage 5: 32</td>
</tr>
<tr>
<td>Stage 3: 50</td>
<td>Stage 6: 75</td>
</tr>
<tr>
<td>Fracture $x$-coordinates (start from heel as zero, m)</td>
<td></td>
</tr>
<tr>
<td>Stage 1: 69</td>
<td>Stage 4: 341</td>
</tr>
<tr>
<td>Stage 2: 179</td>
<td>Stage 5: 434</td>
</tr>
<tr>
<td>Stage 3: 262</td>
<td>Stage 6: 503</td>
</tr>
<tr>
<td>Fracture conductivities $k_{fWY}$ ($10^{-14}$m$^3$)</td>
<td></td>
</tr>
<tr>
<td>Stage 1: 1.52 (50 md-ft)</td>
<td>Stage 4: 4.57 (151 md-ft)</td>
</tr>
<tr>
<td>Stage 2: 3.05 (102 md-ft)</td>
<td>Stage 5: 7.62 (253 md-ft)</td>
</tr>
<tr>
<td>Stage 3: 3.05 (102 md-ft)</td>
<td>Stage 6: 3.00 (100 md-ft)</td>
</tr>
</tbody>
</table>


Figure 5. 3-Dimensionless pressure $p_D$ distribution in the reservoir with a 6-stage fractured horizontal well at (a) $t_D=0.0025$, (b) $t_D=0.025$, (c) $t_D=0.1$ and (d) $t_D=0.5$ (original in color).
sub-system boundaries also move along the wellbore during production. But the change of their locations is very small. Therefore, the reservoir in Fig. 5.3 can be simplified as six sub-systems with fixed straight boundaries.

Take Sub-system 2 (130m-220m in \(x\)-direction) for example. In Sub-system 2, most of the fluid beyond the fracture tip converges towards the tip. On the left and right parts of Sub-system 2, most of the fluid flows perpendicularly to the hydraulic fracture. Inside the fracture, the fluid moves towards the horizontal wellbore \( (y=80m) \). The fluid flows in other sub-systems are quite similar to that in Sub-system 2. It is concluded that every sub-system can be considered as combinations of several flow regions. And each flow region has one kind of simple fluid flow.

5.2.2 Basic Flow Regions

In a reservoir stimulated with MFHWs, each sub-system is regarded as combinations of different flow regions. According to Fig. 3, there are in total five kinds of basic flow regions. These five flow regions are (1) source/sink region, (2) radial flow region, (3) \(x\)-direction linear flow region, (4) \(y\)-direction linear flow region and (5) hydraulic fracture region.

5.2.2.1 Source/Sink Region

Fig. 5.4a shows the basic flow region with a continuous source/sink inside. All boundaries of this region are closed. If a source with rate \( q \) exists at Point A \((x_A, y_A)\), the dimensionless pressure distribution in the flow region can be described with the partial differential equation (PDE):

\[
\frac{\partial^2 \tilde{p}_D}{\partial x_D^2} + \frac{\partial^2 \tilde{p}_D}{\partial y_D^2} = \frac{1+\beta}{\eta} \left( u \tilde{p}_D - p_D (\epsilon_D = 0) \right)
\]
Figure 5.4-(a) Source/sink Region; (b) Radial flow region.

Figure 5.4-(c) ±x-direction linear flow region.

Figure 5.4-(d) ±y-direction linear flow region. (e) Hydraulic fracture region (original in color).
with the initial pressure distribution as

\[ p_D(t_D = 0) = f(x_D, y_D). \]

(5.1b)

inner boundary condition as

\[ \frac{\partial \bar{p}_D}{\partial x_D} + \frac{\partial \bar{p}_D}{\partial y_D} \bigg|_{x_D=x_{AD} \to x_{AD} \to y_{AD}} = \bar{q}_D. \]

(5.1c)

and closed outer boundary conditions:

\[ \frac{\partial \bar{p}_D}{\partial x_D} \bigg|_{x=0, x=a} = 0, \]

(5.1d)

\[ \frac{\partial \bar{p}_D}{\partial y_D} \bigg|_{y=0, y=b} = 0. \]

(5.1e)

The \( u \) is Laplace variable corresponding to the time variable \( t \). The \( \beta \) represent the contribution of extra source/sink term \( R \), i.e. gas adsorption/desorption, to the fluid flow: \( \beta = \frac{\partial R}{\partial p_D} \). But Eq. 5.1a with \( \beta \) is valid only if the \( \beta \) is independent of the unknown variable \( p \). Definitions of other dimensionless variables can be found in Appendix A.

**Appendix A.** Eq. 5.1b applies a continuous function \( f(x_D, y_D) \) to represent the pressure distribution at time \( t=0 \). Eq. 5.1a’s solution includes the complementary solution to its corresponding homogeneous equation and a particular solution. According to the Eq. 5.1a’s general solution, the dimensionless pressure at Point B \((x_B, y_B)\) in Laplace domain is

\[ \bar{p}_D(x_{BD}, y_{BD}) = \alpha \bar{q}_D \int_0^\infty S_x(x_{AD}, x_{BD}) S_y(y_{AD}, y_{BD}) e^{-\mu u t} d\tau_D + Q(x_D, y_D). \]

(5.2)

**Appendix D** lists detailed definitions of \( \alpha, S_x \) and \( S_y \) in the complementary solution of Eq. 5.1. Here \( Q(x_D, y_D) \) is the particular solution that satisfies

\[ \frac{\partial^2 Q}{\partial x_D^2} + \frac{\partial^2 Q}{\partial y_D^2} = (1 + \beta) \frac{u Q - f(x_D, y_D)}{\eta}. \]
5.2.2.2 Radial Flow Region

Fig. 5.4b shows the basic radial flow region. The fluid flow in this rectangular-shaped region behaves as radial flow. The $r_\text{i}$ and $r_\text{e}$ are hypothetical inner and outer boundaries. The radial flow equation is applied:

$$\frac{1}{r_D} \frac{\partial}{\partial r_D} \left( r_D \frac{\partial \tilde{p}_D}{\partial r_D} \right) = \frac{1+\beta}{\eta} \left( u\tilde{p}_D - p_D(t_D = 0) \right), r_{D_1} \leq r_D \leq r_{D_2}. \tag{5.3}$$

The initial pressure distribution can be represented by $p_D(t_D = 0) = f(r_D)$. The general solution of Eq. 5.3 is

$$\tilde{p}_D = A_r I_0 \left( \frac{u}{\sqrt{\eta}} r_D \right) + B_r K_0 \left( \frac{u}{\sqrt{\eta}} r_D \right) + Q(r_D). \tag{5.4}$$

Here $I_0$ and $K_0$ are the first and second kind of zero-order modified Bessel equations, respectively. The coefficients $A_r$ and $B_r$ are listed in Appendix D. The particular solution $Q(r_D)$ should follow

$$\frac{1}{r_D} \frac{\partial}{\partial r_D} \left( r_D \frac{\partial q}{\partial r_D} \right) = (1 + \beta) \frac{uQ-f(r_D)}{\eta}.$$  

5.2.2.3 X-direction Linear Flow Region

Fig. 5.4c shows the flow region with the linear flow in $\pm x$-direction. The linear flow equation can give the pressure distribution in this region:

$$\frac{\partial^2 \tilde{p}_D}{\partial x_D^2} = C(u, \beta, \eta) \tilde{p}_D - (1 + \beta) \frac{p_D(t_D = 0)}{\eta}. \tag{5.5}$$

Similar to the radial flow region, the initial pressure distribution in this linear flow region can be written as $p_D(t_D = 0) = f(x_D)$. There are four boundaries I-IV for this linear flow region in Fig. 5.4c. The $x$-direction linear flow from other regions enters this region through Boundary III and leaves this region through Boundary I. Boundaries II and IV receive $y$-direction linear flow from other adjacent regions. The general solution of Eq. 5.5 becomes

$$\tilde{p}_D = A_x \cosh[-|x_D - x_{1D}|\sqrt{C}] + B_x \sinh[-|x_D - x_{1D}|\sqrt{C}] + Q(x_D). \tag{5.6}$$
The coefficients $A$, $B$, and $C$ are derived in Appendix D. The particular solution $Q(x_D)$ should satisfy \( \frac{\partial^2 Q}{\partial x_D^2} = C(u, \beta, \eta)Q - (1 + \beta) \frac{f(x_D)}{\eta}. \)

### 5.2.2.4 $Y$-direction Linear Flow Region

Fig. 5.4d shows the basic region with the linear flow in $\pm y$-direction. The linear flow equation is similar to Eq. 5.5:

\[
\frac{\partial^2 \tilde{p}_D}{\partial y_D^2} = D(u, \beta, \eta)\tilde{p}_D - (1 + \beta) \frac{p_D(t_D=0)}{\eta}. \]

The $y$-direction linear flow from other regions enters this region through Boundary III and leaves through Boundary I. Boundaries II and IV receive $x$-direction linear flow from other adjacent regions. The general solution of Eq. 5.7 is

\[
\tilde{p}_D = A_y \cosh[-|y_D - y_{1D}|\sqrt{D}] + B_y \sinh[-|y_D - y_{1D}|\sqrt{D}] + Q(y_D). \]

The derivation of coefficient $A_y$, $B_y$ and $D$ also refers to the Appendix D. The particular solution $Q(y_D)$ can be found in \( \frac{\partial^2 Q}{\partial y_D^2} = D(u, \beta, \eta)Q - (1 + \beta) \frac{f(x_D)}{\eta} \) with the initial pressure distribution $p_D(t_D = 0) = f(y_D)$.

### 5.2.2.5 Hydraulic Fracture Region

Fig. 5.4e shows the hydraulic fracture region. The governing equation for the linear flow in a hydraulic fracture (propagation direction: $y$-direction) is

\[
\frac{\partial^2 \tilde{p}_{FD}}{\partial y_D^2} = H(u, \beta, \eta)\tilde{p}_{FD} + f(u). \]

The general solution for Eq. 5.9 is

\[
\tilde{p}_D = A_F \cosh[-|y_D - y_{1D}|\sqrt{H}] + B_F \sinh[-|y_D - y_{1D}|\sqrt{H}] + Q(u). \]

Here $H$ (in Appendix D) is influenced by flow regions on the left and right side of the hydraulic fracture. The coefficients $A_F$ and $B_F$ are determined by the
boundary conditions at the fracture/wellbore intersection and fracture tip. The particular solution \( Q(u) \) should satisfy \( H(u, \beta, \eta)Q(y_D) + f(y_D) = 0 \).

Transient dual-porosity modification can be introduced to any kind of the five flow regions. The Laplace variable \( u \) in all above equations is replaced by \( s \) to represent the transient dual-porosity:

\[
s = u \left( 1 + \sqrt{\frac{\lambda \omega}{3u}} \tanh \left( \frac{3\omega u}{\lambda} \right) \right).
\]

Serra et al. (1983) and Brown et al. (2011) give detailed definitions of the storativity ratio \( \omega \) and flow capacity ratio \( \lambda \).

