Eye Love Arithmetic: An Inversion and Associativity Eye Tracking Study

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Abstract

Arithmetic is important for developing the cognitive and problem-solving skills that are fundamental for higher levels of math. As such, it is important that individuals understand arithmetic concepts such as inversion and associativity, which can be reflected in how they solve three-term arithmetic problems. If an adult solves an inversion problem like 27 + 46 – 46 by cancelling the 46s, it suggests they understand inversion and have used an inversion shortcut. Similarly, when adults solve an associativity problem like 3 × 26 ÷ 13 by first computing 26 ÷ 13, they have used an associativity shortcut. To deconstruct why some individuals are better at using shortcuts, the current study used an eye tracker to generate heat maps and compare the visual attention of shortcut users to shortcut non-users. Participants (n =22) solved 32 three-term arithmetic problems while their eye fixations were tracked. Half of the problems were inversion, and the other half were associativity. Problems differed by operators (additive or multiplicative) and their format (conducive or non-conducive). Results support previous findings that adults are more accurate and use more shortcuts on inversion, additive, and conducive problems than associativity, multiplicative, and non-conducive problems. When comparing the eye movements of shortcut users to shortcut non-users, the heat maps indicate that participants focused on different areas. Further visual and statistical analyses are needed to compare the eye movements of shortcut users to shortcut non-users. Continuing to study the visual attention of shortcut users might explain why they perform well on these problems.

Keywords: arithmetic, inversion, associativity, shortcuts, attention, eye tracking

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Arithmetic involves working with the operators and is an important precursor for learning subsequent mathematics topics such as algebra (Booth & Koedinger, 2008). To understand the skills of proficient arithmetic individuals, researchers study adults' conceptual knowledge of two related arithmetic concepts —inversion and associativity— by measuring their calculation accuracy, execution function, and problem-solving strategies (Eaves et al., 2022). Inversion is understanding that addition and subtraction, and multiplication and division, are opposing operators (Baroody & Dowker, 2003), while associativity is understanding that solving the problem in any order results in the same answer (Canobi et al., 1998). Based on adults' accuracy and shortcut use, individuals have a better concept of inversion than associativity (Robinson & LeFevre, 2012).

Researchers also manipulate the problem operators and format to understand how this influences problem-solving (Robinson & Ninowski, 2003). Execution function skills, such as attention, are thought to influence problem-solving, but there is insufficient evidence to support this domain-general skill (Siegler & Araya, 2005). Eye tracking, which measures attention (Hartmann, 2015), has never been used for inversion and associativity problems, and the current study incorporated this device to understand the role of attention in solving arithmetic problems.

Inversion Problems

Inversion is when same-value terms with opposing operators "cancel out" in an arithmetic problem (Starkey & Gelman, 1982). Inversion problems written as $a + b - b$ and $d \times e \div e$ are called additive and multiplicative inversion problems, respectively (Eaves et al., 2021; Robinson & Ninowski, 2003). An individual who understands the inversion concept will notice that adding '*b'* and subtracting '*b'* cancels out, leaving '*a,'* the answer (Bisanz & LeFevre, 1990). Similarly, multiplying by ‘*e’* and dividing by ‘*e’* cancels out, resulting in ‘*d’* (Bisanz & LeFevre, 1990). When individuals solve inversion problems by cancelling out like-terms, they have used the inversion shortcut and are called shortcut users (Eaves et al., 2021). However, some adults problem-solve left-to-right (e.g., solving $a + b$ first in the problem $a + b - b$) and are called shortcut non-users (Eaves et al., 2021).

Although additive and multiplicative inversion problems measure the same concept, researchers have found that adults are better at additive problems, which suggests that operators influence problem-solving (Robinson & LeFevre, 2012; Robinson & Ninowski, 2003). For example, in additive inversion problems, adults use shortcuts 90-95% of the time, while for multiplicative problems, they use shortcuts 70-75% of the time (Eaves et al., 2021). This difference in shortcut use could be attributable to different concepts stored in memory, as additive inversion problems require recognizing that $b - b = 0$, while multiplicative inversion problems require recognizing $e \div e = 1$ (Robinson & LeFevre, 2012).

Inversion problems are also differentiated by their format. The examples $a + b - b$ and $d \times e \div e$ are conducive, meaning the terms that “cancel out” are located next to one another (Eaves et al., 2021). On the other hand, non-conducive problems are written as $b + a - b$ or $e \times d \div e$ and require scanning the entire problem to find the terms that “cancel out” and use a shortcut (Eaves et al., 2021). Regardless of the problem format, individuals with a strong concept of inversion will use the shortcut on conducive and non-conducive problems (Eaves et al., 2021).

Associativity Problems

Associativity is when reordering the operators in an arithmetic problem does not change the final answer (Canobi et al., 1998). Associativity problems written as $a + b - c$ or $d \times e \div f$ are called additive and multiplicative associativity problems, respectively (Robinson & Ninowski, 2003). Evidence of individuals' understanding of the associativity concept is suggested through their calculation order (Klein & Bisanz, 2000). For example, while solving an additive associativity problem, if an individual solves the subexpression '*b – c'* and then adds that value to *'a*,' they have used the associativity shortcut (Klein & Bisanz, 2000). Moreover, if an individual first solves the subexpression '*e* ÷ *f*,' then multiples the quotient by '*d,'* they have used the associativity shortcut in a multiplicative associativity problem (Robinson & Ninowski, 2003). The associativity shortcut is beneficial because it allows individuals to simplify a part of the problem instead of computing left-to-right (Eaves et al., 2021).

Adults are also better at additive associativity problems than multiplicative associativity problems (Robinson & Ninowski, 2003). For example, in a study by Robinson and Ninowski (2003), adults' shortcut use was 58% for additive associativity problems, while for multiplicative associativity problems, their shortcut use was 33%. Performing better on additive problems is also observed in inversion problems, which suggests that, in general, adults are better at additive than multiplicative arithmetic problems (Robinson & LeFevre, 2012). This difference in performance could be attributable to increased familiarity with addition and subtraction concepts as they are learned earlier than multiplication and division concepts (Dubé & Robinson, 2017). Moreover, division is the least understood operator and is taught using multiplication, addition, and subtraction (Robinson & Ninowski, 2003).

The associativity problems $a + b - c$ and $d \times e \div f$ are conducive, and the associativity problems $b + a - c$ and $e \times d \div f$ are non-conducive (Eaves et al., 2021). Multiplicative associativity studies by Edwards (2013) and Robinson and Ninowski (2003) found that shortcut use was higher for conducive than non-conducive problems. Robinson and Ninowski (2003) used the non-conducive problem 9 × 22 ÷ 18, knowing that solving 9 ÷ 18 and then multiplying by 22 would result in the fastest computation. However, individuals in this study failed to recognize the associativity shortcut in this format and instead problem-solved left-to-right (Robinson & Ninowski, 2003). Moreover, an additive associativity study by Eaves et al. (2019) found that shortcut users solved more conducive problems than shortcut non-users in a limited time frame. However, for non-conducive problems, there was no difference in the number of solved problems between shortcut users and shortcut non-users (Eaves et al., 2019). The results from these studies suggest that problem format influences individuals' shortcut use.

Comparing Inversion and Associativity Performance

Regardless of changing the problem operators or format, using a shortcut results in faster computation and reflects an individual's understanding of inversion and associativity (Eaves et al., 2021). The literature identifies individuals as part of the dual concept group, meaning they use inversion and associativity shortcuts, or part of the inversion-only group, whereby they only use inversion shortcuts (Dubé & Robinson, 2010; Robinson & Dubé, 2009). An associativity-only group does not exist as comprehension of the associativity concept seems to depend on comprehension of the inversion concept (Dubé & Robinson, 2010; Robinson & Dubé, 2009).