### 5.2.3 Composite Modeling

Combinations of basic flow regions are various in one sub-system. The guideline of coupling any two adjacent regions in one sub-system is that the pressure and/or flow rates at the two regions’ interface should be same. Figs. 5.1b-5.1d gives three kinds of region combinations in the literature. Fig. 5.5 lists another four kinds of region coupling in a sub-system. In Fig. 5.5a, a sub-system is comprised of four regions I-IV. Region I is a source/sink region. Regions II and III are \( x \)-direction linear flow regions. And Region IV is the fracture region. The flow rate out of Point A in Region I is equal to that into the fracture tip of Region IV. And the pressure at Point B in Region I is equal to that at the fracture tip in Region IV. The fluid out of Regions II and III enters the fracture through Planes C and D. The hydraulic fracture Region IV receives fluid from all other three regions and conducts the flow into the wellbore. In Fig. 5.5b, there are also two
Figure 5. Case 1(a), Case 2(b), Case 3(c) and Case 4(d) of region coupling in a sub-system (original in color). Case 1 includes a source/sink region while Case 2 has a radial flow region. Case 3 and 4 have multiple $x$- and $y$-direction linear flow regions.
$x$-direction linear flow regions (Regions II and III) and one hydraulic fracture region (Region IV). Different from Fig. 5.5a, Region I in Fig. 5.5b is a radial flow region. The flow rate and pressure at inner boundary $r = r_w$ in Region I equal those at the fracture tip in Region IV. The flow rate at outer boundary $r = r_e$ is set to be zero. Fig. 5.5c shows that in one sub-system, there could be one source/sink region, one hydraulic fracture region and multiple $x$- and $y$-direction linear flow regions. Fig. 5d demonstrates that a sub-system could also comprise one hydraulic fracture region, multiple $x$-direction linear flow regions and multiple radial flow regions. The Appendix D gives the coupling equations for Figs. 5.5a-5.5d. For the $i$-th of $n$ sub-systems in a reservoir, coupling all flow regions in the $i$-th sub-system gives

$$p_{iFD}(y_{0d}) = A_i q_{id}.$$ ..........................................................(5.12)

The $q_{id}$ is the dimensionless flow rate out of $i$-th hydraulic fracture in $i$-th sub-system. The $\bar{p}_{iFD}$ is the dimensionless bottomhole pressure at the $i$-th fracture/wellbore intersection. Appendix D explains the details of the coefficient $A_i$. It is important for region coupling to choose appropriate Point B $(x_B, y_B)$ in source/sink region and inner/outer boundary $(r_w, r_e)$ in radial flow region. Appendix E lists the procedure of determining these parameters for best model performance.

Coupling of sub-systems can generate solutions for the reservoir-MFHW system. The sub-system boundaries should be determined before sub-system coupling. Theoretically any two adjacent sub-systems should have same pressure all the time at the shared boundary. This provides a basis for the
Figure 5.6 - An illustration of sub-system coupling for the 6-stage MFHW. And in each of the 6 sub-systems, several flow regions are already coupled together.
sub-system boundary determination. In this work sub-system boundaries are chosen to minimize the pressure difference of adjacent sub-systems at the shared boundaries. Appendix E gives the detailed procedure of determining boundaries for sub-systems.

Fig. 5.6 shows one sub-system combination for the 6-stage MFHW in Fig. 5.2. The pressure at all the fracture/wellbore intersections is assumed to be the same under either constant-pressure or constant-rate production conditions. The bottomhole pressure and rate can be calculated by equalizing Eq. 5.12 from each sub-system. When the total production rate $q$ out of an $n$-stage MFHW is constant, $n$ linear equations in Laplace domain can be obtained:

$$
\begin{pmatrix}
A_1 & -A_2 & & & \\
A_2 & -A_3 & & & \\
& A_3 & -A_4 & & \\
& & & \ddots & \\
& & & & A_{n-1} & -A_n \\
1 & 1 & 1 & \cdots & 1 & 1
\end{pmatrix}
\begin{pmatrix}
\bar{q}_{1D} \\
\bar{q}_{2D} \\
\bar{q}_{3D} \\
\vdots \\
\bar{q}_{nD}
\end{pmatrix}
= 
\begin{pmatrix}
0 \\
0 \\
0 \\
0 \\
\bar{q}_D
\end{pmatrix},
\text{..........................(5.13)}
$$

Solutions of Eq. 5.13 give the flow rate $\bar{q}_{iD}$ out of $i$-th hydraulic fracture, the bottomhole pressure $\bar{P}_{iFD}$ of the horizontal wellbore and the pressure distribution of each flow region in the Laplace domain. Stehfest algorithm (1970) can transform the $\bar{q}_{iD}$ and $\bar{P}_{iFD}$ from Laplace domain to the $q_{iD}$ and $P_{iFD}$ in real-time domain.

5.2.4 Methodology Applicability

The composite methodology introduced here is versatile in modeling the fluid flow in reservoirs with reservoir heterogeneity, matrix stress-sensitivity, slip and transition flow, fracture stress-sensitivity, and/or gas adsorption/desorption. For modeling reservoir heterogeneity, each flow region could have a set of reservoir
properties (such as permeability $k$, porosity $\phi$ and compressibility $c_t$) that is different from other flow regions, even in the same sub-system. For the stress-sensitivity and slip/transtion flow, matrix or fracture permeabilities change along with reservoir pore pressure and the PDEs become highly non-linear. It is quite difficult to obtain analytical solutions of these non-linear PDEs. A feasible way is to replace the time variable $t$ in the PDEs with the time difference variable $\Delta t$ for the target flow regions. Then the matrix or fracture permeabilities are assumed to be constant during each $\Delta t$ time step. But the initial pressure at the beginning of each time step, i.e. $\Delta t=0$, could hardly be 0. Therefore the key step when applying the composite methodology in solving stress-sensitivity problems is to achieve the initial pressure distribution function at the start of each $\Delta t$ time step. The procedure of coupling target stress-sensitive or slip-flow regions with regular regions could refer to Chapter 4. With the composite methodology, it also becomes easier to consider the gas adsorption/desorption phenomenon. In the literature, Langmuir adsorption isotherms are widely applied to describe the adsorbed gas mass vs. gas pressure. However, the derivative of adsorbed gas mass over pressure is still related with the pressure itself. So the $\beta$ in aforementioned PDEs is a function of pressure $P$. The choice of time difference step and the assumption of constant $\beta$ during each time step can well solve the problem.

The composite methodology’s accuracy and stability depend on the flow region types and coupling manner. The geometries of reservoir and MFHWs influence the fluid flow characteristics and reservoir pressure distribution. For
example, the outer formation beyond fracture tips may have various region areas and geometries. The linear flow may not fully describe the flow convergence towards hydraulic fracture tips for all types of outer formations. In addition, the radial flow may occur around a MFHW when the horizontal well spacing is too large or when the reservoir stimulated with the single MFHW is too big. But current composite methodology cannot accurately describe such late-stage radial flow regime. For the stress-sensitive reservoirs, composite models' accuracy depends on the time step size. Theoretically smaller steps are better for reservoirs with more stress-sensitive matrix permeability. It can be concluded that specific types of flow region and specific coupling manner should be developed and tested carefully when the composite methodology is utilized to solve the problems under a specific reservoir condition.

5.3 Applicability in Modeling Heterogeneous Reservoirs

Unconventional reservoirs stimulated with MFHWs could have complex heterogeneity conditions. Many MFHWs have heterogeneous completion along the horizontal wellbore. Usually a horizontal wellbore is in thousands of meters long. It is common for a horizontal well to go through zones with different reservoir properties (such as porosity $\phi$, permeability $k$, etc.). Hydraulic fracturing treatments along the horizontal wellbore could reactivate existing natural fractures, which results in higher permeability in the formation adjacent to fractures. The proposed methodology is applicable when the MFHW has a heterogeneous completion. This section focuses on the application of the composite methodology in heterogeneous reservoirs. In the future, this
methodology will be applied to model the flow behavior in reservoirs with MFHWs under the influence of stress-sensitivity, slip flow and gas adsorption/desorption.

5.3.1 Region Types and Coupling Pattern for Heterogeneous Reservoirs

Fig. 5.7 displays the flow chart of a composite model development for a heterogeneous reservoir stimulated by a fractured horizontal well. The input parameters of this composite model include reservoir and hydraulic fracture conditions. When the maximum half-length \( x_{j_{\text{max}}} \) in all hydraulic fractures is too small compared with the reservoir \( (x_{j_{\text{max}}}/(y_{2}-y_{0})<0.1) \), the pseudo-radial flow (late radial flow) will occur in the reservoir and this is not handled by the composite model. Firstly the composite model determines \( x \)-direction linear regions and boundaries \( (x_{L}, x_{R}) \) for all sub-systems. Then for the formation beyond the fracture tip inside each sub-system, source/sink, radial or linear flow regions are chosen according to the ratio \( (x_{R}+x_{L})/(y_{2}-y_{1}) \). The relationship between \( (x_{R}+x_{L})/(y_{2}-y_{1}) \) and flow region selection are based on a series of comparisons between this study and analytical/numerical software results. When the length in one dimension of a zone beyond the fracture tip is much longer than that in the other dimension, radial and source/sink flow regions are no longer applicable. If \( (x_{R}+x_{L})/(y_{2}-y_{1}) \) is smaller than 0.5, \( y \)-direction linear flow regions should be applied beyond fracture tips. And \( x \)-direction linear flow regions should be added beyond fracture tips when \( (x_{R}+x_{L})/(y_{2}-y_{1}) \) is larger than 5. Next the region coupling beyond fracture tips could be further refined according to the reservoir heterogeneity.
Figure 5. The flow chart of the composite model development for heterogeneous reservoirs.
conditions. After all regions are set, $x_B$, $r_w$ and $r_e$ are calculated for each sub-system. **Eq. 5.13** is finally solved for the production rate and pressure distribution in the reservoir.

### 5.3.2 Reservoir Heterogeneity along the Horizontal Wellbore

In heterogeneous unconventional reservoirs, reservoir properties can be different along the horizontal wellbore. Medeiros et al. (2010) used the boundary-element method and Green’s functions to consider the effect of reservoir heterogeneity on the production decline. Different from Medeiros et al.’s method, this composite model is fast in the calculation and applicable to complex heterogeneous conditions. In a composite model, each sub-system can have a set of reservoirs properties that is different from other sub-systems. **Table 5.2** gives the information of the reservoir with a 10-stage fractured horizontal well. Fractures with $x_f$ 70m are equally spaced along the wellbore. A high-permeability zone with length 160m, width 300m and permeability 0.1md exists in the reservoir. And the second hydraulic fracture is located at the center of the high-permeability zone.