Due to a lack of understanding of associativity and failure to recognize the associativity shortcut, adults perform better on inversion than on associativity problems (Robinson & LeFevre, 2012). For example, adults’ shortcut use on additive inversion problems is about 90-95%, and for additive associativity problems, shortcut use is around 50-60% (Eaves et al., 2021). However, researchers believe adults’ knowledge of associativity is greater than what is reflected in experiments (Eaves et al., 2021). For example, some adults may understand the principle but fail to use a shortcut because they are familiar with problem-solving left-to-right or do not want to reorder the operators (Robinson & Dubé, 2012).

The decrease in associativity performance could also be attributable to strategy identification and execution (Eaves et al., 2021). Strategy identification is recognizing the shortcut within the arithmetic problem, while strategy execution is using the appropriate strategy effectively (Eaves et al., 2021). On inversion problems, identification and execution occur simultaneously—when someone identifies the shortcut, no additional calculation is required to cancel out like-terms (Eaves et al., 2021). On associativity problems, strategy identification precedes strategy execution— individuals must attend to the shortcut and then perform a calculation (Eaves et al., 2021). The additional steps required for associativity problems highlight why associativity problems are more complex than inversion problems.

Attention and SCADS

The Strategy Choice and Discovery Simulation (SCADS) model outlines the development of individuals' problem-solving strategies (Shrager & Siegler, 1998). When solving arithmetic problems, individuals automatically choose strategies they are confident performing (Siegler & Araya, 2005). To execute new strategies, individuals must experience an interruption, and attention is the first step in the process (Siegler & Araya, 2005). Attention is prioritizing certain stimuli over others, sometimes in a specific location (Kim & Cave, 1995). For shortcut non-users, problem-solving left-to-right is automatically chosen because this is an effective strategy learned through the order of operations (Eaves et al., 2022). A shortcut non-user must notice the shortcut before they switch to using the shortcut as a strategy (Siegler & Araya, 2005). As highlighted above, this is easier for inversion than associativity problems because of strategy identification and execution (Eaves et al., 2021).

Based on the SCADS model, attention is important for using the shortcut. As such, researchers have altered problem features to divert attention to the shortcut. For example, a study by LeFevre and Robinson (2010) made the second term in the arithmetic problem a two-digit number, leading participants to solve the shortcut first. Researchers assert that the set-up of arithmetic problems influences problem-solving (Landy & Goldstone, 2007), and the SCADS model states that problems with similar perceptual features will be solved similarly (Siegler & Araya, 2005).

As such, three studies have used priming to influence problem-solving. Dubé and Robinson's (2010) study primed individuals with the multiplication or division subexpression and then the entire inversion or associativity problem. They found that presenting the subexpression increased shortcut use on inversion problems but not associativity problems (Dubé & Robinson, 2010). Eaves et al. (2020) replicated Dubé and Robinson's (2010) study but used additive associativity problems. Likewise, they found that priming participants with the subexpression containing the associativity shortcut did not influence shortcut use for associativity problems.

Eaves et al. (2019) found that priming individuals with inversion conducive or inversion non-conducive problems increased shortcut use on associativity problems compared to individuals primed with two-term arithmetic problems. The researchers also found that solving inversion conducive problems facilitated performance on associativity conducive problems. However, there was no difference in performance on associativity non-conducive problems, which suggests that participants primed in the inversion non-conducive group did not experience any advantages.

The results from Dubé and Robinson (2010), Eaves et al. (2020), and Eaves et al. (2019) suggest that attentional cues differ for inversion and associativity problems and conducive and non-conducive problems, respectively. Nonetheless, no studies to date have directly measured participants’ attention while they solve inversion and associativity problems.

Eye Movements

The eye-mind hypothesis asserts that an individual's eye fixations reflect where they focus their attention, and while individuals are retrieving information from long-term memory, there are differences in where they gaze and for how long (Just & Carpenter, 1984). Thus, eye movements are a real-time measure of attention (Hartmann, 2015). To understand the cognitive processes involved in problem-solving, researchers often use eye tracking to measure participants' eye movements (Hartmann, 2015). However, limited studies have used eye tracking for arithmetic problem-solving. A study by Green et al. (2007) looked at how participants solve two-term addition problems, and they found that participants problem-solved left-to-right, which matched their self-reported strategy use. Another study by Zhou et al. (2012) looked at participants’ attention while solving two-term multiplication problems and found that for problems such as 5 × 60, individuals attend to the larger term (i.e., 60) before they attend to the smaller term (i.e., 5).

A study by Curtis et al. (2016) used two-term addition, subtraction, multiplication, and division problems to see how adults’ eye movements differ across the terms and operators. The researchers found that for addition and multiplication problems, the operator received the longest gaze duration compared to the terms, but the time spent on each term was even. They also found this pattern for small subtraction and division problems. However, on large division problems, individuals spent most of their time on the left-most term (i.e., the dividend). Nonetheless, this study highlights that where participants focus in a problem can be influenced by the terms and operators.

One benefit of eye tracking is that there are multiple ways to visualize participants' eye movements. For example, by recording eye fixations, glances, and saccades, heat maps can be generated (Tobii, n.d.-c). A heat map records the number of fixations in an area of an image and then assigns areas a colour ranging from red (i.e., lots of fixations) to green (i.e., few fixations) (Tobii, n.d.-a). Thus, for arithmetic problems, heat maps reflect what terms or operators' participants are fixating on. Eye fixations from multiple participants can be mapped onto one another to generate the average fixations of participants (Tobii, n.d.-c).

The Current Study

Knowing that the role of attention role in solving inversion and associativity problems is understudied and that arithmetic performance differs across problem type (inversion vs. associativity), operation type (additive vs. multiplicative), and format (conducive vs. non-conducive), the current study used eye tracking to measure participants' attention during problem-solving.

Our first research question is how the various problems influence accuracy. We hypothesized that participants would have higher accuracy on additive, inversion, and conducive problems, as previous research has supported this (Eaves et al., 2019; Eaves et al., 2020; Edwards, 2013; Robinson & Ninowski, 2003). Thus, when comparing problems that differ by one level of the independent variable, we hypothesized that the more difficult condition would result in lower accuracy. For example, a multiplicative inversion conducive problem would have lower accuracy than a multiplicative inversion non-conducive problem. Moreover, when these conditions interact, we hypothesized that additive inversion conducive problems would have the highest accuracy, while multiplicative associativity non-conducive problems would have the lowest accuracy.

Our second research question regards how the various problem conditions influence shortcut use. Since accuracy and shortcut use are related, we hypothesized that participants would have higher shortcut use on additive and inversion problems (Dubé & Robinson, 2010; Eaves et al., 2020) and conducive problems (Edwards, 2013; Robinson & Ninowski, 2003). Similar to our hypothesis on accuracy, when looking at the interaction of independent variables, we hypothesized that when comparing problems that differ by one level of the independent variable, the problem condition with the more difficult level of the independent variable would result in lower shortcut use. Again, we hypothesized that additive inversion conducive problems would have the highest shortcut use, while multiplicative associativity non-conducive problems would have the lowest shortcut use.

Our final research question regards the differences in eye fixations between shortcut users and shortcut non-users. By generating heatmaps, we hypothesized that eye movements would differ, and shortcut users' eye fixations would reflect more heat on the shortcut – whether that be cancelling out like-terms (inversion shortcut) or simplifying terms (associativity shortcut) – and less time on the remaining term. In contrast, shortcut non-users' eye movements would be equally distributed across all the problem terms. For example, if we consider the problem 27 + 46 – 46, we hypothesized that a shortcut user would pay more attention to the 46s, while a shortcut non-user would have an equal number of fixations on the three terms.

Method

Participants

A total of 22 undergraduate students (14 identified as female, 7 identified as male, and 1 identified as other) (ages in years: M= 21.27, SD= 4.610) participated in the study. Participants were recruited through the University of Regina Psychology Participant Pool, which allows undergraduate students enrolled in 100 or 200-level psychology courses to earn a 1% bonus in their psychology course for participating in the study. Our study did not require any specific participant gender or sex considerations. However, participants had to have normal or corrected vision (i.e., wearing glasses or contacts).

Materials

Demographic Questionnaire

A demographic questionnaire (see Appendix A) was administered through Qualtrics and asked participants to state their age, gender, year and area of study, frequency of doing math, and enjoyability of math.