**Fig. 5.8** compares the dimensionless pressure $P_D$ and pressure derivative $dP_D/d\ln t_D$ (in constant-rate condition) calculated by the composite model (red dots) and the numerical simulation of Ecrin (solid lines). The good agreement between the two model results verifies that the composite model is applicable when zones with different reservoir characteristics exist along the horizontal wellbore. The composite model’s computation takes 10 seconds which is shorter than 18 seconds of the Ecrin numerical model. The difference of calculation time
Table 5.2 Reservoir characteristics and the MFHW parameters in Section 5.3.2.

| Reservoir lateral length $L$ (m) | 1600 |
| Well Spacing $W$ (m) | 300 |
| Pay zone thickness $H$ (m) | 50 |
| Reservoir permeability $k$ (m$^2$) | $10^{-17}$ (0.01 md) |
| Reservoir compressibility $c_t$ (pa$^{-1}$) | $1.45 	imes 10^{-12}$ (10$^{-6}$ psi$^{-1}$) |
| Reservoir porosity $\phi$ | 0.1 |
| Viscosity $\mu$ (Pa·s) | 0.001 (1 cp) |
| Fracture half-length $x_f$ (m) | 70 |
| Fracture conductivity $k_{wf}$ (m$^3$) | $6 	imes 10^{-14}$ (200 md·ft) |
| Fracture Stages $n$ | 10 |
| Fracture $x$-coordinates (start from heel as zero in $x$-axis, m) | Stage 1: 80 | Stage 6: 880 |
| | Stage 2: 240 | Stage 7: 1040 |
| | Stage 3: 400 | Stage 8: 1200 |
| | Stage 4: 560 | Stage 9: 1360 |
| | Stage 5: 720 | Stage 10: 1520 |
| High-permeability zone center coordinate $(x,y)$ | (240,150) |
| High-permeability zone Length(m)×width(m) | 160×300 |
| High-permeability zone reservoir permeability $k$ (m$^2$) | $10^{-16}$ (0.1 md) |
Figure 5. 8-The comparison of transient pressure behavior of the MFHW in the reservoir with heterogeneity along the wellbore and without heterogeneity (calculated by the composite model and numerical model) (original in color).
between the composite model and Ecrin numerical models becomes larger for MFHWs with more hydraulic fractures.

Fig. 5.8 also compares the transient pressure response with (red dots) and without (black triangles) the high-permeability zone. The high-permeability zone reduces both pressure drop and pressure derivative of the MFHW. And pressure derivative curves are more sensitive to the heterogeneity condition than pressure curves. The influence of high-permeability zone occurs at the middle stage of production in pressure curves while at very early stage in pressure derivative curves of Fig. 5.8. Fig. 5.8 also applies an Ecrin homogeneous numerical model to match the calculated pressure behavior of the heterogeneous case. Dashed lines in Fig. 5.8 are the best-match results of a homogeneous model with reservoir permeability $k_{0.012}$md. There exists obvious discrepancy in pressure derivative curves between the heterogeneous and best-match homogeneous cases when $t_D$ is in $(0.05, 1)$. It is concluded that homogeneous models cannot fully evaluate and predict the production behavior of MFHWs in heterogeneous reservoirs.

5.3.3 Reservoir Heterogeneity around A Hydraulic Fracture

The reservoir heterogeneity around a hydraulic fracture has significant influence on MFHWs’ production behavior. Usually hydraulic fracturing treatments increase the permeability of the formation around hydraulic fractures. But fracturing treatments can also cause damage to the vicinal formation and reduce its permeability. These heterogeneity conditions can be represented in
the composite model by setting reservoir properties different in different regions of one sub-system.

For simplicity, only heterogeneity inside a sub-system is considered in this section. Therefore a single-stage fractured horizontal well is studied in this section and the half of the reservoir is same as Fig. 5.5a. The trilinear and five-region models are also applicable for the single-stage fractured horizontal well. In Fig. 5.5a, the permeability of Regions II and III is set to be different from that of Region I. Table 5.3 lists the detailed reservoir and well information. Fig. 5.9 shows the dimensionless pressure \( P_D \) calculated by the composite model, trilinear model (Brown et al. 2011), five-region model (Stalgorova and Mattar 2013) and Ecrin numerical model.

When the reservoir permeability is higher around the fracture, the results of our composite model (dotted red line), the trilinear model (triangles) and five-region model (circles) agree well with the numerical model result (rectangles). And both the composite model and Ecrin use 5 seconds in computation. When the reservoir permeability is lower around the fracture, neither the trilinear model (“\( \times \)”) nor the five-region model (“\( + \)”) can match the numerical results after \( t_D > 1 \) while the composite model (solid blue line) shows good agreement with the pressure curve calculated by Kappa Ecrin (diamond). Fig. 5.10 gives the reason by showing the comparison of pressure distribution between this composite model and Ecrin numerical model. When the reservoir around the hydraulic fracture has lower permeability, the composite model gives a pressure distribution very close to that of numerical solutions at \( t_D = 10 \).
Table 5.3 Heterogeneous reservoir and wellbore parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reservoir lateral length (m)</td>
<td>160</td>
</tr>
<tr>
<td>Well Spacing (m)</td>
<td>300</td>
</tr>
<tr>
<td>Pay zone thickness (m)</td>
<td>50</td>
</tr>
<tr>
<td>Fracture half length (m)</td>
<td>70</td>
</tr>
<tr>
<td>Reservoir permeability in Region I (m$^2$)</td>
<td>$10^{-17}$ (0.01 md)</td>
</tr>
<tr>
<td>Higher reservoir permeability in Regions II and III (m$^2$)</td>
<td>$10^{-16}$ (0.1 md)</td>
</tr>
<tr>
<td>Lower reservoir permeability in Regions II and III (m$^2$)</td>
<td>$10^{-18}$ (0.001 md)</td>
</tr>
<tr>
<td>Reservoir compressibility (pa$^{-1}$)</td>
<td>$1.45 \times 10^{-10}$ (10$^{-6}$ psi$^{-1}$)</td>
</tr>
<tr>
<td>Fracture conductivity (m$^3$)</td>
<td>$6 \times 10^{-14}$ (200 md·ft)</td>
</tr>
<tr>
<td>Fracture Stages</td>
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</tr>
</tbody>
</table>
Figure 5. 9-Transient pressure behavior of the MFHW in the reservoir with heterogeneity around the hydraulic fracture calculated by the composite model, the Ecrin numerical model, trilinear model (Brown et al. 2011) and five-region model (Stalgorova and Mattar 2013).
Figure 5. 10a-Dimensionless pressure $p_D$ profiles beyond the hydraulic fracture tip (calculated in the Kappa Ecrin numerical model) at $t_D=10$ (original in color).

Figure 5.10b-Dimensionless pressure $p_D$ profiles beyond the hydraulic fracture tip (calculated in the composite model) at $t_D=10$ (original in color).
Figure 5.10c- Dimensionless pressure $p_D$ profiles in the inner reservoir around the fracture (calculated in the Kappa Ecrin numerical model) at $t_D=10$ (original in color).

Figure 5.10d- Dimensionless pressure $p_D$ profiles in the inner reservoir around the fracture (calculated in the composite model) at $t_D=10$ (original in color).
The model can describe more complex heterogeneity around a hydraulic fracture. As shown in Figs. 5.5c and 5.5d, there could be more than two regions on the left or right of a hydraulic fracture. And the composite model works well when each of these regions has a different combination of reservoir properties such as permeability $k$, porosity $\phi$ and compressibility $c_t$, etc.

5.3.4 Field Example

The production data in the field example are taken from Wei et al. (2015) for a fractured horizontal well in Sichuan Basin, China. The well was completed with 12-stage fracturing treatments and each stage was perforated with four clusters. Table 5.4 lists the reservoir and wellbore information. Fig. 5.11 shows the production pressure and rates during 809 days’ production. For accurate production analysis, Wei et al. calculated the material balance time $t_{mba}$ and the modified pseudopressure $m(p)$ with the method of Nobakh et al. (2011). The effects of adsorbed gas and apparent permeability in the shale reservoir were considered in $t_{mba}$ and $m(p)$. Furthermore, the pseudopressure data were normalized in the transient pressure analysis to include the variation of production rate with time.

Fig. 5.12a shows the rate-normalized pseudopressure (RNP) data vs. $t_{mba}$. In Fig. 5.12a, three flow regimes can be identified: (1) linear flow from $t_{mba}=1$ day to $t_{mba}=41$ day, (2) transitional flow from $t_{mba}=42$ day to $t_{mba}=310$ day and (3) linear flow from $t_{mba}=310$ day to $t_{mba}=2130$ day. Such production behavior demonstrates the existence of reservoir heterogeneity—the reservoir diffusivity in the vicinity of hydraulic fractures is lower than that of the formation a little far from fractures.
Table 5. Well and reservoir data of the field example (Wei et al., 2015).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal well length (ft)</td>
<td>3540</td>
</tr>
<tr>
<td>Reservoir porosity</td>
<td>0.08</td>
</tr>
<tr>
<td>Pay zone thickness (ft)</td>
<td>130</td>
</tr>
<tr>
<td>Initial reservoir pressure (psi)</td>
<td>2012</td>
</tr>
<tr>
<td>Fracture stage</td>
<td>12</td>
</tr>
<tr>
<td>Initial gas compressibility (psi⁻¹)</td>
<td>0.000476</td>
</tr>
<tr>
<td>Initial gas deviation factor</td>
<td>0.7915</td>
</tr>
<tr>
<td>Initial gas viscosity, cp</td>
<td>0.01731</td>
</tr>
<tr>
<td>Reservoir temperature, K</td>
<td>364.5</td>
</tr>
</tbody>
</table>
Figure 5. Production rate and bottomhole pressure history profile for the MFHW in Sichuan Basin, China (Wei et al., 2015).
Figure 5. 12a-Rate normalized pseudopressure (RNP) data and the best-matching curve generated based on the composite model (original in color).

Figure 5.12b-Each sub-system is composed of eight regions and regions adjacent to the hydraulic fracture have different permeability than that of regions far from the fracture (original in color).
One of the reasons for this phenomenon could be the damage to the vicinal reservoir during hydraulic fracturing treatments. And the reservoir diffusivity becomes higher at further regions when the stimulation effect of hydraulic fracturing exceeds damage effect.

A single hydraulic fracture is assumed to exist at one stage. And a 12-stage fractured horizontal well model is applied for the type curve matching. The composite model has 12 sub-systems and conditions of all sub-systems are assumed to be the same. Each sub-system is divided into eight regions (Fig. 5.12b). The porosity in all regions (except hydraulic fracture region) is assumed to be the same. Fig. 5.12a shows the matching results for the RNP data. Overall the match is satisfactory. The fracture conductivity $k_{wf}$ is determined as 100 md·ft. The width of damaged zone is estimated as 25ft. The ratio of fracture vicinity’s reservoir permeability over stimulated reservoir permeability is 0.125. Fig. 5.12a also shows the production behavior of the fractured horizontal well when the reservoir is assumed as homogeneous (dashed line). The calculated RNP with homogeneous reservoir assumption gravely deviates from the field data in transitional and linear flow regimes.

The field example demonstrates the composite model’s applicability in production analysis of MFHWs in heterogeneous reservoirs. Type curve matching based on composite models in this study depends on the quality and quantity of available production and well data. The analysis results may become different if more field data are provided.
5.4 Conclusions

In this chapter, a practical semi-analytical, composite methodology and composite models are developed for heterogeneous reservoirs with MFHWs. The principal contributions in this work are summarized below:

1. The composite methodology considers characteristics of pressure distribution in reservoirs stimulated with MFHWs and extends and generalizes the multi-linear models in the literature. The composite models are fast in calculation and good alternatives to multi-linear, semi-analytical/analytical and numerical models.