Arithmetic Problems

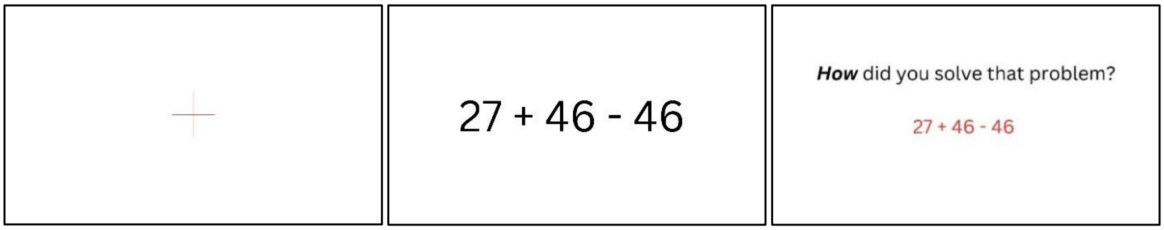
Participants solved 32 three-term arithmetic problems (see Appendix B) that differed by operation type, problem type, and format. The 16 additive problems were modelled after Eaves et al. (2019), and 16 multiplicative problems were modelled after Robinson et al. (2003) and Dubé (2014). Each problem set was further differentiated by problem type: 8 inversion problems and 8 associativity problems. For each problem type, 4 problems were conducive, and 4 were non-conducive. Differentiating problems by operation type, problem type, and format resulted in eight arithmetic problem conditions. Originally, participants were going to solve either the additive problem set first, or the multiplicative problem set first. However, the problem sets were split up so that participants started solving half of one problem set, then switched to the other problem set, and then finished with the starting problem set. For example, a participant would start with 8 additive problems, then solve 16 problems from the multiplicative set, and end with 8 additive problems. Or participants would start with 8 multiplicative problems, then solve 16 problems from the additive set, and end with 8 multiplicative problems. However, flip-flopping between problems does not control for the influence of problem order, so this independent variable was dropped from our analysis.

Procedure

The procedure took approximately 30 minutes to an hour for each participant. First, participants scanned the Qualtrics link and gave their consent to take part in the study. Participants then completed the demographic questionnaire. After they completed the demographic questionnaire, participants sat in front of a laptop with the eye tracker. To ensure confidentiality, participants were given a participant number to keep track of their math and eye data. Once participants were comfortable, the Tobii Pro Nano eye tracker was calibrated, which required participants to follow the movement of dots on the laptop screen with their eyes. Once a baseline of participants' eye movements had been established, participants started the study. Questions were presented through a series of PowerPoint slides that contained a fixation cross slide, arithmetic problem slide, and post-problem strategy slide (see Figure 1). The sequence was as follows: first, the fixation cross slide appeared, then participants hit the space bar. Second, the arithmetic problem slide was shown, and participants stated their answers aloud, and then hit the space bar. Third, a post-problem strategy report slide was shown, and the researcher asked the participants to explain how they got their answers. The researcher documented participants' answers (accuracy) and how they solved the problem (strategies) on a premade data sheet with the participants' numbers. This process was repeated for all 32 arithmetic problems. Once participants were done, they were debriefed and thanked for participating in the study.

Figure 1

Sequence of Slides



Note. Each arithmetic problem had a fixation cross slide, an arithmetic problem slide, and post-problem strategy report slide.

Measures

Accuracy

Participants answers were recorded for each of the 32 problems by the researcher. After the experiment, the researcher coded the answers as right or wrong.

Problem-Solving Strategies

After participants had stated their answers to an arithmetic problem, the post-problem strategy report slide was shown, asking participants how they solved the problem. If participants did not know how to respond, the researcher asked participants the order in which they problem solved (e.g., "What part of the problem did you solve first?"). Regardless of the operation type, problem type, or problem format, if participants solved a problem using a shortcut, they were labelled as a shortcut user for that problem. If participants problem solved from left-to-right or solved the problem using an alternative strategy, they were labelled as a shortcut non-user for that problem. For example, when solving the problem 26 × 3 ÷ 26, if participants stated that they cancelled out the 26s first, they were coded as shortcut users. If a participant said that they solved 26 × 3 first, which is common for a left-to-right problem solver, or if they solved the subexpression 3 ÷ 26 first, which is a more difficult strategy, they were coded as shortcut non-users. Participants' strategies for the 32 problems were recorded on a data sheet by the researcher.

Eye Movements

Eye movements were tracked using the Tobii Pro Nano eye tracker with Tobii Pro Lab software, which was placed below the laptop screen (Tobii, n.d.-b). This eye tracker uses non-invasive infrared technology, making no physical contact with participants (Strohmaier et al., 2020). As participants viewed the arithmetic problem slide, their eye fixations were recorded, which allowed the software to generate heat maps for that specific problem. Thus, participants' eye fixations were recorded for each of the 32 problems. Generating heat maps for problems allowed us to compare the visual attention of shortcut users to non-users and see if there was a difference in where participants were allocating their time within the problem. Only problems with an equal number of shortcut users to non-users were used for comparison (*n* = 3).

Results

Accuracy

We hypothesized that additive, inversion, and conducive problems had higher accuracy and that when comparing problems that differed by one or more levels of the independent variable (e.g., additive inversion conducive problems compared to additive inversion non-conducive problems), the problem with more difficult independent variables (the additive inversion problems in non-conducive format) would result in lower accuracy. The proportion of accuracy responses was calculated for each problem type (e.g., additive inversion conducive) for each participant.

The accuracy data was analyzed using a 2 (operation: additive, multiplicative) x 2 (problem: inversion, associativity) x 2(format: conducive, non-conducive) analysis of variance (ANOVA), and the results indicated that there were three main effects, one for each of the independent variables (see Figure 2). The operation type (additive or multiplicative) was significant, $F\ (1, 21) = 19.225$, $p < .001$, with an effect size of $\eta_p^2 = 0.478$, which is a large effect, indicating that accuracy was higher on additive (M= 0.9260, SE=0.018) than multiplicative (M= 0.764, SE=0.040) problems. Problem type was also significant, $F\ (1, 21) = 27.611$, $p < .001$, with an effect size of $\eta_p^2 = 0.568$, which is a large effect, indicating that accuracy was higher on inversion (M=0.943, SE=0.016) than on associativity problems (M=0.747, SE=0.041). Lastly, there was a main effect for format, $F\ (1, 21) = 11.541$, $p < .003$, with an effect size of $\eta_p^2 = 0.355$, which is a large effect, indicating that accuracy was higher on conducive (M =0.895, SE =0.023) than non-conducive problems (M =0.795, SE=0.034).

These main effects support our first hypothesis that adults are better at additive, inversion, and conducive arithmetic problems. The ANOVA also revealed interaction effects of how the combination of independent variables influenced accuracy. There was a significant interaction between operation and problem type, $F\ (1, 21) = 13.306$, $p = .002$, $\eta_p^2{=}0.388$. Post hoc pairwise comparisons were performed and indicated that when looking at associativity problems, there was a significant difference ($p < .001$) between additive associativity problems (M=0.886, SE=0.026) and multiplicative associativity problems (M=0.608, SE=0.069), which indicated that additive associativity problems had higher accuracy. However, when comparing inversion problems, there was not a significant difference between the accuracy on additive and multiplicative inversion problems (p =.104). When comparing additive problems, there was a significant difference (p =.005) between additive inversion problems (M=0.966, SE=0.017) and additive associativity problems (M=0.885, SE=0.026), which indicated that additive inversion problems had higher accuracy. Lastly, when comparing multiplicative problems, there was a significant difference (*p* <.001) between multiplicative inversion problems (M=0.920, SE=0.024) and multiplicative associativity problems (M=0.608, SE=0.069), which indicated that multiplicative inversion problems had higher accuracy.

The interaction between operation and problem type supports our hypothesis that regardless of the operation, associativity problems are more difficult, and, thus, accuracy is lower. However, for inversion problems, when changing the operation type, participant accuracy did not differ by a significant amount. This interaction effect suggests that operation has less of an influence on inversion problems than it does on multiplicative problems.