2. The composite methodology is validated and applicable for heterogeneous completion along the horizontal wellbore, reservoir heterogeneity along the horizontal wellbore and reservoir heterogeneity around hydraulic fractures.

3. Although nonuniqueness is a problem in heterogeneous reservoirs, matching production data with type curves from the composite model gives good estimations of reservoir and fracture properties.

4. The composite methodology is applicable in incorporating complex flow phenomena such as stress-sensitivity, gas adsorption/desorption and slip/transition flow.
The fluid flow becomes a complex multi-scale process in low-permeability unconventional reservoirs stimulated with MFHWs. It requires the thorough knowledge of both unconventional reservoirs rocks and the flow behavior to develop feasible ways of enhancing hydrocarbon recovery. Recently the unconventional reservoirs become a possible choice for geologic carbon storage. Studies of flow characteristics in this thesis would also provide important information for the CO$_2$ transport in unconventional reservoirs. Studies in this thesis can be concluded that

- Shale intrinsic permeability is determined by the multi-scale pore/throat size distributions and interconnectivity.
- Gas flow permeability in shale is higher than the intrinsic permeability even at elevated reservoir pressure but the linear Klinkenberg equation cannot fully describe the behavior of rarefied gas flow permeability vs. pore pressure.
- A higher-order equation of permeability vs. pressure is developed based on pore network flow modeling and can be applied in reservoir-scale modeling/simulations.
- Stress-sensitive hydraulic fractures have varying compressibilities when the effective pressure increases. The semi-analytical modeling demonstrates the influence of bottomhole pressure profiles on the calculated production rates.
- A composite methodology is validated to model MFHWs’ production behavior in complex heterogeneous reservoirs.
In Chapter 2 nm-μm-scale pore network models are developed to represent pore space structure inside selected shale samples. Coupling of μm-scale OM-Clay-Interparticle Pore space and nm-scale pores creates multi-scale pore networks with smooth, continuous pore/throat size distributions from nanometer to micrometer. The interconnectivities and PSDs of μm-scale space and nm-scale pore space codetermine the shale intrinsic permeability in the range of (1nD, 1mD). With the tremendous differences in sizes, the nm-scale pore volume may still be smaller than that of μm-scale pores even when the nm-scale pores/throats dominate the shale pore space. Shale would have higher intrinsic permeability with higher μm-scale interconnectivity, larger μm-scale space, higher nm-scale interconnectivity, larger nm-scale pores as well as higher nm-μm interconnectivity.

The Chapters 3 describes the high-pressure rarefied gas flow in both a single-conduit and multi-scale pore networks. The volumetric rate out of a single nm-scale conduit is higher with considering the rarefied gas flow mechanism. The conduit cross-section shape has significant influence on its conductivity. New unified models/equations are developed for the rarefied flow rate of a cylindrical, rectangular or triangular nm-scale conduit, which is fully applied in the network flow modeling. The pore size distribution and morphology demonstrate strong effects on pore networks’ apparent permeabilities. A 2nd-order equation is developed to match the permeabilities ratios in the reservoir pressure range (1MPa, 30MPa). Furthermore, power-law relationships are found
in the shale intrinsic permeability vs. coefficients of the $2^{nd}$-order equation, which can be applied in the reservoir-scale modeling.

The study in **Chapter 4** moves to a higher scale-the reservoir-scale or the single-well-scale. For low-permeability unconventional reservoirs stimulated with MFHWs, considering the hydraulic fractures’ stress-sensitivity is crucial for optimizing unconventional reservoir hydrocarbon production. Chapter 4 develops an equation of fracture compressibility vs. pore pressure and models the flow in reservoir and stress-sensitive hydraulic fractures with a semi-analytical scheme. It is concluded that the fracture stress-sensitivity would deviate the slope of $1/q$ vs. $t$ from the regular $\frac{1}{2}$ value in non-stress-sensitive cases. The behaviors of $q$ vs. $t$ are examined under the influence of fracture compressibility characteristics and drawdown pressure profiles. Suggestions are provided to reduce the possibility and minimize the adverse effects of fracture stress-sensitivity.

**Chapters 2 and 3** discuss the influence of rarefied gas flow in multi-scale pore structures in shale reservoirs. **Chapter 4** talks about the existence of hydraulic fracture stress-sensitivity in stimulated unconventional reservoirs. In fact, more mechanisms including reservoir heterogeneity, rock matrix stress-sensitivity and gas adsorption/desorption, play important roles in low-permeability reservoirs. **Chapter 5** shows a composite methodology that is versatile in taking the aforementioned mechanisms into reservoir-scale modeling. The composite methodology originates from the “decomposition” idea and couple simple flow equations for the complex flow in the whole reservoir.
system. The composite methodology’s applicability in modeling reservoir heterogeneity is validated in Chapter 5.

In Chapters 2 and 3 the permeability equations are derived based on the bulk real gas properties. But in the confined nm-scale pore space of low-permeability rocks, the real gas properties deviate from bulk characteristics. Systematic work should be completed in the future work to consider the real gas properties in confined space. Chapter 5 only validates the composite methodology’s applicability in heterogeneity. In the future the methodologies’ applicability in modeling the slip/transition flow, matrix stress-sensitivity and gas adsorption/desorption.
APPENDIX A

Dimensionless pressure:

\[
p_D = \begin{cases} 
\frac{2\pi k_{ref} H}{q_{ref} B_o \mu} (p_{ini} - p), & \text{for oil} \\
\frac{2\pi k_{ref} H}{q_{ref} B_g \mu} (m(p_{ini}) - m(p))m(p) = 2 \int_{p_{ini}}^{p} \frac{p}{\mu(p) \varepsilon(p)} dp, & \text{for gas}
\end{cases}
\] ..............................................(A-1)

The \( B_o \) and \( B_g \) are the formation volume factor for oil and gas respectively. In Chapter 5, \( k_{ref} \) is \( 10^{-17} \text{m}^2 \) (0.01md) and \( q_{ref} \) is 0.0002m\(^3\)/s (610 ft\(^3\)/day).

Dimensionless time:

\[
t_D = \frac{\eta_{ref} t}{L_{ref}^2}, \quad \eta_{ref} = \frac{k_{ref}}{\phi_{ref} \mu c_{ref}} .
\] .................................................................(A-2)

In Chapter 5, the \( \phi_{ref} \) is 0.1, the \( c_{ref} \) is \( 1.45 \times 10^{-10} \text{Pa}^{-1} \) (10\(^{-6}\) psi\(^{-1}\) and the reference length \( L_{ref} \) is chosen to be 300m (984ft).

Dimensionless rate:

\[
q_D = q / q_{ref} .
\] ...................................................................................................................(A-3)

Dimensionless distances:

\[
x_D = \frac{x}{L_{ref}},
\]
\[
y_D = \frac{y}{L_{ref}},
\] ....................................................................................................................(A-4)

\[
w_{jd} = \frac{w_f}{L_{ref}},
\]
\[
r_D = \frac{r}{L_{ref}} .
\]

Dimensionless diffusivity:
\[ \eta = \left( \frac{k}{\phi \mu C_r} \right) \frac{1}{\eta_{ref}}. \] \hspace{1cm} \text{(A-5)}

In Chapter 4, the \( \eta \) is selected as 1.

Dimensionless fracture conductivity:

\[ F_{CD} = \frac{k_f w_f}{k_{ref} L_{ref}}. \] \hspace{1cm} \text{(A-6)}
APPENDIX B

It is assumed that a \( m \)-stage fractured horizontal well is located in the center of a cuboid-shaped reservoir with all-closed boundaries. The reservoir’s length, width, and net-pay thickness are \( A \) in the \( x \)-direction, \( B \) in the \( y \)-direction and \( H \) in the \( z \)-direction, respectively. And all hydraulic fractures fully penetrate the pay zone.

B.1 Flow Modeling In Hydraulic Fractures

The dimensionless flow equation of the \( j \)-th hydraulic fracture is

\[
\frac{\partial}{\partial y_D} \left( \frac{\hat{p}_{jD,i}}{\partial y_D} \right) + \frac{q_{rf,i} \left( p_{jD,i} \right)}{F_{CD,i} \left( p_{jD,i} \right)} = \frac{1}{F_{pj}} \frac{\hat{p}_{jD,i} (y_D)}{\partial t_D}, \quad j=1, 2 \cdots m \ldots (B-1)
\]

The \( q_{rf} \) is the flow rate from the reservoir matrix to the fracture. Eq. B-1 is a non-linear partial differential equation (PDE) since both \( q_{rf,i} \) and \( F_{CD,i} \) are functions of \( p_{jD} \) and \( p_{jD} \) is function of \( t_D, x_D \) and \( y_D \). In order to solve Eq. B-1, the hydraulic fracture is divided into \( n \) segments and time domain is divided into discrete time steps. During the \( k \)-th time step and in \( i \)-th fracture segment, the governing equation becomes

\[
\frac{\partial}{\partial y_D} \left( \frac{\hat{p}_{jD,i}}{\partial y_D} \right) + \frac{q_{rf,i} \left( p_{jD,i} \right)}{F_{CD,i} \left( p_{jD,i} \right)} = \frac{1}{F_{pj}} \frac{\hat{p}_{jD,i} (y_D)}{\partial t_D}, \quad y_{D,i-1,j} \leq y_D \leq y_{D,i,j}, t_D^{k-1} < t_D \leq t_D^k \ldots (B-2a)
\]

\[
\Delta t_D = t_D - t_D^{k-1} \ldots (B-2b)
\]

\[
p_{jD,i} (\Delta t_D = 0) = p_{jD,i} (t_D = t_D^{k-1}) \ldots (B-2c)
\]

Here both \( q_{rf} \) and \( F_{CD} \) become constants. For this \( i \)-th fracture segment, the inflow from and outflow into adjacent segments are

\[
q_{Dout} = q_{FDi-1,j} \ldots (B-3a)
\]
\( q_{\text{din}} = q_{fDi,j} \), .................................................................(B3b)

The Laplace transformation (Riley et al., 2010) of dimensionless pressure \( p_{fDi,j} \) (in the time domain \( \Delta t_D \)) into \( \tilde{p}^*_j \) in Laplace domain makes Eq. B-2a an ordinary differential equation (ODE):

\[
\begin{align*}
\frac{\partial}{\partial y_D} \left( \frac{\partial \tilde{p}^*_j}{\partial y_D} \right) + \frac{q_{fDi,j}}{F_{CDi,j}} = \frac{1}{F_{\eta,j}} \left( u^* \tilde{p}^*_j - p_{fDi,j} \left( t_D^{k-1} \right) \right), \quad y_{Di-1,j} \leq y_D \leq y_{Di,j}, \quad ....(B-4)
\end{align*}
\]

This equation is a non-homogeneous equation when \( p_{fDi}(t^{k-1}) \neq 0 \). Its solution is the summation of the complementary solution to its corresponding homogeneous equation and a particular solution. When \( q_{fi,j} \) and \( q_{in} \) are considered as source flow and \( q_{out} \) are sink flow (van Kruysdijk, 1988), the complementary solution \( \tilde{p}_C^* \) for the homogeneous part in Eq. B-4 is

\[
\tilde{p}_C^*_{fDi,j}(y_D, u^*) = E_{i,j} \tilde{p}^*_{fDi,j} + F_{i,j} \tilde{p}^*_{fDi-1,j} + G_{i,j} \tilde{p}^*_{fDi,j}, \quad y_{Di-1,j} \leq y_D \leq y_{Di,j}, \quad 1 \leq i \leq n \), 
\]

...........................................................................................................................................................

\[
\begin{align*}
E_{i,j} = & \frac{1}{F_{CDi,j}} \left[ \left( y_{Di} - y_D \right) \frac{u^*}{\sqrt{F_{\eta,j}}} \right]^{k}, \quad 
F_{i,j} = - \frac{1}{F_{CDi,j}} \left[ \left( y_{Di-1,j} - y_D \right) \frac{u^*}{\sqrt{F_{\eta,j}}} \right]^{k}, \quad 
G_{i,j} = - \frac{F_{\eta,j}^{k}}{F_{CDi,j}^{k} u^*}, \quad ................................................................. \quad (B-5b)
\end{align*}
\]

The pressure distribution inside the fracture segment at the end of \( k-1 \)-th time
step is represented by third-order polynomials $O(y_D)$. The particular solution $\bar{p}_S^*$ of Eq. B-4 satisfies

$$\frac{\partial}{\partial y_D} \left( \frac{\partial \bar{p}_s^{* fD_{i,j}}}{\partial y_D} \right) + \frac{q_{r fD_{i,j}}}{F_{CD_{i,j}}} = \frac{1}{F_{g_{i,j}}} \left( u \bar{p}_s^{* fD_{i,j}} - O(y_D) \right). \quad \text{(B-6)}$$

The particular solution $\bar{p}_S^*$ is represented by third-order polynomials $Q(y_D)$ for each fracture segment during each time step. Accordingly the full solution to Eq. B-4 is

$$\bar{p}_{j,0i,j}(y_D, u^*) = E_{i,j} \bar{q}_{j,0i,j}^* + F_{i,j} \bar{q}_{j,0i-1,j}^* + G_{i,j} \bar{q}_{j,0i,j}^* + Q(y_D), y_{D-1,j} \leq y_D \leq y_{D,j}. \quad \text{(B-7)}$$

**B.2 Flow Modeling In Reservoir**

The governing equation of fluid flow in the reservoir is

$$\frac{\partial^2 p_D}{\partial x_D^2} + \frac{\partial^2 p_D}{\partial y_D^2} = \frac{\partial p_D}{\partial t_D}, \quad \text{................................................................. (B-8)}$$

with inner boundary conditions,

$$\frac{\partial p_D}{\partial x_D} \bigg|_{x_D=x_{fD_j}} = q_{r fD_j}(y_D, t_D), j = 1,2,3 \ldots m \text{ for the } j\text{-th hydraulic fracture, (B-9)}$$

outer boundary conditions,

$$\frac{\partial p_D}{\partial x_D} \bigg|_{x_D=0,A_p} = 0, \quad \text{................................................................. (B-10a)}$$

$$\frac{\partial p_D}{\partial y_D} \bigg|_{y_D=0,B_p} = 0, \quad \text{................................................................. (B-10b)}$$

and initial condition as

$$p_D(x_D, y_D, t_D = 0) = 0. \quad \text{................................................................. (B-11)}$$
The Laplace transformation of \( p_D \) in the time domain \( t_D \) into \( \bar{p}_D \) in the Laplace domain makes Eq. B-8 an ODE:

\[
\frac{\partial^2 \bar{p}_D}{\partial x_D^2} + \frac{\partial^2 \bar{p}_D}{\partial y_D^2} = u \bar{p}_D \quad \text{...................................... (B-12)}
\]

The solution for Eq. B-12 is based on the Green’s function and source/sink functions in Laplace domain. Compared with the whole big reservoir, each fracture segment is reduced to a finite plane with locations \((x_{Di}, y_{Di} - y_{Di,j})\) in the reservoir. The source function in Laplace domain that describes the pressure response caused by the \( i \)-th fracture segment of \( j \)-th hydraulic fracture \((x_{Di}, y_{Di-1,j} - y_{Di,j})\) is

\[
\bar{p}_D(x_D, y_D, u) = \bar{q}_{Di,j} \int_0^\infty S_{yi} S_{yi} e^{-u t_D} d t_D, \quad \text{.......................................... (B-13a)}
\]

where

\[
S_{yi} = \frac{1}{A} \left\{ 1 + \sum_{n=1}^\infty \left[ \cos \left( \frac{n \pi (x_D - x_{Di})}{A_D} \right) + \cos \left( \frac{n \pi (x_D + x_{Di})}{A_D} \right) \right] \right\} \exp \left( -\frac{n^2 \pi^2 (t_D - \tau_D)^2}{A_D^2} \right), \quad \text{........................................ (B-13b)}
\]

and

\[
S_{yi} = \frac{1}{2} \left\{ \text{erf} \left( \frac{y_{Di,j} - y_D - 2nB_D}{2\sqrt{t_D - \tau_D}} \right) - \text{erf} \left( \frac{y_{Di-1,j} - y_D - 2nB_D}{2\sqrt{t_D - \tau_D}} \right) + \text{erf} \left( \frac{y_{Di,j} + y_D - 2nB_D}{2\sqrt{t_D - \tau_D}} \right) - \text{erf} \left( \frac{y_{Di-1,j} + y_D - 2nB_D}{2\sqrt{t_D - \tau_D}} \right) \right\} \quad \text{........................................ (B-13c)}
\]

When \( y \)-direction reservoir permeability \( k_y \) is different from \( x \)-direction permeability \( k \), the \( t_D \) in Eq. B-13c is replaced by \( t_D k / k \). The influence of fracture segments follows the superposition principle. When each hydraulic fracture is
divided into \( n \) segments, the pressure response caused by all fracture segments is

\[
\bar{p}_D(x_D, y_D, u) = \sum_{j=1}^{m} \sum_{i=1}^{n} q_{ij} \phi_{ij} \int_0^\infty S_{ij} S_{ji} e^{-\alpha_{ij} t} dt \quad \cdots \cdots \text{(B-14)}
\]
APPENDIX C

The flow modeling of fractures can be coupled with the flow in reservoir based on the pressure- and rate-continuity conditions at interfaces of fractures and the reservoir. Considering Eqs. B-7 and B-14, the pressure and rate at the interface of \( i \)-th segment of \( j \)-th hydraulic fracture/reservoir satisfy

\[
L^{-1}(q_{jD,i}) = L^{-1}(\tilde{q}_{jD,i}). \quad \text{(C-1)}
\]

\[
L^{-1}(p_{jD,i}) = L^{-1}(\tilde{p}_D(x_{jD})). \quad \text{(C-2)}
\]

Extra attention should be paid to above equations since the pressure and rates on the left-hand side are in different Laplace domains from the right-hand side. The Stehfest algorithm for the inverse Laplace transformation shows (Zeng, 2008)

\[
p_D(t) = \frac{\ln 2}{t_D} \sum_{h} V_{jD} \tilde{p}_D(u_{jD}). \quad \text{(C-3a)}
\]

\[
p_{jD,i}(\Delta t) = \frac{\ln 2}{\Delta t_D} \sum_{h} V_{jD} \tilde{p}_{jD,i}(u_{jD}). \quad \text{(C-3b)}
\]

Equalizing Eqs. C-3a and C-3b gives

\[
\frac{1}{t_D} \tilde{p}_D(u_{jD}) = \frac{1}{\Delta t_D} \tilde{p}_{jD,i}(u_{jD}). \quad \text{(C-4)}
\]

Similarly, the production rates at the interface is derived as

\[
\frac{1}{t_D} \tilde{q}_{jD}(u_{jD}) = \frac{1}{\Delta t_D} \tilde{q}_{jD}(u_{jD}). \quad \text{(C-5)}
\]

According to Eqs. C-4 and C-5, the dimensionless pressure in Laplace domains at the interface of \( i \)-th segment of \( j \)-th fracture/reservoir should satisfy
\[
\begin{align*}
\left( E_{i,j} - F_{i,j} \right) q_{jD,i,j} + F_{i,j} q_{jD,i-1,j} + G_{i,j} \bar{q}_{jD,i,j} - \frac{Q_D(y_D)}{\Delta t_D^k}, i=1, 2\cdots n, j=1, 2\cdots m. \quad \text{.....(C-6)}
\end{align*}
\]

In summary, the pressure- and rate-continuity conditions generate \(n \times m\) equations similar to Eq. C-6 with \(2m^*n\) unknown variables \(q^*_j\) and \(q_{jD}\). The couplings of any two adjacent fracture segments at interfaces provide another \((n-1) \times m\) equations:

\[
\begin{align*}
\left( E_{i,j} - F_{i+1,j} \right) \bar{q}_{jD,i,j} + F_{i,j} \bar{q}_{jD,i-1,j} \right. & \quad \left. - E_{i+1,j} \bar{q}_{jD,i+1,j} \right.
\end{align*}
\]

\[
\begin{align*}
+ \frac{\Delta t_D^k}{t_D^k} \left( G_{i,j} \bar{q}_{jD,i,j} - G_{i+1,j} \bar{q}_{jD,i+1,j} \right) \right)
\end{align*}
\]

\[
\begin{align*}
= Q_{i+1,j}(y_D) - Q_{i,j}(y_D) \quad i=1,2\cdots n-1, \quad j=1,2\cdots m. \quad \text{.....(C-7)}
\end{align*}
\]

When the bottomhole pressure \(p_{wf}\) is given as boundary conditions, the pressure at wellbore/\(j\)-th hydraulic fracture interface should be

\[
\begin{align*}
E_{n,j} \bar{q}_{jD,n,j} + F_{n,j} \bar{q}_{jD,n-1,j} + G_{n,j} \bar{q}_{jD,n,j} - \frac{Q_{n,j}(y_D)}{\Delta t_D^k} + Q_{n,j}(y_D) = \bar{p}_{wf}^*, j=1, 2\cdots m. \quad \text{.....(C-8)}
\end{align*}
\]

Here the \(\bar{p}_{wf}^*\) is in the Laplace domain with Laplace variable \(u^*\). There exist \(m\) equations like Eq. B-8 for all hydraulic fractures. When the \(Q(y_D)\) is removed from Eqs. C-6 and C-7, the resulting errors can be larger when both the pressure \(p_{wfD}\) and production rate \(q_{jD}\) out of fractures change over time. In addition, it is difficult to obtain the analytical \(\bar{p}_{wfD}(u)\) expressions for many complex curves of \(p_{wfD}\) vs. \(t_D\) in real field cases. This problem is solved in Eq. C-8 with \(\bar{p}_{wfD}^*\). Here the \(p_{wfD}\) is considered as constant during \(k\)-th time step.
At last, a linear system of $2m*n$ equations (Eqs. C-6, C-7 and C-8) is established for $2m\cdot n$ unknown variables for $k$-th time step:

$$A \begin{bmatrix} q^* \\ q_{\rho D} \\ q_{\eta D} \end{bmatrix} = B.$$  (B-9)
APPENDIX D

Take the region coupling in Fig. 5.5 in Chapter 5 as an example to explain the details of solutions and coefficients in each type of flow regions.