There was also a significant interaction between operation and format, F (1, 21) = 9.536, p =.006, $\eta_p^2 = 0.312$. Post hoc pairwise comparisons were performed and indicated that when looking at conducive problems, there was a significant difference (*p* =.026) between additive conducive problems (M=0.943, SE=0.021) and multiplicative conducive problems (M=0.847, SE= 0.052), indicating that there was higher accuracy on additive conducive problems. Moreover, when looking at non-conducive problems, there was a significant difference (*p* < .001) between additive non-conducive problems (M= 0.909, SE=0.0204) and multiplicative non-conducive problems (M=0.682, SE=0.052), indicating that there was higher accuracy on additive non-conducive problems. However, when comparing additive problems, there was not a significant difference between accuracy on additive conducive problems and additive non-conducive problems (*p* =.229). When comparing multiplicative problems, there was a significant difference (*p* < .001) between accuracy on multiplicative conducive problems (M=0.847, SE=

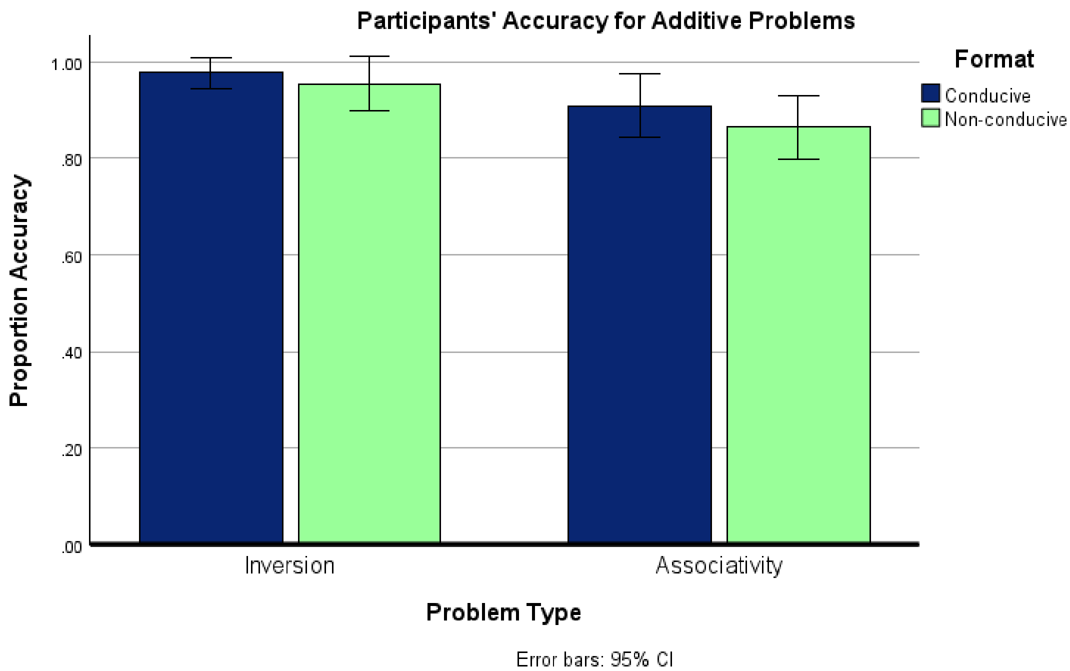
0.052) and multiplicative non-conducive problems (M=0.682, SE=0.052), indicating that there was higher accuracy on multiplicative conducive problems.

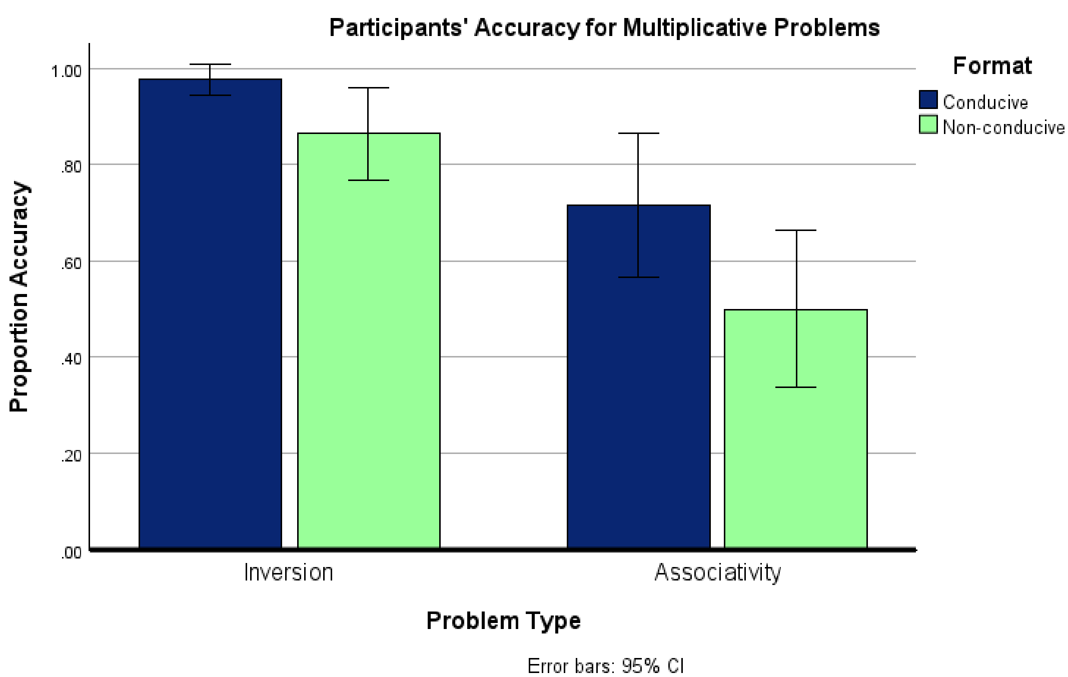
The interaction between operation and problem type supports our hypothesis that problems of the same format, but different operation, yield higher accuracy for the additive problems. More interestingly, when looking strictly at multiplicative problems, format influences accuracy in that conducive problems are more accurate than non-conducive problems. However, for additive problems, this is not the case, as additive conducive and non-conducive problems do not differ significantly in accuracy. This interaction effect suggests that format matters for multiplicative problems but not for additive problems.

The interaction between problem type and format was not significant, F (1, 21) = 2.432, p = .134, η_p^2= 0.104. This finding did not support our hypothesis. However, it suggests that when comparing problems of the same problem type but in different formats, or when comparing problems of the same format but different problem type, accuracy is similar. Moreover, the interaction between operation, problem type, and format was not significant, F (1, 21) = 1.094, p =.308, η_p^2 = 0.049. As we hypothesized, additive inversion conducive problems had the highest accuracy (M = 0.9773, SE = 0.07356), while multiplicative associativity non-conducive problems had the lowest accuracy (M = 0.5000, SE = 0.37001). However, since the three-way interactions were not significant, the differences between problem types that differed by one level of the independent variable were not compared.