D.1 Region coupling in Figs. 5.5a and 5.5b

D.1.1 Region I

If Region I is a source/sink region, the line source/sink’s location should coincide with the fracture tip’s location \((x_{LD}, y_{1D})\). But the line source/sink cannot be located on region boundaries for possible computation. The source/sink line is assumed to be located at Point A \((x_{LD}, y_{1D}+10^6)\), which is close enough to the fracture tip. When a line source/sink with rate \(q_{ID}\) exists at Point A in Region I (Fig. 5.5a), the pressure at Point B \((x_{BD}, y_{1D})\) becomes

\[
\bar{p}_{ID} (x_{BD}, y_{1D}) = \alpha \bar{q}_{ID} \int_0^\infty S_x S_y e^{-\nu \gamma D \tau_D} d\tau_D + Q(x_{BD}, y_{1D}, u) \cdot \alpha = 2\pi \nu_r \frac{\phi_{ref} C_{ref}}{\phi_{1ccl}}
\]

\[
S_x = \frac{1}{2(|x_L|+|x_R|) \sqrt{\pi \eta \frac{\eta_1}{\eta_1 + \eta} (t_D - \tau_D)}} \sum_{n=-\infty}^{\infty} \left\{ \exp \left[ \frac{-(x_{BD} - x_{LD} + n)^2}{\frac{\eta_1}{\eta_1 + \eta} (t_D - \tau_D)} \right] + \exp \left[ \frac{-(x_{BD} + x_{LD} + n)^2}{\frac{\eta_1}{\eta_1 + \eta} (t_D - \tau_D)} \right] \right\},
\]

\[
S_y = \frac{1}{2(y_2 - y_1) \sqrt{\pi \eta \frac{\eta_1}{\eta_1 + \eta} (t_D - \tau_D)}} \sum_{n=-\infty}^{\infty} \left\{ \exp \left[ \frac{-(10^{-6} + n)^2}{\frac{\eta_1}{\eta_1 + \eta} (t_D - \tau_D)} \right] + \exp \left[ \frac{-(2y_{1D} + 10^{-6} + n)^2}{\frac{\eta_1}{\eta_1 + \eta} (t_D - \tau_D)} \right] \right\},
\]

\[
\bar{q}_{ID} \text{ is the flow rate from Region I to Region IV. And the pressure at Point B } (x_{BD}, y_{1D}) \text{ is assumed to be equal to the pressure at the fracture tip. The selection of Point B coordinates are discussed in Appendix E.}
\]
If the radial flow rather than source/sink flow exists in Region I (Fig. 5.5b),
the flow equation becomes
\[
\frac{1}{r_D} \frac{\partial}{\partial r_D} \left( r_D \frac{\partial \tilde{p}_{1D}}{\partial r_D} \right) = \frac{1+\beta}{\eta} \left( u \tilde{p}_{1D} - p_{1D}(t_D = 0) \right)
\]  
(D-4a)
with the initial pressure distribution that is approximated by
\[
p_{1D}(t_D = 0) = a \ln r_D + b.
\]  
(D-4b)
No-flow boundary condition at \( r_D = r_{eD} \):
\[
\frac{\partial \tilde{p}_{1D}}{\partial r_D} \bigg|_{r_D=r_{eD}} = 0,
\]  
(D-5)
Pressure-continuity boundary condition between Regions I and IV (Fig.5.5b):
\[
\overline{p}_{1D}(r_{eD}) = \overline{p}_{FD} \bigg|_{r_D=r_{eD}},
\]  
(D-6)
The solution for Eq. B-4 is
\[
\tilde{p}_{1D} = A_r I_0 \left( \frac{u(1+\beta)}{\eta} r_D \right) + B_r K_0 \left( \frac{u(1+\beta)}{\eta} r_D \right) + \frac{a}{u} \ln r_D + \frac{b}{u},
\]  
(D-7)
\[
A_r = \varepsilon \tilde{p}_{FD}(y_{1D}) + \gamma_A(u),
\]
\[
\gamma_A(u) = \frac{a}{u} \frac{1}{\eta} \frac{1}{r_D} \left( \frac{u(1+\beta)}{\eta} \right)^2 K_0 \left( \frac{u(1+\beta)}{\eta} r_D \right) + \frac{a}{u} \ln r_D K_1 \left( \frac{u(1+\beta)}{\eta} r_D \right) + \frac{b}{u} K_1 \left( \frac{u(1+\beta)}{\eta} r_D \right),
\]
\[
\varepsilon = \frac{1}{\beta K_0 \left( \frac{u(1+\beta)}{\eta} r_D \right) + l_0 \left( \frac{u(1+\beta)}{\eta} r_D \right)}.
\]  
(D-8)
\[
B_r = \varepsilon \beta \tilde{p}_{FD}(y_{1D}) + \gamma_B(u),
\]
\[
\gamma_B(u) = \frac{a}{u} \frac{1}{\eta} \frac{1}{r_D} \left( \frac{u(1+\beta)}{\eta} \right)^2 K_0 \left( \frac{u(1+\beta)}{\eta} r_D \right) - \frac{a}{u} \ln r_D K_1 \left( \frac{u(1+\beta)}{\eta} r_D \right) + \frac{b}{u} \ln r_D K_1 \left( \frac{u(1+\beta)}{\eta} r_D \right) + \frac{b}{u} \ln r_D K_1 \left( \frac{u(1+\beta)}{\eta} r_D \right).
\]
\[
\beta = \frac{1}{\beta K_1 \left( \frac{u(1+\beta)}{\eta} r_D \right) + l_1 \left( \frac{u(1+\beta)}{\eta} r_D \right)}.
\]  
(D-9)
\( I_\alpha \) and \( K_\alpha \) are the first kind and second kind of \( \alpha \)-order modified Bessel equations, respectively. The flux into the fracture tip from Region I to Region IV (fracture region) is proportional to

\[
\frac{\partial \bar{p}_{\text{ID}}}{\partial r_D}\bigg|_{r_W} = A_r \sqrt{\frac{u(1+\beta)}{\eta}} I_1 \left( \sqrt{\frac{u(1+\beta)}{\eta}} r_W \right) - B_r \sqrt{\frac{u(1+\beta)}{\eta}} K_1 \left( \sqrt{\frac{u(1+\beta)}{\eta}} r_W \right) + \frac{a}{u r_W} .
\]

(D-10)

The wellbore radius \( r_w \) and outer boundary radius \( r_e \) are determined according to Appendix E.

**D.1.2 Regions II and III**

The fluid flow in Region II (Fig. 5.5a or 5.5b) is 1D linear flow:

\[
\frac{\partial^2 \bar{p}_{\text{II}}}{\partial x_D^2} = \frac{u(1+\beta)}{\eta} \bar{p}_D - \frac{(1+\beta)}{\eta} p_D(t_D = 0) .
\]

(D-11a)

with the initial pressure distribution that is approximated by

\[
p_{\text{II}}(t_D = 0) = a_{\text{II}} x_D + b_{\text{II}} .
\]

(D-11b)

No-flow boundary condition at \( x_{RD} \):

\[
\frac{\partial \bar{p}_{\text{II}}}{\partial x_D}\bigg|_{x_{RD}} = 0 .
\]

(D-12)

Pressure-continuity boundary condition between Regions II and IV:

\[
\bar{p}_{\text{II}}\bigg|_{x_D = \frac{x_{WD}}{2}} = \bar{p}_{\text{IVD}}\bigg|_{x_D = \frac{x_{WD}}{2}} .
\]

(D-13)

The solution of Eq. D-11 is

\[
\bar{p}_{\text{II}} = \frac{\bar{p}_{\text{WD}}\left(\frac{x_{WD}}{2}\right)}{\cosh\left(\frac{\bar{p}_{\text{WD}}}{2} x_{RD}\right) \sqrt{\frac{u(1+\beta)}{\eta_{\text{II}}}}} \cosh \left( x_D - x_{RD} \sqrt{\frac{u(1+\beta)}{\eta_{\text{II}}}} \right) + Q_{\text{II}}(x_D, u) .
\]

(D-14a)
\[ Q_{II}(x_D, u) = T_{II}(u) \cosh \left[ (x_D - x_{RD}) \sqrt{\frac{u(1+\beta)}{\eta_{II}}} \right] + R_{II}(u) \sinh \left[ (x_D - x_{RD}) \sqrt{\frac{u(1+\beta)}{\eta_{II}}} \right] + \frac{a_{II}}{u(1+\beta)} x_D + \frac{b_{II}}{u(1+\beta)} \]  

\[ T_{II}(u) = \frac{-a_{II} \left( \frac{w_{FD}}{2} - x_{RD} \right) \frac{\eta_{II}}{u(1+\beta)}}{\cosh \left[ (\frac{w_{FD}}{2} - x_{RD}) \sqrt{\frac{u(1+\beta)}{\eta_{II}}} \right]} \]  

\[ R_{II}(u) = - \frac{a_{II}}{u(1+\beta) \sqrt{\frac{u(1+\beta)}{\eta_{II}}}} \]  

The flux between Regions II and IV is proportional to

\[ \frac{\partial \eta_{IVD}}{\partial x_D} \left|_{w_{FD}} \right. = \tilde{p}_{IVD} \left( \frac{w_{FD}}{2} \right) \sqrt{\frac{u(1+\beta)}{\eta_{II}}} \tanh \left[ \left( \frac{w_{FD}}{2} - x_{RD} \right) \sqrt{\frac{u(1+\beta)}{\eta_{II}}} \right] + G_{II}(u). \quad \text{(D-15a)} \]

\[ G_{II}(u) = \frac{\sqrt{u(1+\beta)}}{\eta_{II}} \sinh \left[ \left( \frac{w_{FD}}{2} - x_{RD} \right) \sqrt{\frac{u(1+\beta)}{\eta_{II}}} \right] + R_{II}(u) \sqrt{\frac{u(1+\beta)}{\eta_{II}}} \cosh \left[ \left( \frac{w_{FD}}{2} - x_{RD} \right) \sqrt{\frac{u(1+\beta)}{\eta_{II}}} \right] + \eta_{11} + \alpha_1 u + 1\beta. \quad \text{...............................................(D-15b)} \]

The solution of Region III (Fig. 5.5a or 5.5b) is similar to that of Region II:

\[ \tilde{p}_{III} = \frac{\tilde{p}_{IVD} \left( \frac{-w_{FD}}{2} \right) \cosh \left[ \left( x_{LD} - x_{D} \right) \sqrt{\frac{u(1+\beta)}{\eta_{II}}} \right]}{\cosh \left[ \left( \frac{w_{FD}}{2} + x_{LD} \right) \sqrt{\frac{u(1+\beta)}{\eta_{II}}} \right]} + Q_{III}(x_D, u). \quad \text{........ (D-16)} \]