Figure 2

Accuracy on additive and multiplicative problems





Problem-Solving Strategies

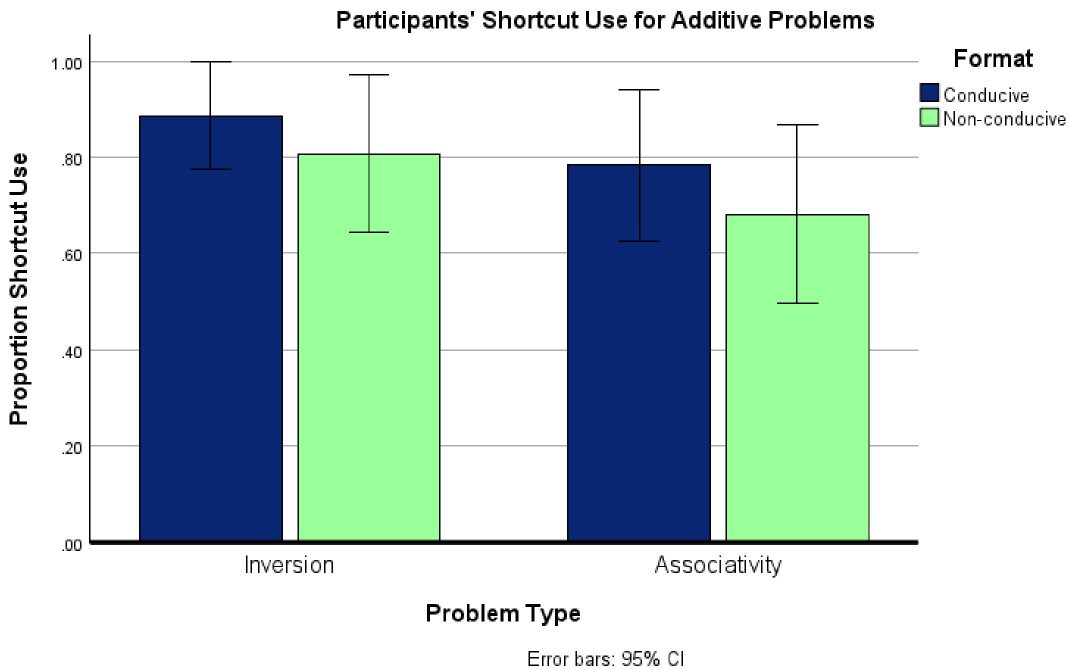
We hypothesized that additive, inversion, and conducive problems would have higher shortcut use, and when comparing problems that differed by one or more levels of the independent variable, the problem with more difficult independent variables would result in lower shortcut use. To answer this question, shortcut use was recorded. The proportion of shortcut use was calculated for each problem type (e.g., additive inversion conducive) for each participant.

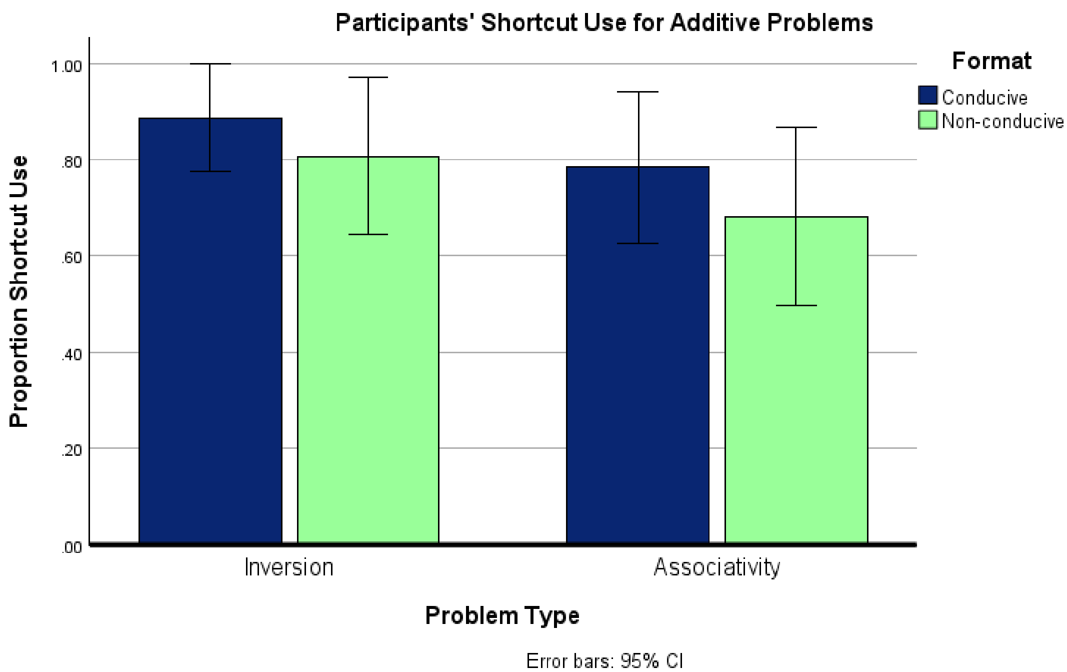
The strategy data was analyzed using a 2 (operation: additive, multiplicative) x 2 (problem: inversion, associativity) x 2(format: conducive, non-conducive) analysis of variance (ANOVA) and revealed three main effects (see Figure 3). The operation type (additive or multiplicative) indicated that there was more shortcut use on additive (M=0.790, SE= 0.064) than multiplicative problems (M=0.639, SE=0.062), F (1, 21) = 7.032, p =.015, η_p^2=0.25. There was also a main effect for problem type, F (1, 21) = 14.538, p =.001, η_p^2= 0.409, which revealed that shortcut use was higher on inversion (M =0.795, SE= 0.058) than associativity problems (M=0.634, SE=0.062). Lastly, there was a main effect for format, F (1, 21) = 7.187, p = .014, η_p^2=0.255, which revealed that shortcut use was higher on conducive (M=0.778, SE=0.056) than non-conducive problems (M=0.651, SE=0.066).

These main effects support our hypothesis that additive, inversion, and conducive problems have higher shortcut use. Moreover, the additive inversion conducive problems had the highest accuracy, while the multiplicative associativity non-conducive problems had the lowest accuracy. However, there were no significant interactions between the independent variables, which suggests that when comparing problems that differ by one or more levels of the independent variable, there is no difference in shortcut use.

Figure 3

Shortcut use on additive and multiplicative problems





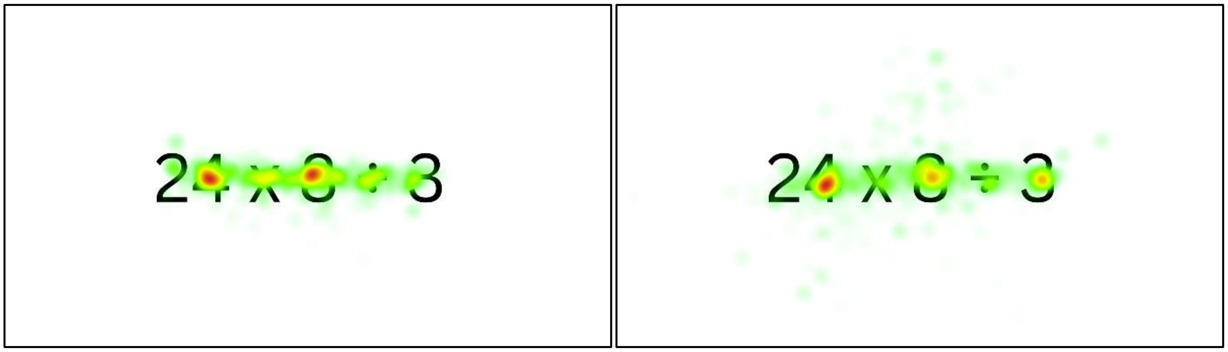
Comparison between shortcut users and shortcut non-users

To compare the visual attention of shortcut users to shortcut non-users, we generated heat maps using the Tobii Pro Lab software. We hypothesized that shortcut users would have more attention on the shortcut terms and less on the remaining term, while shortcut non-users would have attention equally distributed on all terms. The following problems were chosen: 26 × 3 ÷ 26, 24 × 3 ÷ 8, and 4 × 24 ÷ 6. All of these problems were multiplicative: 26 × 3 ÷ 26 is an inversion non-conducive problem, 24 × 3 ÷ 8 is an associativity non-conducive problem, and 4 × 24 ÷ 6 is an associativity conducive problem. Although we intended to generate more heat maps, the following problems were chosen because they have an equal number of shortcut users to shortcut non-users –that is, 11 participants were shortcut users, and 11 were shortcut non-users for these problems. However, using only multiplicative problems was beneficial as it allowed us to compare an inversion problem to associativity problems in differing formats.

The heat maps generated for the problem 24 × 8 ÷ 3 are shown in Figure 4. When comparing the heat maps, shortcut users spent less time on the 3. Since shortcut users used a shortcut (e.g., 24 ÷ 3 = 8), the 3 required less of their attention, and once they computed the answer to the shortcut, they spent their remaining attention on computing the answer to the rest of the problem (e.g., 8 × 8 = 64). Shortcut non-users, on the other hand, spent more time on the 3 and, as we hypothesized, had similar distributions of heat on all the terms.

Figure 4

Heatmaps for 24 × 8 ÷ 3

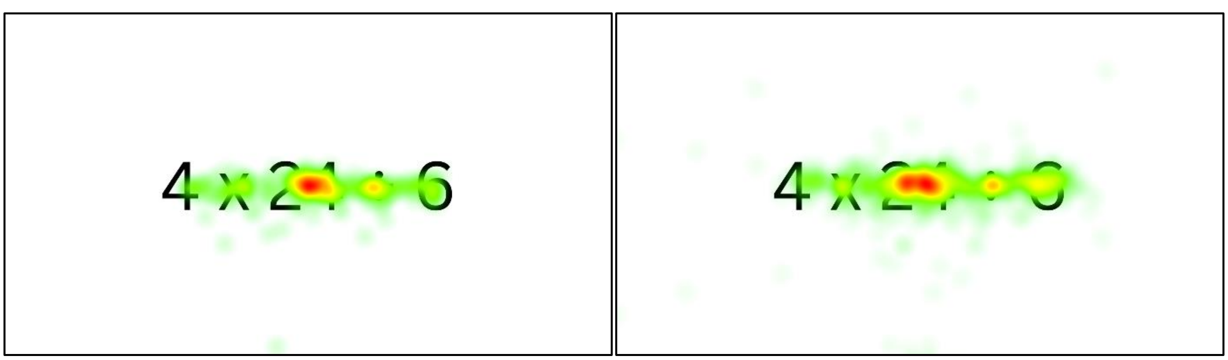


Note. Shortcut users are on the left. Shortcut non-users are on the right.

The heats maps for the problem 4 × 24 ÷ 6 are shown in Figure 5. The heat maps of the two groups are similar, with more fixations on the 24. However, the shortcut users pay less attention to the 6 compared to the shortcut non-users. This heat map is also interesting because shortcut non-users do not have their attention distributed on all the terms. There seems to be more of a fixation on the 24 and 6, but not the 4.

Figure 5

Heatmaps for 4× 24 ÷ 6

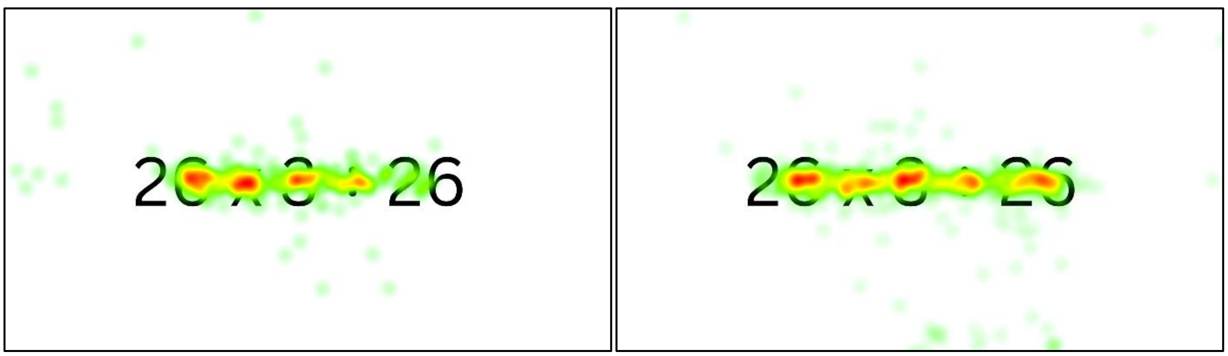


Note. Shortcut users are on the left. Shortcut non-users are on the right.

Lastly, the heatmaps for the problem 26 × 3 ÷ 26 (see Figure 6) illustrate that participants spent a significant amount of time on the left-most 26 and 3, but shortcut users spent less time on the right-most 26. Since shortcut users used a shortcut (e.g. 26 ÷ 26), the right-most 26 requires less of their attention since it was cancelled out. They then spent their remaining attention on computing the answer to the problem (e.g., 3). Shortcut non-users, on the other hand, spent more time on the right-most 26 and, as we hypothesized, had similar distributions of heat on all the terms.

Figure 6

Heatmaps for 26 × 3 ÷ 26


.

Note. Shortcut users are on the left. Shortcut non-users are on the right.

Interestingly, although these three problems differed either in their problem type or format, the same pattern was found: shortcut users had less fixations on the right-hand term. Although we hypothesized that shortcut users would spend more time attending to the shortcut, the results from our heat maps suggest the opposite. If anything, participants spent less time on the shortcut, which, for the non-conducive problems, required looking at the right-most term. The problem 4 × 24 ÷ 6 is more complicated since it has a double-digit in the middle, which could automatically divert attention to the centre of the problem. This argument could also be made for the problems 24 × 8 ÷ 3 and 26 × 3 ÷ 26, in that one might assume that participants had lots of heat on the 24 and left-most 26. However, because both problems are non-conducive and

26 × 3 ÷ 26 has another double-digit number, similar eye patterns were made for these two problems.

Discussion

Our first hypothesis — that accuracy would be higher on additive, inversion, and conducive problems than multiplicative, associativity, and non-conducive problems, respectively — was supported by our data and supports previous research (Eaves et al., 2021; Edwards, 2013; Robinson & Ninowski, 2003). We also hypothesized that when comparing problems that differed by one or more levels of the independent variable, the problem with the more difficult independent variable would have lower accuracy (e.g., comparing additive inversion problems to additive associativity problems). When looking at Figure 2, the additive inversion conducive problems had the highest accuracy, while the multiplicative associativity non-conducive problems had the lowest accuracy. However, this hypothesis was partially supported by the accuracy data as some interactions were insignificant.

The significant interactions echoed previous findings by Robinson and Ninowski (2003) that adult's accuracy was higher for additive problems compared to multiplicative problems. However, there was an exception— changing the operators did not influence accuracy on inversion problems, as both additive inversion problems and multiplicative inversion problems had similar accuracy. This finding suggests that changing operators for associativity problems has more of an impact on accuracy than changing the operators for inversion problems. Moreover, when problems differed by their problem type, inversion problems had higher accuracy than associativity problems, which supports previous research (Eaves et al., 2021). Changing the problem format influenced accuracy for multiplicative problems — multiplicative conducive yielded higher accuracy than multiplicative non-conducive. However, for additive problems there was not a significant difference in accuracy between conducive and non-conducive problems. This finding suggests that differing formats have a larger influence on accuracy for multiplicative problems than additive problems.

When comparing problems with the same problem type but different format (i.e., inversion conducive compared to inversion non-conducive or associativity conducive compared to associativity non-conducive), there is not a significant difference in accuracy. This is also true when comparing problems with the same format but different problem type (i.e., inversion conducive to associativity conducive or inversion non-conducive to associativity non-conducive). If accuracy scores are similar on inversion conducive problems and associativity conducive problems, it might have something to do with the similar perceptual features of the problem, which is what Eaves et al. (2019) found.

Our second hypothesis —that shortcut use would be higher on additive, inversion, and conducive problems —was supported by our data. We also hypothesized that when comparing problems that differed by one or more levels of the independent variable, the problem with the more difficult independent variable would have lower shortcut use. As shown by Figure 3, additive inversion conducive problems had the highest shortcut use, while multiplicative associativity non-conducive problems had the lowest shortcut use. However, when comparing problems that differ by one of the independent variables, there is not a significant difference in shortcut use. Knowing that shortcut use yields more accurate answers (Eaves et al., 2021), one might assume we would have found more interactions with our shortcut use data. However, we know that some shortcut non-users, who problem-solve left-to-right, have efficient calculation skills and are as accurate as shortcut users despite taking longer (Newton et al., 2010).