The flux between Regions III and IV (Fig. 5.5a or 5.5b) is proportional to

\[ \frac{\partial \tilde{p}_{III}}{\partial x_D} \left|_{w_{FD}} \right. = -\tilde{p}_{IVD} \left( \frac{-w_{FD}}{2} \right) \sqrt{\frac{u(1+\beta)}{\eta_{II}}} \tanh \left[ \left( x_{LD} - \frac{w_{FD}}{2} \right) \sqrt{\frac{u(1+\beta)}{\eta_{II}}} \right] + G_{III}(u). \quad \text{......................................................(D-17)} \]

**D.1.3 Region IV**

Considering the flux from Regions II and III, the flow equation for Region IV
(Fig. 5.5a or 5.5b) with the initial pressure inside a hydraulic fracture as \( p_{FD,0} \) becomes

\[
\frac{\partial^2 p_{FD}}{\partial y_D^2} = H(u, \beta, \eta)p_{FD} + f(u).
\]  \( \text{(D-18)} \)

with

\[
H = \frac{u(1+\beta)}{\eta_F} - \frac{k_H}{w_{FD}k_f} \sqrt{\frac{u(1+\beta)}{\eta_H}} \tanh \left[ \left( x_D - x_{RD} \right) \sqrt{\frac{u(1+\beta)}{\eta_H}} \right] - \frac{k_H}{w_{FD}k_f} \sqrt{\frac{u(1+\beta)}{\eta_H}} \tanh \left[ \left( x_{LD} - x_D \right) 1+\beta \right] \eta_H.
\]  \( \text{(D-19a)} \)

and

\[
f(u) = \frac{k_H}{w_{FD}k_f} (G_H(u) + G_H(u)).
\]  \( \text{(D-19b)} \)

The \( H \) is derived based on the method developed by Stalgorova and Mattar (2013). Flux-continuity boundary condition across the fracture tip:

\[
\frac{\partial p_{FD}}{\partial y_D} \bigg|_{y_{ID}} = -\frac{2\pi q_{ID}}{F_{CD}} \text{ in Fig. 5.5a or } \frac{\partial p_{FD}}{\partial y_D} \bigg|_{y_{ID}} = \delta_i \frac{\partial p_{FD}}{\partial y_D} \bigg|_{r_{ID}} \frac{2\pi k_i r_{ID}}{k_f w_{FD}} \text{ in Fig. 5.5b } \text{ (D-20)}
\]

Constant-rate boundary condition at the hydraulic fracture/wellbore intersection:

\[
\frac{\partial p_{FD}}{\partial y_D} \bigg|_{y_{ID}} = -\frac{2\pi q_D}{F_{CD}}. \quad \text{ (D-22)}
\]

According to Eqs. D-1, D-10, D-15 and D-17, the solution for the sub-system in Figs. 5.5a and 5.5b is

\[
p_{FD} = A_F \cosh[(y_D - y_{1D})\sqrt{H}] + B_F \sinh[(y_D - y_{1D})\sqrt{H}] - \frac{f(u)}{H}, \quad \text{(D-23)}
\]
\[ A_F = \frac{-2\pi}{D_{CD}} \frac{\xi(u)}{\sqrt{H}} + \frac{q_D}{D_{CD}} + C(u). \]

\[ B_F = -\frac{2\pi}{D_{CD}} \frac{\xi(u)}{\sqrt{H}} A_F + \beta(u), \]

\[ \xi(u) = \frac{1}{\alpha \int_0^\infty s_x s_y e^{-\alpha D} dD}. \]

\[ C(u) = -\frac{\beta(u) \cosh[-y_{1D} \sqrt{H}]}{\sqrt{H} \sinh[-y_{1D} \sqrt{H}] + \frac{2\pi}{D_{CD}} \xi(u) \cosh[-y_{1D} \sqrt{H}]} . \]

\[ \beta(u) = \frac{2\pi}{D_{CD}} \frac{\xi(u)}{\sqrt{H}} \left( \frac{f(u)}{H} + Q(x_{BD}, y_{1D}) \right) \text{ for Fig. 5.5a} \quad \text{............... (D-24)} \]

\[ A_F = \frac{-2\pi}{D_{CD}} \frac{q_D}{\sqrt{H}} \frac{\xi(u)}{\sqrt{H}} + C_2(u), \]

\[ B_F = \frac{C_3(u)}{\sqrt{H}} A_F - \frac{f(u)}{H} + C_4(u) \]

\[ C_1(u) = \frac{a}{u} \ln r_{WD} + b, \]

\[ C_2(u) = \frac{-c^2(u)}{\sqrt{H} \sinh[-y_{1D} \sqrt{H}] + \frac{2\pi}{D_{CD}} \xi(u) \cosh[-y_{1D} \sqrt{H}]}. \]

\[ C_3 = \frac{1}{2} \varepsilon I_1 \left( \frac{u(1+\beta)}{\eta_1} r_{WD} \right) - \frac{1}{2} \varepsilon K_1 \left( \frac{u(1+\beta)}{\eta_1} r_{WD} \right), \]

\[ C_4 = \frac{1}{2} \gamma A(u) I_1 \left( \frac{u(1+\beta)}{\eta_1} r_{WD} \right) - \frac{1}{2} \gamma B(u) K_1 \left( \frac{u(1+\beta)}{\eta_1} r_{WD} \right) + C_1(u), \]

\[ C_5 = \left( -\frac{f(u)}{H} C_3 + C_4 \right) \cosh[-y_{1D} \sqrt{H}] \text{ for Fig. 5.5b} \quad \text{.................. (D-25)} \]

**D.2 Region coupling in Figs. 5.5c and 5.5d**

In Fig. 5.5c, there are \( m \) \( x \)-direction linear regions on the right of fracture, \( m-1 \) \( y \)-direction linear regions on the right of fracture, \( n \) \( x \)-direction linear regions on the left of fracture and \( n-1 \) \( y \)-direction linear regions on the left of fracture. With the flow contribution from \( j \)-th \( y \)-direction linear region and \( j-1 \)-th \( x \)-direction
linear region, the Eq. D-14 for the $j$-th $x$-direction linear region on the right of
fracture (represented by the subscript $r$) becomes

$$
\tilde{p}_{iD}(x_D) = \left( \frac{\tilde{p}_{i+1D}(x_{rID}) - O_i(x_{rID},u)}{C_i} \right) \cosh \left[ \left( x_D - x_{r(i-1)D} \right) \sqrt{E_i} \right]
+ \left( \frac{\tilde{p}_{i+1D}(x_{rID}) - O_i(x_{rID},u)}{C_i} \right) D_i \sinh \left[ \left( x_D - x_{r(i-1)D} \right) \sqrt{E_i} \right] + O_i(x_D,u)
$$

\hspace{100pt} \text{.................................................................}(D-26)

$$
C_i = \cosh \left[ \left( x_{rID} - x_{r(i-1)D} \right) \sqrt{E_i} \right] + D_i \sinh \left[ \left( x_{rID} - x_{r(i-1)D} \right) \sqrt{E_i} \right], \quad i > 0. \quad \text{....}(D-27)
\hspace{100pt} \text{.................................................................}

\hspace{100pt} \text{.................................................................}

$$
D_i =

\frac{k_{i-1}}{k_i} \sqrt{\frac{E_{(i-1)}}{E_i}} \sinh \left[ \left( x_{r(i-1)D} - x_{r(i-2)D} \right) \sqrt{E_{(i-1)}} \right] + D_{i-1}(u) \cosh \left[ \left( x_{r(i-1)D} - x_{r(i-2)D} \right) \sqrt{E_{(i-1)}} \right], \quad i > 1,
$$

$$
D_1 = 0. \quad \text{.................................................................}(D-28)
$$

$$
O_i(x_D,u) = T_i(u) \cosh \left[ \left( x_D - x_{r(i-1)D} \right) \sqrt{E_i} \right] - O_{i-1}(x_{r(i-1)D},u)
\hspace{100pt} \text{.................................................................}
$$

$$
\frac{k_{i-1}}{k_i} \sqrt{\frac{E_{(i-1)}}{E_i}} \sinh \left[ \left( x_{r(i-1)D} - x_{r(i-2)D} \right) \sqrt{E_{(i-1)}} \right] + D_{i-1}(u) \cosh \left[ \left( x_{r(i-1)D} - x_{r(i-2)D} \right) \sqrt{E_{(i-1)}} \right]
\hspace{100pt} \text{.................................................................}
$$

$$
\sinh \left[ \left( x_D - x_{r(i-1)D} \right) \sqrt{E_i} \right] + R_i(x_{r(i-1)D},u) \sinh \left[ \left( x_D - x_{r(i-1)D} \right) \sqrt{E_i} \right] + \frac{a_i}{\eta_i E_i} x_D
\hspace{100pt} \text{.................................................................}(D-29)
\hspace{100pt} \text{.................................................................}
$$

\hspace{100pt} \text{.................................................................}

\hspace{100pt} \text{.................................................................}

\hspace{100pt} \text{.................................................................}

\hspace{100pt} \text{.................................................................}
\[
Q_i(x_D,u) = T_i(u) \sqrt{E_i \sinh[(x_D - x_{r(i-1)D}) \sqrt{E_i}]} - \\
O_{i-1}(x_{r(i-1)D}, u) \frac{k_{(i-1)}}{k_i} \sqrt{\frac{E_{(i-1)}}{E_i}} \sqrt{E_i} \\
\sinh[(x_{r(i-1)D} - x_{r(i-2)D}) \sqrt{E_{(i-1)}}] + D_{i-1}(u) \cosh[(x_{r(i-1)D} - x_{r(i-2)D}) \sqrt{E_{(i-1)}}] \\
\cosh[(x_{r(i-1)D} - x_{r(i-2)D}) \sqrt{E_{(i-1)}}] + D_{i-1}(u) \sinh[(x_{r(i-1)D} - x_{r(i-2)D}) \sqrt{E_{(i-1)}}] \\
\cosh[(x_D - x_{r(i-1)D}) \sqrt{E_i}] + R_i(x_{r(i-1)D}, u) \sqrt{E_i} \cosh[(x_D - x_{r(i-1)D}) \sqrt{E_i}] + \frac{a_i}{\eta_i E_i}, i > 1, \\
Q_1(x_{rD}, u) = T_1(u) \sqrt{E_1 \sinh[(x_{rD} - x_{r0D}) \sqrt{E_1}]} + R_1(u) \sqrt{E_1} \cosh[(x_{rD} - x_{r0D}) \sqrt{E_1}] \\
+ \frac{a_1}{\eta_1 E_1}, i = 1.
\]