Our final hypothesis —that shortcut users would have more attention on the terms used for a shortcut strategy and less on the remaining term, while shortcut non-users would have attention equally distributed on all terms — was exploratory and partially supported by the heat maps we generated. For the non-conducive problems, such as 24 × 8 ÷ 3 and 26 × 3 ÷ 26, we found that shortcut users had less attention on the right-most term (e.g., the 3 in the problem 24 × 8 ÷ 3 and the right-most 26 in the problem 26 × 3 ÷ 26). This was surprising because using a shortcut for both problems requires computing 24 ÷ 3 first or computing 26 ÷ 26 first. However, based on shortcut users' eye fixations, they had less attention on the right-most term, which suggests that they may need less attention to compute the shortcut. Other researchers have found that adults who excel on inversion and associativity problems exhibit strong domain-specific skills such as calculation accuracy (Newton et al., 2010). So, it could be that their enhanced calculation skills allowed them to compute the shortcut quickly, spending the rest of their time computing the final answer. For these non-conducive problems, shortcut non-users' eye fixations supported our hypothesis— their eye movements had an equal amount of attention on each term.

We also generated heat maps for the conducive problem 4 × 24 ÷ 6, which deserves further inspection. For this problem, shortcut users and shortcut non-users had similar eye fixations, whereby both groups had more attention on the 24. The issue with this problem is that it has a double-digit in the centre, which can automatically divert attention, and this has been illustrated in other studies (LeFevre & Robinson, 2010; Zhou et al., 2012). Shortcut users still have less attention on the 6, or the right-most term, which is similar to what we found for the other problems.

Implications

Given that accuracy and shortcut use serve as important implicit and explicit measures, continuing to integrate eye tracking into these studies might reveal other explanations for shortcut users' efficient problem-solving. For example, our study found that when comparing problems in the same format but different problem types, there was not a significant difference in accuracy or shortcut use. If we consider the heat maps for 24 × 8 ÷ 3 (see Figure 4) and 26 × 3 ÷ 26 (see Figure 6), shortcut users made similar eye fixations for both problems, as did shortcut non-users. Thus, the heat maps provide a visual explanation for this finding.

As the first eye tracking inversion and associativity study, our results highlight that attention could help us understand how to increase adults' performance and understanding of associativity. While other studies have tried to increase attention toward the shortcut before presenting the entire problem (Dubé & Robinson, 2010; Eaves et al., 2019; Eaves et al., 2020), our study analyzed individuals' attention during the presentation of the entire problem. We found that shortcut users had less fixations on the shortcut for inversion and associativity problems. This finding suggests that both inversion and associativity shortcuts require less of shortcut users' attention since they are familiar.

The priming studies mentioned above found that attentional cues were beneficial for inversion problems but not associativity problems. Several researchers have found that priming is beneficial for adults who have prior conceptual knowledge (Jiang et al., 2014). Thus, using perceptual features to increase associativity shortcuts might not work. Eaves et al. (2020) also state that attention might be more beneficial for inversion shortcuts than associativity shortcuts. However, given that associativity is related to inversion, developing attention could indirectly help individuals solve associativity problems. Therefore, developing visual and spatial attention in children and adolescents might facilitate the learning of associativity after inversion, which, in the long term, could enhance the learning of algebra and more advanced mathematics topics.

Limitations

Although this is the first inversion and associativity eye tracking study, there are several limitations. The lack of significant two-way and three-way interactions for accuracy and shortcut use could be attributable to our small sample size. As for the procedure, placing the fixation cross in the centre of the screen before the problem appeared presents another issue – it might inflate the fixations on the middle term once the arithmetic problem slide appears. To overcome this issue, other studies have placed a fixation cue in one of the four corners before each problem (Curtis et al., 2016).

Further, our problem order (i.e., starting with half of one problem set, doing the other problem set, then finishing with the original problem set) did not allow us to see if presenting additive problems before multiplicative problems facilitates shortcut use, which has been used in other studies (Robinson & Ninowksi, 2003; Robinson et al., 2006). Since we are concerned with visual attention, and we know that adults perform better on inversion problems, future problems could be ordered by problem type, such that presenting inversion problems before associativity problems would facilitate performance as it has with other studies (Eaves et al., 2019). Eye movements could then be compared to see if groups that were first shown inversion problems had different fixations on associativity problems than groups that were first shown associativity problems and then inversion problems. Nonetheless, the ability to switch from problems that differ by operation or problem type and perform well might be related to another domain-general skill called task-switching (Eaves et al., 2020).

Aside from the problem order, a limitation of the post-problem strategy slide, asking participants, “How did you solve the problem?” was that some participants had trouble explaining their problem-solving strategy. When this was the case, the researcher prompted participants and asked, “What part of the problem did you solve first?” For inversion problems, shortcut users stated that they “cancelled out like-terms,” while shortcut non-users described that they worked left-to-right. Notably, it was easier for participants to describe how they solved inversion problems than associativity problems, which indirectly reflects their lack of understanding of associativity. Some participants would say, “this part was easier to solve first” or “there was a small difference between those two numbers.” For example, for the associativity problem 24 + 49 – 47, solving the subexpression 49 – 47 yields a small positive number while computing left-to-right is more difficult. Since the small difference incentivizes the participants to attend to this section of the problem first, it is hard to determine if they understand associativity or solved the problem this way out of convenience. If it was out of convenience, it does reflect less rigidity in their problem-solving abilities. For the associativity problems in our study, all the shortcuts yielded a small positive number ranging from 2 to 5. Regardless, it would be interesting to see the point by which participants are no longer incentivized to use a shortcut. Nonetheless, self-reported strategies should continue to be used in conjunction with accuracy and implicit measures, as repeatedly asking adults how they solved a problem can bias them or influence future problem-solving (Haider et al., 2014; Siegler & Stern, 1998).

Capturing visual attention through heat maps also had several drawbacks. The heat maps are generated by layering the eye fixations of several participants (Tobii, n.d.-a). To generate equally balanced images for comparison, we used problems that had the same number of shortcut users to shortcut non-users. As such, only 3 of the 32 problems were used for comparative analysis. Some problems have a high proportion of shortcut users, especially among adults, which makes it hard to compare shortcut users to shortcut non-users. Moreover, because individuals in our sample were not consistent with their shortcut use, only a few participants used shortcuts on every problem. As the literature notes, individuals who use shortcuts on inversion and associativity problems are dual concept, while most of our sample was inversion-only as they only used shortcuts for inversion problems (Dubé & Robinson, 2010; Robinson & Dubé, 2009). It would be interesting to analyze dual concept individuals' eye movements and compare them to inversion-only individuals for all 32 problems. However, given that there are not many dual concept individuals, we could also compare how these individuals' eye movements differ across the various problems.

Future Directions

Given that the role of visual attention in solving inversion and associativity problems remains unsupported, future studies should continue to use eye tracking to understand if this domain-general skill influences problem-solving. Aside from generating heat maps, there are several ways to capture and measure visual attention with eye tracking. For example, Curtis et al. (2016) incorporated Areas of Interest (AOIs), whereby each term and operator is designated an area. The AOIs could be used to calculate the number of fixations on a specific term or operator. For our study, which involves three-term arithmetic problems, five areas of interest would be created. Since this type of analysis can be performed post-hoc, we plan to continue our study and use AOIs to measure the fixations between shortcut users and shortcut non-users. The AOIs could also be used for other comparisons as well.

As indicated by our results, when comparing problems that differ by operator, those that are additive have higher accuracy than those that are multiplicative. Thus, we could compare participants' visual attention on these problems by generating AOIs for additive and multiplicative problems and comparing the number of fixations on the terms and operators. We might find that when solving additive problems, participants have less fixations on the addition and subtraction operator, but when solving multiplicative problems, participants have more fixations on the multiplication and division operator. This might reflect that participants get more distracted and lose confidence due to decreased familiarity with multiplication and division.

Conclusion

Our inversion and associativity study was the first to integrate an eye tracker to compare shortcut users' and shortcut non-users' visual attention. The results from our study reinforce that adults are more accurate and use more shortcuts on inversion problems than associativity problems. Moreover, our study also found that shortcut users and shortcut non-users attend to different parts of the arithmetic problem, whereby shortcut users spend less time attending to the shortcut. These differences in visual attention suggest that this domain-general skill might facilitate performance on associativity problems. As such, there should be increased efforts to develop attention as it could help children and adolescents learn associativity, which in the long run will ease the transition towards learning more difficult mathematical concepts.