...........................................................................................................(D-30)

\[
T_i(u) = - \frac{a_i}{\eta_i E_i} x_{r(i-1)D} - \frac{S_i(u)}{\eta_i E_i}, i > 1
\]

\[
T_1(u) = - \frac{a_1}{\eta_1 E_1} x_{r1D} - \frac{S_1(u) \sinh[(x_{r1D} - x_{r0D}) \sqrt{E_1}]}{\cosh[(x_{r1D} - x_{r0D}) \sqrt{E_1}]}, i = 1 \quad ...........................................(D-31)
\]

\[
R_i(x_{r(i-1)D}, u) = \frac{k_{(i-1)}Q_{(i-1)}}{k_i \sqrt{E_i}} \frac{(x_{r(i-1)D}D_u)}{d} - \frac{a_i}{\eta_i E_i \sqrt{E_i}}, i > 1, \\
R_1(u) = - \frac{a_1}{\eta_1 E_1 \sqrt{E_1}}, i = 1. \quad ...........................................(D-32)
\]

\[
E_i(u) = \frac{u}{\eta_i} - \frac{k_{iu}}{k_i y_{1D} \sqrt{\eta_i u}} \tanh \left[ (y_{1D} - y_{2D}) \sqrt{\frac{u}{\eta_i u}} \right], i > 0 \quad ...........................................(D-33)
\]

\[
S_i(u) = b_i + \frac{k_{i-1}}{k_i} \eta_i \left[ T_{i-1}(u) \sqrt{E_{i-1} \sinh[(x_{r(i-1)D} - x_{r(i-2)D}) \sqrt{E_{i-1}}]} \\
+ R_{i-1}(u) \sqrt{E_{i-1}} \cosh[(x_{r(i-1)D} - x_{r(i-2)D}) \sqrt{E_{i-1}}] + \frac{a_{i-1}}{u(1 + \beta)} \right], i > 1 \\
S_1 = b_1. \quad ...........................................(D-34)
\]
The pressure equations for linear regions on the left of fracture are similar to Eqs. D-26-D-34. In Fig. 5.5d, there could be \( m \) \( (m \geq 1) \) radial flow regions in one sub-system. The pressure and its radial gradient in every radial flow region are defined in details by Kuchuk and Habashy (1995). In Fig. 5.5d, there could be \( n \) \( x \)-direction linear flow regions on the right of the hydraulic fracture. The derivation for the region coupling in Fig. 5.5d can follow the procedure of region coupling in Fig. 5.5c.
APPENDIX E

E.1 Determination of $x_B$

When source/sink functions are applied, the Point B $(x_B, y_1)$ becomes important for our model’s accuracy. An implicit assumption in our model is that no fluid flow across the interfaces between source/sink regions and adjacent linear flow regions. Therefore, the best $x_B$ should minimize the pressure difference across the interfaces. Take Fig. 5.5a for an example. The $x_{BD}$ is determined as below:

1. Calculate the pressure at the interface of Regions I and II.

The pressure at the center point of the interface provides the basis for determining $x_{RD}$. In Fig. 5.5a, the pressure at $(x_{RD}/2, y_{1D})$ in Region I is calculated as:

$$p_{1D} \left( \frac{x_{RD}}{2}, u \right) = \frac{\bar{p}_{FD}(y_{1D}) - Q(x_{BD}, y_{1D})}{S_x(x_{BD})S_y(y_{1D})e^{-u \tau_D}} \int_0^\infty S_x \left( \frac{x_{RD}}{2} \right) S_y(y_{1D})e^{-u \tau_D}d\tau_D + Q_I \left( \frac{x_{RD}}{2}, y_{1D}, u \right)$$

.................................................................................................................................(E-1)

The pressure at $(x_{RD}/2, y_{1D})$ in Region II is

$$p_{II} \left( \frac{x_{RD}}{2}, u \right) = \frac{\bar{p}_{FD}(y_{1D})}{\cosh \left( \frac{u_{FD}}{2} - x_{RD} \right) \sqrt{\frac{u(1+\beta)}{\eta_I}}} \cosh \left( \frac{-x_{RD}}{2} \sqrt{\frac{u(1+\beta)}{\eta_I}} \right) + Q_{II} \left( \frac{x_{RD}}{2}, u \right)$$

.................................................................................................................................(E-2)

The pressure at the fracture tip $\bar{p}_{FD}(y_{1D})$ is calculated in Eq. D-23 and the calculated $\bar{p}_{FD}(y_{1D})$ changes along with selected $x_{BD}$ and $u$.

2. Calculate the relative pressure difference at the center point in two regions.
In Fig. 5a, the pressure difference at \((x_{RD}/2, y_{1D})\) between Regions I and II is
\[
\Delta \bar{p}_D = \bar{p}_{ID} \left( \frac{x_{RD}}{2}, u \right) - \bar{p}_{IID} \left( \frac{x_{RD}}{2}, u \right) = G(x_{BD}, u), \quad \text{..............................(E-3)}
\]
\[
G(x_{BD}, u) = \frac{\bar{p}_{FD}(y_{1D}) - Q(x_{BD}, y_{1D})}{\int_0^\infty S_x(x_{BD})S_y(y_{1D})e^{-ut_D}d\tau_D} \int_0^\infty S_x \left( \frac{x_{RD}}{2} \right) S_y(y_{1D})e^{-ut_D}d\tau_D +
\]
\[
Q \left( \frac{x_{RD}}{2}, y_{1D}, u \right) - \frac{\bar{p}_{FD}(y_{1D})}{\cosh \left( \frac{u(x_{BD})}{2} - x_{RD} \right) \sqrt{u(1+\beta)}} \cosh \left[ \left( \frac{-x_{RD}}{2} \right) \sqrt{\frac{u(1+\beta)}{\eta_{II}}} \right] + Q_{II} \left( \frac{x_{RD}}{2}, u \right). \quad \text{....(E-4)}
\]

The pressure difference \(\Delta \bar{p}_D\) is dependent on \(x_{BD}\) and \(u\) when all other model parameters are set in sub-systems.

(3) Choose appropriate \(x_{BD}\) to minimize the pressure difference \(\Delta \bar{p}_D\) over the range \((u_2, u_1)\) in Laplace domain.

\[
\Delta \bar{p}_{avg}(x_{BD}) = \min \left\{ \int_{u_2}^{u_1} \left| G(x_{BD}, u) \right| du \middle| \frac{u_2}{u_1 - u_2} \right\}. \quad \text{..............................(E-5)}
\]

\(\Delta \bar{p}_{avg}\) is the minimum average relative pressure difference over time. In the Stehfest algorithm, the Laplace variable \(u\) is defined as \(ln2/t_D\). The \(x_{BD}\) value influences the well’s pressure behavior before the boundary effect occurs. It is summarized that for most cases with the dimensionless definitions in Appendix A, the boundary effect occurs after \(t_D\) reaches 0.01. Accordingly, the lower limit \(u_2\) in Eq. E-5 is chosen as \(ln2/0.01\). The upper limit \(u_1\) is set to be \(ln2/t_{Dmin}\). Here \(t_{Dmin}\) is the minimum production time point in model calculation, which can be chosen by users.
E.2 Determination of $r_e$

When radial flow equations are applied to a region outside of fracture tips, the hypothetical outer boundary $r_e$ is determined based on the region geometry. For a rectangular region, the outer boundary for radial flow is assumed as

$$r_e = \sqrt{\frac{2W_{\text{Region}} \times L_{\text{Region}}}{\pi}}.$$ ..........................(E-6)

Here $W_{\text{Region}}$ and $L_{\text{Region}}$ are the region’s width and length, respectively.

E.3 Determination of $r_w$

When radial flow equations are applied beyond fracture tips, the inner boundary $r_w$ has significant influence on our composite model’s stability and accuracy. The first choice of $r_w$ is half of the hydraulic fracture width, $w_f/2$. And half of the production rate from a vertical well with wellbore radius $w_f/2$ in a $2W_{\text{Region}} \times L_{\text{Region}}$ region is assumed as the flow rate from the radial flow region to the fracture tip. But this choice underestimates the rate towards a fracture tip. Therefore, the $r_w$ in the composite model is usually larger than $w_f/2$. In addition, the best $r_w$ should minimize the pressure difference across the interface between radial flow regions and adjacent linear flow regions. Take the sub-system in Fig. 5b for an example to show the procedure of deciding $r_w$:

1) Calculate the pressure at the interface of Regions I and II.

The pressure at the center point of the interface provides the basis for determining $r_w$. The pressure at $(x_{RD}/2, y_{1D})$ in Region I of Fig. 5.5b is

$$\bar{p}_{1D}(r_{WD}, u) = A_r I_0 \left( \frac{u(1+\beta)}{\eta_l} r_{WD} \right) + B_r K_0 \left( \frac{u(1+\beta)}{\eta_l} r_{WD} \right) + \frac{a}{u} n r_{WD} + \frac{b}{u} . .... \ \ (E-7)$$

The pressure at $(x_{RD}/2, y_{1D})$ in Region II is...
\[ \hat{p}_{II}(\frac{x_{RD}}{2}, u) = \frac{\bar{p}_{FD}(y_{ID})}{\cosh[\left(\frac{w_{FD}}{2} - x_{RD}\right)\sqrt{\frac{u(1+\beta)}{\eta_I}}]} \cosh\left[\left(\frac{-x_{RD}}{2}\right)\sqrt{\frac{u(1+\beta)}{\eta_H}}\right] + Q_{II}(\frac{x_{RD}}{2}, u) \] ........(E-8)

The \( A_r \) and \( B_r \) depend on the value of pressure at the fracture tip \( \bar{p}_{FD}(y_{ID}) \). And the \( \bar{p}_{FD}(y_{ID}) \) can calculated in Eq. D-23 and becomes a function of the selected \( r_{wD} \).

(2) Calculate the pressure difference at the center point of the interface.

In Fig. 5.5b, the pressure difference at \( (x_{RD}/2, y_{ID}) \) between Regions I and II is

\[ \Delta \hat{p}_D = \hat{p}_{ID}(r_{wD}, u) - \hat{p}_{II}(\frac{x_{RD}}{2}, u) = G(r_{wD}, u), \] .................................(E-9)

\[
G(r_{wD}, u) = A_r I_0\left(\sqrt{\frac{u(1+\beta)}{\eta_I}} r_{wD}\right) + B_r K_0\left(\sqrt{\frac{u(1+\beta)}{\eta_I}} r_{wD}\right) + \frac{a}{u} \ln r_{wD} + \frac{b}{u} - \frac{\bar{p}_{FD}(y_{ID})}{\cosh[\left(\frac{w_{FD}}{2} - x_{RD}\right)\sqrt{\frac{u(1+\beta)}{\eta_I}}]} \cosh\left[\left(\frac{-x_{RD}}{2}\right)\sqrt{\frac{u(1+\beta)}{\eta_H}}\right] - Q_{II}(\frac{x_{RD}}{2}, u) \]. .................................(E-10)

The pressure difference \( \Delta \hat{p}_D \) is dependent on \( r_w \) and \( u \) when all other model parameters are set in sub-systems.

(3) Choose appropriate \( r_w \) to minimize the pressure difference \( \Delta \bar{p}_D \) over the range \((u_2, u_1)\) in Laplace domain.

\[ \Delta \bar{p}_{avgD} = \min_{u_2} \left\{ \int_{u_2}^{u_1} G(r_{wD}, u) du \right\} \] .................................(E-11)

The \( r_w \) influences the well’s pressure behavior before the boundary effect occurs.

Similar to Eq. E-5, the lower limit \( u_2 \) in Eq. E-11 is chosen as \( \ln 2/0.01 \).


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