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Appendix A

An eye tracking study: Video games, arithmetic problems, and drawing

Pp#. ______ Date: ______ Time: ______ Researcher: _______

Researchers:
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This consent form is only part of the process of informed consent. If you want more details about something mentioned here, or information not included here, you should feel free to ask. Please take time to read this carefully.

Purpose(s) and Objective(s) of the Research:
Eyetrackers can be used for many purposes including knowing where participants look when they are gazing at a screen when solving arithmetic problems, and how participants can use an eyetracker to play video games or draw with their eyes. The present study examines 1) whether playing a video game with your eyes gives you better eye control while drawing with your eyes, 2) whether your eye movements while solving arithmetic problems vary across individuals. You will be randomly assigned to one of two conditions. In the first condition participants will play a video game with their eyes and then will be asked to draw a picture with their eyes. In the second condition participants will solve arithmetic problems while their eye movements are being measured, and then will be asked to draw a picture with their eyes. How we solve arithmetic problems is a key research topic in mathematical cognition and our eye movements during arithmetic eye solving may differ depending on the type and format of the arithmetic problems presented.

Procedures:
This study will take place in the IMPACT Lab in Campion College. The total time required for your participation will be approximately 60 minutes.
Upon arriving, you will be asked to fill out an online demographic questionnaire via Qualtrics software.
You will be situated in front of a screen with an eye-tracker, and either presented with a video game to play with your eyes, or presented with several arithmetic problems, and asked to solve each problem one at a time and report how you solved the problem (this part will be audio recorded).
Finally, you will be asked to draw simple pictures with your eyes.
If you have any questions before you begin the study, feel free to ask the researchers.
Potential Risks and Benefits:
There are no known or anticipated risks or benefits to you by participating in this research. However, you may experience mild eye fatigue from looking at the computer screen so we will ask you if you need to take a break at any point in the study.

Compensation:
In exchange for your participation, you will receive 2 bonus marks towards your final grade in a psychology class that is registered in the Participant Pool.

Confidentiality:

Participant anonymity is limited as the researchers are also the experimenters. However, participant confidentiality will be protected – no link will be made between the data collected and the participant's identity. Only the researchers will have access to data. The Participant Pool Coordinator may also know identities of participants (signing up through the SONA).
Survey options in Qualtrics are set to anonymize data, and no information linking you to your survey data will be stored.
Security options in Qualtrics have been set to include data encryption to increase data security and confidentiality.
Although participants may be students of the Research Supervisor or Participant Pool Coordinator, or a classmate of the student researcher, all data will be identified only by participant number and grouped together so that you will not be identifiable in any way.

Storage of Data:
The data obtained in this study will be stored by participant number only and will be comprised of non-identifiable numerical behavioural data only and there will be no way to link you with specific data or any other data. All data will be securely stored by Dr. Katherine Robinson. Data sheets and audio recordings coded by participant number will be destroyed once data coding and reliability checks have been completed. The online Qualtrics data will be transferred and deleted from the server as soon as data collection is complete.

The non-identifiable numerical behavioural data will be made available upon request to other researchers who are interested in similar research questions. It is possible that your data will be re-used by researchers outside of Canada who are not bound by Canadian Tri-agency regulations concerning the ethical re-use of data. As the data is non-identifiable numerical behavioural data, only quantitative re-analysis of the data will be possible. Sharing the data has the potential benefit of furthering scientific understanding without the need to collect additional data.

The non-identifiable numerical behavioural data will be stored on the Open Science Framework and made available to researchers upon request. The OSF is a free open-source software project that facilitates open collaboration in science research.

Right to Withdraw:
Participation is voluntary, and you may withdraw from the study at any time during your session by letting the researcher know you want to withdraw—there will be no repercussions if you withdraw (e.g., the bonus mark will still be given). However, once you leave the lab it will no longer be possible to withdraw your data as there will be no way to link any of your data to you specifically.

Follow up:
If you wish to learn about the group results of this study once it is complete, you can find N. McCullough's full honours thesis in the OURspace repository at the UofR library (https://ourspace.uregina.ca). Please feel free to contact the researchers and have results sent via email, visit the IMPACT lab, view conference presentations, or read published results in an academic journal.

Questions or Concerns:
If you have any further questions or want clarification regarding this research and/or your participation, contact: Alexandra Apesland or Natalia McCullough (student researchers; mathcog.lab@uregina.ca) or Katherine Robinson (principal investigator; katherine.robinson@uregina.ca). This project has been approved on ethical grounds by the University of Regina Research Ethics Board on February 06, 2024. Any

questions regarding your rights as a participant may be addressed to the committee at (306-585-4775 or research.ethics@uregina.ca). Out of town participants may call collect.

Consent:

Since your name and contact information will not be kept alongside your non-identifiable numerical behavioural data, it will not be possible to ask for your consent to use your data in the future. If you do not wish for your data to be re-used by other researchers in the future, you should not proceed with the study.

By clicking on "I consent" below, this indicates that you have read and understand the description provided; you have had an opportunity to ask questions and your questions have been answered, and that you consent to participate in the research project.

A paper copy of this Consent Form will be given to you for your records.

Do you consent

○ Yes

○ No

An eye tracking study: Video games, arithmetic problems, and drawing

Pp#. ______ Date: ______ Time: ______ Researcher: _______

Q1. What is your current age?

____

Q2. What is your gender?

____

Q3. What year of study are you in?

○ First year

○ Second Year

○ Third Year

○ Fourth Year

○ Fifth Year and Beyond

Q4. What area of study are you in?

- ○ Psychology
- ○ Other

Q4.1. What is your area of study?

____

Q5. Have you used an eye-tracker before?

- ○ Yes
- ○ No

Q6. How often do you play video games?

- ○ Almost daily
- ○ A few times a week
- ○ A few times a month
- ○ Hardly ever / never

Q7. How much do you enjoy playing video games?

- ○ Like a great deal
- ○ Like somewhat
- ○ Neither like nor dislike
- ○ Dislike somewhat
- ○ Dislike a great deal

Q8. Do you think video games are educational?

- ○ Yes
- ○ No

Q9. How often do you draw?

- ○ Almost daily
- ○ A few times a week
- ○ A few times a month
- ○ Hardly ever / never

Q10. How much do you enjoy drawing?

- ○ Like a great deal
- ○ Like somewhat
- ○ Neither like nor dislike
- ○ Dislike somewhat
- ○ Dislike a great deal

Q11. How often do you do math?

- ○ Almost daily
- ○ A few times a week
- ○ A few times a month
- ○ Hardly ever / never

Q12. How much do you enjoy doing math?

- ○ Like a great deal
- ○ Like somewhat
- ○ Neither like nor dislike
- ○ Dislike somewhat
- ○ Dislike a great deal

Appendix B

Conducive	Non-Conducive
Inversion Additive	
27 + 46 – 46	43 + 28 – 43
26 + 38 – 38	36 + 25 – 36
23 + 39 – 39	47 + 24 – 47
22 + 43 - 43	35 + 29 - 35
Inversion Multiplicative	
6 × 21 ÷ 21	28 × 7 ÷ 28
4 × 27 ÷ 27	24 × 4 ÷ 24
3 × 24 ÷ 24	26 × 3 ÷ 26
9 × 25 ÷ 25	22 × 8 ÷ 22
Associativity Additive	
24 + 49 - 47	39 + 23 – 35
29 + 35 - 32	43 + 22 – 41
25 + 38 - 36	46 + 27 – 44
28 + 48 - 43	38 + 26 – 33
Associativity Multiplicative	
3 × 26 ÷ 13	24 × 8 ÷ 3
6 × 28 ÷ 7	25 × 9 ÷ 5
8 × 22 ÷ 11	21 × 6 ÷ 7
4 × 24 ÷ 6	27 × 7 ÷ 9